MET 330

TEST 1 - SPRING 2019

AL McCLENNEY

MARCH 11, 2019

PURPOSE

a) FOR THE SYSTEM SHOWN IN THE DRAWINGS, 60° WATER IS DELIVERED AT A RATE OF 75 gpm FROM A PRESSURIZED STORAGE TANK TO A TRAPEZOIDAL OPEN CHANNEL. IF THE PIPE CARRYING THE WATER IS $1\frac{1}{2}$ in SCHEDULE 40 STEEL PIPE RUNNING A DISTANCE OF 300 ft, DETERMINE THE AIR PRESSURE REQUIRED ABOVE THE TANK FOR THIS TO OCCUR. NEGLECT MINOR LOSSES DUE TO THE VALVE OR ELBOW, BUT CONSIDER ENERGY LOSSES DUE TO THE STRAIGHT PIPE.

b) CALCULATE THE WATER DEPTH (Y) IN THE OPEN CHANNEL WITH LATERAL WALLS ANGLED AT 60° AND A WIDTH AT THE TOP OF THE WATER (T) AS $T = 2,309Y$.

c) WHAT WOULD THE RELEVANT TOTAL HORIZONTAL AND VERTICAL FORCES BE IN THE WHOLE SYSTEM PIPE-ELBOW FOR THE SUPPORT DESIGN?

d) FOR THE BLIND FLANGE AT THE LEFT-HAND-SIDE OF THE TANK, COMPUTE THE ACTING FORCE AND ITS LOCATION ON THE FLANGE.

e) IF A SQUARE HICKORY LOG WERE TO BE CARRIED BY THE OPEN CHANNEL, WHAT IS THE LARGEST WOOD LOG THE CHANNEL COULD CARRY? KNOWING THE DENSITY OF HICKORY WOOD IS 830 kg/m^3, WOULD THE LOG BE STABLE?

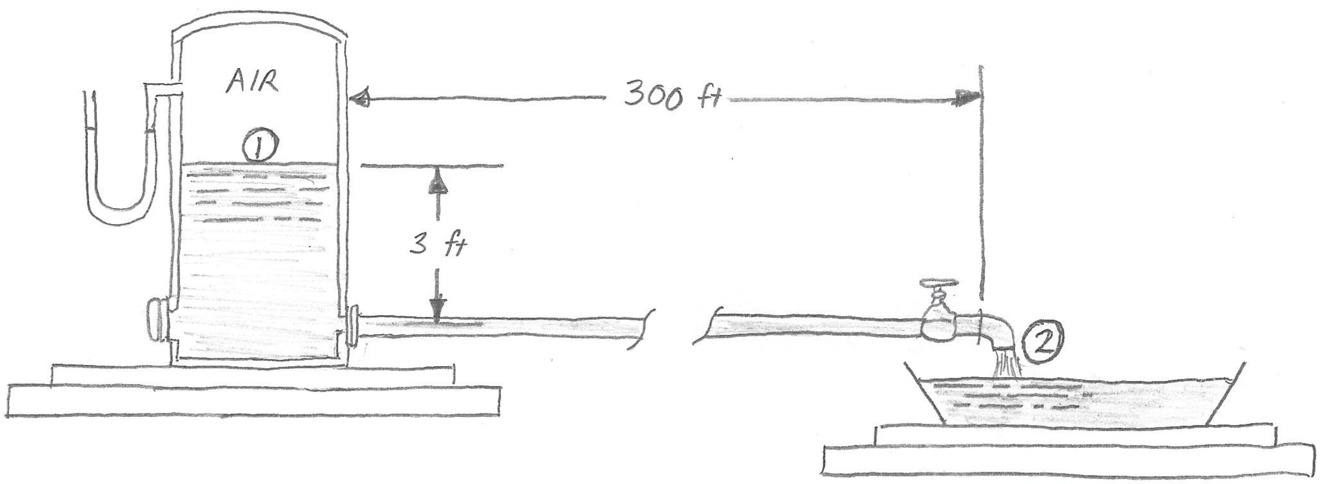
f) IF USING A MERCURY U-TUBE MANOMETER TO MONITOR AIR PRESSURE, WHAT SHOULD BE THE MINIMUM DIMENSION OF THE U-TUBE AS IT IS MEASURED FROM THE CONNECTION AT THE TANK TO THE LOWEST POINT OF THE U-TUBE.

