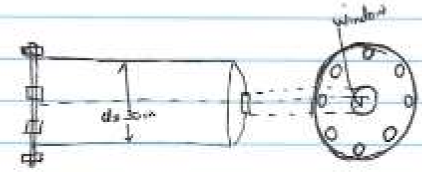
# Homework 2.1

This week was a short week because one of our class days was canceled due to a holiday, however we still learned a lot from the lecture we did have and the examples. We learned how fluids act as a force on flat surfaces, angled surfaces, and curved surfaces. The examples showed multiple ways and orientations of how fluids apply force to a surface or wall. We also learned how to find the center point of the moment of force acting on those surfaces. Another topic that was discussed this week was buoyancy. Part of this was learning the equations related to buoyancy and how to manipulate the equations to find information on the properties of the floating object and the fluid.

Homework Problems

4.2

Problem 4.2




Calculate the total force that must be resisted by the bolts in the flange.

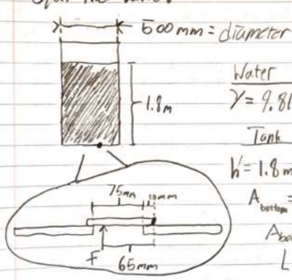
$$F = P_{gage} \times A$$

$$= 14.4\ lb/in^2 \times \frac{\pi}{4}(30\ in)^2$$

$$\underline{F = 10{,}178.76\ lb}$$

4.10

4-10) A simple shower for remote locations is designed with a cylindrical tank 500 mm in diameter and 1.8 m high. The water flows through a flapper valve in the bottom through a 75 mm opening. The flapper must be pushed upward to open the valve. How much force is required to open the valve?




Water

$\gamma = 9.81\ kN/m^3$

Tank

$h = 1.8\ m$

$A_{bottom} = \pi r^2 \Rightarrow \pi \cdot 250^2 \Rightarrow$

$A_{bottom} = 196{,}349.54\ mm^2$

$\llcorner = 0.196\ m^2$

$A_{flap} = \pi \cdot (37.5^2) = 4417.86\ mm^2$

$\llcorner = 0.00441786\ m^2$

Pressure at bottom of tank

$$\{P = \gamma_{water} \cdot h\} \Rightarrow P_{bot} = 9.81\ \frac{kN}{m^3} \cdot 1.8\ m$$

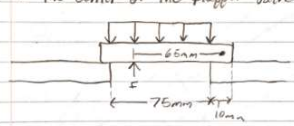
$$\Rightarrow P_{bot} = 17.7\ kPa$$

Force acting on unsupported portion of valve

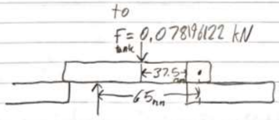
$$\{F = P \cdot A\} \Rightarrow F = P_{bot} \cdot A_{flap} = 17.7\ \frac{kN}{m^2} \cdot 0.00441786\ m^2$$

$$\