g) IF A FLOW NOZZLE IS USED FOR MEASURING FLOW, AND HAS A RATIO OF 0.5 TO THE PIPE DIAMETER, WHAT WOULD BE THE PRESSURE DROP ACROSS THE NOZZLE?

h) WHAT WOULD BE THE PRESSURE INCREMENT IF THE VALVE IN THE PIPE WERE CLOSED SUDDENLY? IF THE MODULUS OF ELASTICITY OF STEEL IS 200 GPa, WOULD THERE BE ANY CHANGE OF CAVITATION IN THE SYSTEM?

i) IF A LOG HALF THE SIZE OF THE LARGET LOG WERE CARRIED BY THE CHANNEL, CALCULATE THE LARGEST DRAG FORCE IT WOULD EXPERIENCE IF IT WERE STUCK IN THE CHANNEL.

DRAWINGS & DIAGRAMS



SOURCES

MOTT, R. & UNTENER, J. (2015). APPLIED FLUID MECHANICS, 7TH EDITION. BOSTON: PEARSON.

DESIGN CONSIDERATIONS

- CONSTANT PROPERTIES
- INCOMPRESSIBLE FLUID
- STEADY STATE
- CONSTANT TEMPERATURE

DATA & VARIABLES

FOR $1\frac{1}{2}$ in SCHEDULE 40 STEEL PIPE:

$D_{PIPE} = 0.1342$ ft $= 40.9$ mm $= 0.0409$ m (FROM TABLE F-1, PAGE 500)

$A_{PIPE} = 0.01414$ ft^2 $= 1.314 \times 10^{-3}$ m^2

TEMPERATURE OF WATER $= T = 60°F = 15.6°C$

FLOW RATE OF WATER $= Q = 75$ gpm $= 0.00473$ m^3/s

HEIGHT OF WATER IN TANK (FROM OUTLET) $= 3$ ft $= 0.9144$ m

OPEN CHANNEL:

LENGTH OF CHANNEL $= L = 300$ ft $= 91.44$ m

ANGLE OF LATERAL WALLS $= 60°$

WIDTH AT THE TOP OF THE WATER (IN CHANNEL) $= 2.309y$

CHANNEL SLOPE $= 0.1$ PERCENT $= 0.001$

CHANNEL MATERIAL = UNFINISHED CONCRETE

MANNING'S NUMBER (UNFINISHE CONCRETE) $= n = 0.017$ (TABLE 14.1, PAGE 277)

DENSITY OF HICKORY WOOD $= 830$ kg/m^3

NOZZLE DIAMETER TO PIPE DIAMETER RATIO $= 0.5$

MODULUS OF ELASTICITY OF STEEL $= E_{STEEL} = 200$ GPa $= 200 \times 10^9$ Pa

SPECIFIC WEIGHT OF WATER $= \gamma_W = 9.81 \times 10^3$ N/m^3 (AT 15°C)

GRAVITY $= g = 9.81$ m/s^2

PIPE ROUGHNESS $= \varepsilon = 4.6 \times 10^{-5}$ m (TABLE 8.2, PAGE 185)

SPECIFIC GRAVITY OF HICKORY WOOD $= sg_{HICKORY} = 0.72$

PROCEDURE

a) TO DETERMINE THE AIR PRESSURE REQUIRED ABOVE THE WATER IN THE TANK, I WILL UTILIZE BERNOULLI'S EQUATION. THE VELOCITY HEAD AT THE SURFACE OF THE TANK IS CONSIDERED TO BE ZERO AND IT CAN BE CANCELLED FROM BERNOULLI'S EQUATION. ALSO, WHEN THE FLUID AT THE OUTLET STREAM IS EXPOSED TO THE ATMOSPHERE, THE PRESSURE IS ZERO AND THE PRESSURE HEAD TERM CAN ALSO BE CANCELLED FROM BERNOULLI'S EQUATION, UTILIZING THE MOODY CHART AND DARCY'S EQUATION, THE AIR PRESSURE REQUIRED ABOVE THE WATER IN THE TANK IS ULTIMATELY FOUND,

b) USING TABLE 14.3 (PAGE 383), BOTH A AND R MUST BE EXPRESSED IN TERMS OF DIMENSION Y; WHERE THE AREA (A) IS EQUAL TO $1.73y^2$, THE WETTING PERIMETER (WP) EQUALS $3.46y$, AND THE HYDRALIC RADIUS (R) EQUALS $y/2$.
WE WILL BE ABLE TO SUBSTITUTE USING THE NORMAL DISCHARGE EQUATION AND ITS VARIENT $\longleftrightarrow Q = \left(\frac{1.00}{n}\right) A R^{2/3} S^{1/2} \longrightarrow AR^{2/3} = \frac{nQ}{S^{1/2}}$

c) A FREE BODY DIAGRAM ON THE FLUID IN THE ELBOW WILL BE MADE. THE FORCES P_1A_1 AND P_2A_2 ARE THE FORCES DUE TO THE FLUID PRESSURE. THE TWO DIRECTIONS WILL BE ANALYZED SEPARATELY.

d) TO COMPUTE THE FORCE ACTING UPON THE BLIND FLANGE AT THE LEFT-HAND-SIDE OF THE TANK, WE WILL USE THE AIR PRESSURE ABOVE THE WATER AND THE DIAMETER OUR $1\frac{1}{2}$ in SCHEDULE 40 STEEL PIPE

PROCEDURES (CONTINUED)

e) THE DENSITY OF OUR HICKORY WOOD IS 830 kg/m^3 AND THE DENSITY OF WATER IS 1000 kg/m^3. SINCE THE HICKORY WOOD IS LESS DENSE THAN WATER, IT SHOULD CERTAINLY FLOAT IN OUR OPEN CHANNEL. BUOYANT FORCE IS EQUAL TO THE WEIGHT OF FLUID DISPLACED:

WEIGHT, w = (MASS, m)(GRAVITY, g)

AND SINCE DENSITY, $\rho = \dfrac{\text{MASS, } m}{\text{VOLUME, } V}$, THEN

MASS = (DENSITY)(VOLUME)

THUS, THE WEIGHT OF THE DISPLACED FLUID IS DIRECTLY PROPORTIONAL TO THE VOLUME OF THE DISPLACED FLUID.

WE DETERMINE THE VOLUME AVAILABLE IN OUR OPEN CHANNEL AND WE WILL BE ABLE TO DETERMINE THE LARGEST HICKORY LOG THAT THE OPEN CHANNEL COULD CARRY.

f) MERCURY IS USED IN MANOMETERS BECAUSE ITS HIGH DENSITY MEANS THE HEIGHT OF THE COLUMN CAN BE KEPT TO A REASONABLE SIZE. OUR U-TUBE MANOMETER WILL USE THE GAMMA-h EQUATION TO SET ELEVATION. MANOMETERS MEASURE PRESSURE BY MEASURING HEIGHT DIFFERENCES. THE PRESSURE IN OUR STORAGE TANK WAS DETERMINED TO BE 316 kPa. ATMOSPHERIC PRESSURE IS 101.3 kPa. SINCE THE PRESSURE IN THE TANK IS GREATER, THE MERCURY IS FORCED UPWARD ON THE LEFT SIDE OF THE MANOMETER.

g) USING OUR KNOWN DIAMETER OF PIPE AND THE PIPE RATIO, WE ARE ABLE TO DETERMINE OUR NOZZLE DIAMETER.

NEXT, WE WILL DETERMINE THE DISCHARGE COEFFICIENT, AND FROM THIS DETERMINE OUR PRESSURE DROP ACROSS THE NOZZLE.

PROCEDURES

h) STRUCTURAL INTEGRITY OF THE PIPE COULD BE COMPROMISED IF THE VALVE WERE CLOSED QUICKLY. THIS EVENT, CALLED "WATER HAMMER" IS CAUSED BY RAPID VALVE ACTUATION. USING HOOKE'S LAW AND THE MODULUS OF ELASTICITY, WE CAN DETERMINE IF ANY DAMAGE HAS BEEN DONE TO OUR $1\frac{1}{2}$ in SCHEDULE 40 STEEL PIPE.

i) IF A LOG GETS STUCK IN THE OPEN CHANNEL, WE WILL UTILIZE THE DRAG FORCE EQUATION, MAXIMUM CROSS-SECTIONAL AREA OF OUR LOG, AND THE DRAG COEFFICIENT TO DETERMINE THE LARGEST DRAG FORCE.

CALCULATIONS

a) $$\frac{P_1}{\gamma_w} + Z_1 + \cancel{\frac{\upsilon_1^2}{2g}} - h_L = \cancel{\frac{P_2}{\gamma_w}} + Z_2 + \frac{\upsilon_2^2}{2g}$$ (BERNOULLI'S EQUATION)

$\frac{\upsilon_1^2}{2g}$: 0 (LARGE TANK)

$\frac{P_2}{\gamma_w}$: 0 ATM (OUTLET STREAM)

$$P_1 = \gamma_w \left[(Z_2 - Z_1) + \frac{\upsilon_2^2}{2g} + h_L \right]$$

$$\upsilon_2 = \frac{Q}{A_2} = \frac{0.00473 \text{ m}^3/\text{s}}{1.314 \times 10^{-3} \text{ m}^2} = 3.6 \text{ m/s}$$

$$\frac{\upsilon_2^2}{2g} = \frac{(3.6 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \frac{12.96 \text{ m/s}}{19.62 \text{ m/s}^2} = 0.6606 \text{ m}$$

KINEMATIC VISCOSITY (TABLE A.1, PAGE 488) $= \nu = 1.15 \times 10^{-6} \text{ m}^2/\text{s}$

REYNOLDS NUMBER $= N_R = \frac{\upsilon D}{\nu} = \frac{(3.6 \text{ m/s})(0.0409 \text{ m})}{1.15 \times 10^{-6} \text{ m}^2/\text{s}} = 128035$

BECAUSE N_R IS MORE THAN 4000, THE FLOW IS <u>TURBULENT.</u>

$$\frac{D}{\varepsilon} = \frac{0.0409 \text{ m}}{4.6 \times 10^{-5} \text{ m}} = 889 = \text{RELATIVE ROUGHNESS}$$

FRICTION FACTOR $= f = 0.022$ (SEE INCLUDED MOODY CHART)

ENERGY LOSS DUE TO FRICTION USING DARCY'S EQUATION:

$$h_L = f \frac{L}{D} \frac{\upsilon^2}{2g} = 0.022 \left(\frac{91.44 \text{ m}}{0.0409 \text{ m}} \right) \left[\frac{(3.6 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} \right] = 32.5 \text{ m}$$

$$P_1 = 9.81 \times 10^3 \text{ N/m}^3 \left[(-0.9144 \text{ m}) + (0.6606 \text{ m}) + (32.5 \text{ m}) \right]$$

$P_1 = 316 \text{ kN/m}^2 = 316 \text{ kPa}$ = AIR PRESSURE REQUIRED ABOVE THE WATER IN THE TANK

a) GRAPH – MOODY'S DIAGRAM

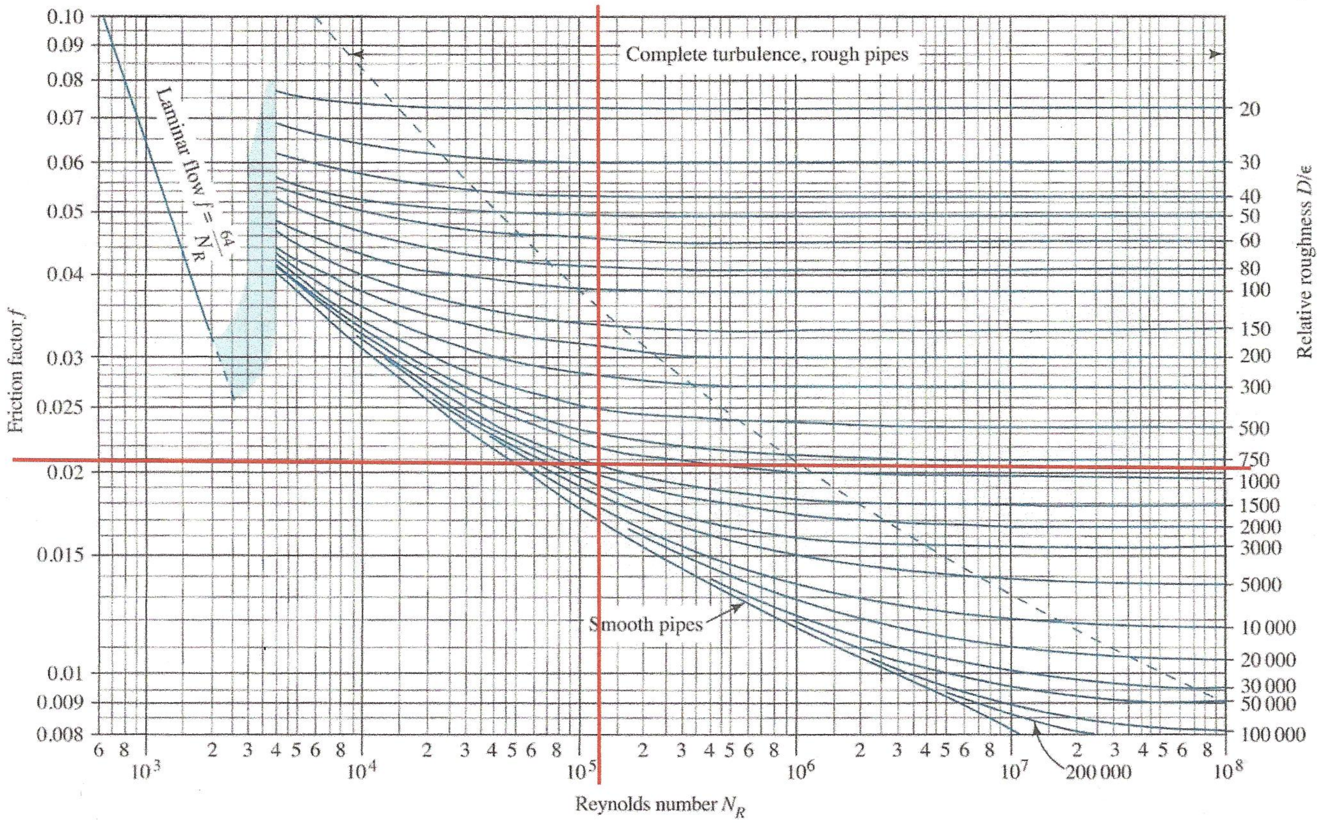


FIGURE 8.7 Moody's diagram. (*Source:* Pao, R.H.E. *Fluid Mechanics*, p. 284. Copyright 9c. 1961. Reprinted by permission of the author.)

CALCULATIONS (CONTINUED)

b) $AR^{2/3} = \frac{nQ}{1.00 S^{1/2}} = \frac{(0.017)(0.00473\ m^3/s)}{1.00\ (0.001)^{1/2}} = \frac{8.04 \times 10^{-5}\ m^3/s}{0.0316}$

$AR^{2/3} = 0.00254$ $\longrightarrow$ A AND R MUST BE EXPRESSED IN TERMS OF THE DIMENSION Y

ALGEBRAIC SOLUTION FOR Y IS NOT SIMPLY DONE.
A TRIAL-AND-ERROR APPROACH CAN BE USED.
THE RESULTS ARE AS FOLLOWS:

Table 1: Determination of the Depth of the Open Trapezoid Channel

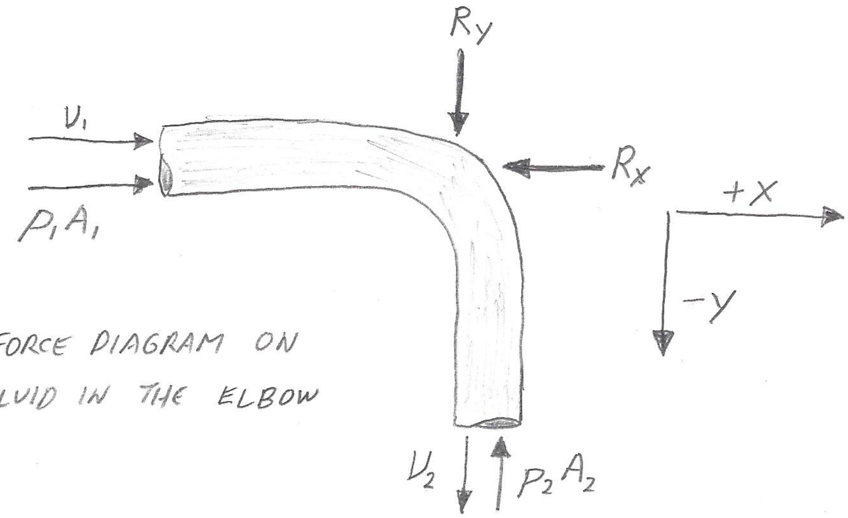
Goal = 0.00254

Trial and Error					
y(m)	***A(m²) =1.73y²***	***R(m) = y/2***	$R^{2/3}$	$AR^{2/3}$	**Required change in *y***
0.100	0.0173	0.0500	0.1357	0.00234	*y* is too low
0.101	0.0176	0.0505	0.1368	0.00241	*y* is too low
0.102	0.0179	0.0510	0.1378	0.00248	*y* is too low
0.103	0.0184	0.0515	0.1387	0.00254	Perfect!

THEREFORE, THE CHANNEL DEPTH WOULD BE 0.103 m
WHEN THE DISCHARGE IS 0.00473 m^3/s

CALCULATIONS (CONTINUED)

c) FREE BODY DIAGRAM:



FORCE DIAGRAM ON FLUID IN THE ELBOW

WE FIND THE NET EXTERNAL FORCE IN THE X DIRECTION BY THE EQUATION:

$$F_x = \rho Q(V_{2x} - V_{1x})$$

WE KNOW THAT:

$$F_x = R_x - P_1A_1$$

$$V_{2x} = 0$$

$$V_{1x} = -V_1$$

FROM OUR DATA:

$P_1 = 316 \text{ kPa} = 316 \times 10^3 \text{ N/m}^2$

$\rho = 1000 \text{ kg/m}^2$

$A_{PIPE} = 1.314 \times 10^{-3} \text{ m}^2$

$Q = 0.00473 \text{ m}^3\text{/s}$

THEN, WE HAVE

$$R_x - P_1A_1 = \rho Q[0-(-V_1)]$$

$$R_x = \rho Q V_1 + P_1A_1$$

$$V_1 = \frac{Q}{A_1} = \frac{0.00473 \text{ m}^3\text{/s}}{1.314 \times 10^{-3} \text{ m}^2} = 3.6 \text{ m/s}$$

$$\rho Q V_1 = \left(\frac{1000 \text{ kg}}{\text{m}^3}\right)\left(\frac{0.00473 \text{ m}^3}{\text{s}}\right)\left(\frac{3.6 \text{ m}}{\text{s}}\right) = 17.03 \text{ kg}\cdot\text{m/s}^2 = 17.03 \text{ N}$$

$$P_1A_1 = \left(\frac{316 \times 10^3 \text{ N}}{\text{m}^2}\right)(1.314 \times 10^{-3} \text{ m}^2) = 415.2 \text{ N}$$

$$R_x = (17.03 \text{ N} + 415.2 \text{ N}) = 432.2 \text{ N}$$ (RESULTANT FORCE IN X DIRECTION)

CONTINUED

CALCULATIONS (C, CONTINUED)

c) IN THE Y DIRECTION:

$$F_y = \rho Q(V_{2y} - V_{1y})$$

WE KNOW THAT:

$$F_y = -R_y - p_2 A_2$$

$$V_{2y} = -V_2$$

$$V_{1y} = 0$$

RESULTANT FORCE:

$$R_y + p_2 A_2 = \rho Q V_2$$

$$R_y = -\rho Q V_2 - p_2 A_2$$

$$\rho Q V_2 = \left(\frac{1000\ kg}{m^3}\right)\left(\frac{0.00473\ m^3}{s}\right)\left(\frac{3.6\ m}{s}\right) = 17.03\ kg \cdot m/s^2 = 17.03\ N$$

$$p_2 A_2 = \left(\frac{316 \times 10^3\ N}{m^2}\right)\left(1.314 \times 10^{-3}\ m^2\right) = 415.2\ N$$

IF ENERGY LOSSES IN THE ELBOW ARE NEGLECTED, $V_2 = V_1$ AND $P_2 = P_1$ BECAUSE THE SIZES OF THE INLET AND OUTLET ARE EQUAL.

$$R_y = -17.03\ N - 415.2\ N$$

$R_y = -432.2\ N$ (RESULTANT FORCE IN THE Y DIRECTION)

THE FORCES R_x AND R_y ARE THE REACTIONS CAUSED AT THE ELBOW AS THE FLUID TURNS 90°.

MY CIVIL ENGINEER COLLEAGUE WILL USE THESE RESULTS TO DESIGN SUPPORTS FOR THE SYSTEM.

CALCULATIONS (CONTINUED)

d) FORCE ON THE FLANGE $= F_{FLANGE} = P \times A$

P_{TANK} = PRESSURE AT THE TANK $= 316 \times 10^3\ N/m^2$

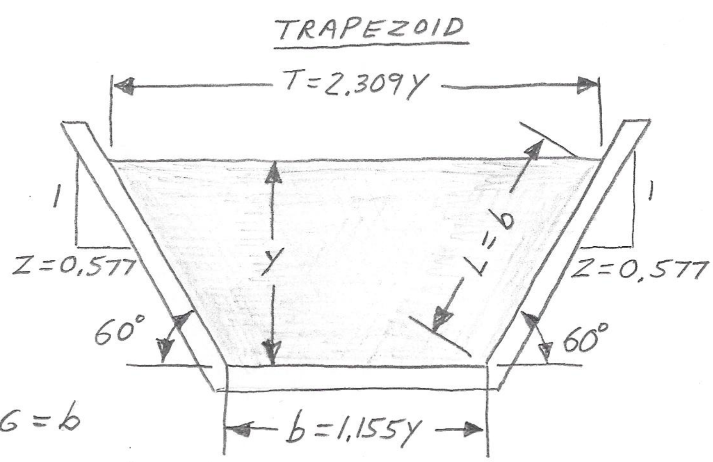
A_{FLANGE} = AREA OF FLANGE = AREA OF PIPE $= 1.314 \times 10^{-3}\ m^2$

$$F_{FLANGE} = P_{TANK} \times A_{FLANGE}$$

$$F_{FLANGE} = \left(\frac{316 \times 10^3\ N}{m^2}\right) \times (1.314 \times 10^{-3}\ m^2) = 415.2\ N$$

THE FLANGE FORCE IS STRONGEST ON THE BOLTS OF THE FLANGE, AND IS DIVIDED EVENLY AMONG THE TOTAL NUMBER OF BOLTS.

e) IT IS GIVEN THAT IN REGARDS TO THE LOG SIZE, THE CROSS SECTION IS A SQUARE, THE SIDE OF THE HICKORY LOG IS LIMITED TO THE WIDTH OF THE BASE OF THE CHANNEL, THE CHANNEL BASE, $b = 1.155y$



WE WILL LET THE SIDE OF THE LOG $= b$

THE VOLUME OF THE LOG CANNOT EXCEED $b \times b \times b = b^3$

ACCORDING TO TABLE 14.3 (PAGE 383) $b = 1.155y$

WE DETERMINED THAT $Y = 0.103\ m$ (IN PART b)

THUS, VOLUME OF THE FLOATING HICKORY LOG (MAX.):

$$V = b^3$$

$$V = (1.155\,y)^3$$

$$V = [(1.155)(0.103\ m)]^3 = 0.118965^3\ m$$

$$V = 0.00168\ m^3$$

$$m = (\text{DENSITY})(\text{VOLUME}) = (830\ kg/m^3)(0.00168\ m^3) = 1.397\ kg$$

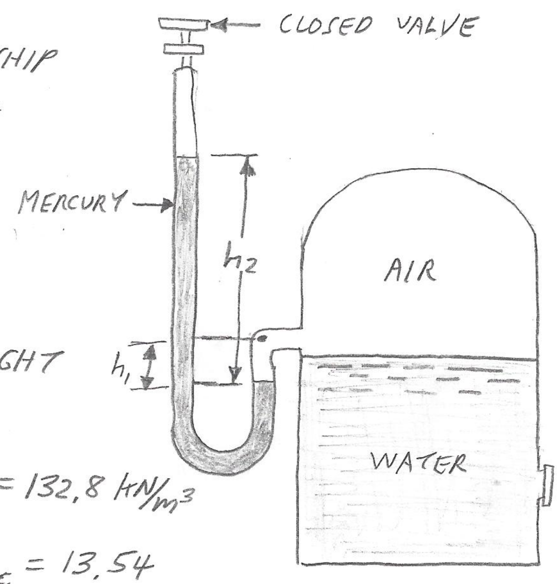
MAXIMUM LARGEST HICKORY LOG $= m = 1.397\ kg$

CALCULATIONS (e, CONTINUED)

e) THE CENTER OF GRAVITY OF THE SQUARE CROSS SECTION OF THE LOG IS ITS GEOMETRIC CENTER.
A FLOATING BODY IS STABLE IF ITS CENTER OF GRAVITY IS BELOW THE METACENTER.

$MB = I/V_d$ $\quad$ (I = INERTIA)

THE LOG IS STABLE.



f) FOR THE MANOMETER, THE RELATIONSHIP BETWEEN A CHANGE IN ELEVATION IN A LIQUID (h) AND A CHANGE IN PRESSURE (Δp) STATED AS:

$$\Delta p = \gamma h$$

WHERE GAMMA (γ) IS THE SPECIFIC WEIGHT OF THE LIQUID.

FOR MERCURY $\longrightarrow$ SPECIFIC WEIGHT $= \gamma_{MERC} = 132.8\ kN/m^3$

$sg_{MERC} = 13.54$

$$P_A = P_{ATM} + \gamma_2 h_2 - \gamma_1 h_1$$

P_A = PRESSURE IN THE TANK $= 316\ kPa$

$P_{ATM} = 101.3\ kPa$

$$316\ kPa = 101.3\ kPa + \frac{132.8\ kN}{m^3}(h_2 - h_1)\,m$$

$$1.62\ m = (h_2 - h_1) = \Delta h$$

g) N_D = NOZZLE DIAMETER
P_D = PIPE DIAMETER $= 0.0409\ m$
RATIO $= 0.5 = \beta$

$$\frac{N_D}{P_D} = \beta \longrightarrow 0.5 = \frac{N_D}{0.0409\ m}$$

$$N_D = 0.02045\ m$$

$$N_{AREA} = 3.28 \times 10^{-4}\ m^2$$

CALCULATIONS (g, CONTINUED)

g) DETERMINING THE DISCHARGE COEFFICIENT FOR THE FLOW NOZZLE:

$$C = 0.9975 - 6.53\sqrt{\beta / N_R}$$

$$C = 0.9975 - 6.53\sqrt{0.5/128035}$$

$$C = 0.985$$

$$V_1 = \frac{Q}{A} = \frac{0.00473\ m^3/s}{3.28 \times 10^{-4}\ m^2}$$

$$N_{AREA} = 3.28 \times 10^{-4}\ m^2$$

SOLVING FOR ΔP ACROSS THE NOZZLE:

$$\Delta p = [V_1/C]^2 [(A_1/A_2)^2 - 1][\gamma/2g]$$

$$\Delta p = [14.42\ m/s / 0.985]^2 [(3.28 \times 10^{-4}\ m^2 / 1.314 \times 10^{-3}\ m^2)^2 - 1]\left[\frac{9.81 \times 10^3\ N/m^3}{9.81\ m/s^2}\right]$$

h) HOOKE'S LAW:

$$F = k\Delta L$$

ΔL = CHANGE CAUSED BY DEFORMATION

Y = YOUNG'S MODULE OF ELASTICITY $\quad \Delta L = \frac{1}{Y}\frac{F}{A}L_0$

Y = 200 GPa

THERE WOULD MOST LIKELY BE A CHANGE OF CAVITATION IN OUR SYSTEM.

i) DRAG FORCE ON THE LOG:

$$F_D = \text{DRAG} = C_D(\rho v^2/2)A$$

C_D = DRAG COEFFICIENT = 1.60 (FROM TABLE 17.1, PAGE 440)

ρ = DENSITY OF WATER

THE DRAG COEFFICIENT DEPENDS ON THE REYNOLD'S NUMBER

$$F_D = C_D(\rho v^2/2)A$$

$$N_R = \frac{vL}{\nu}$$

SUMMARY:

- FRICTION IS NEGLIGIBLE FOR THE SYSTEM.
- CHANNEL SIZE MAY BE BETTER IF INCREASED IN SIZE FOR IMPROVED FLOW.
- HICKORY WOOD MAY REQUIRE INCREASED PRESSURE FROM THE TANK DUE TO ITS HIGHER DENSITY COMPARED TO OTHER WOOD TYPES.

MATERIALS:

- STORAGE TANK
- $1\frac{1}{2}$ in SCHEDULE 40 STEEL PIPE (300 ft)
- MANOMETER (MERCURY)
- TRAPEZOID OPEN CHANNEL (UNFINISHED CONCRETE)
- GATE VALVE
- WATER
- BLIND FLANGE
- PIPE SUPPORTS

ANALYSIS:

- USE OF THE TRAPEZOIDAL OPEN CHANNEL IS MOST EFFICIENT.
- THE U-TUBE MANOMETER MAY BE BETTER SERVED AS A BOURDEN GAUGE DUE TO AN OUTSIDE LOCATION.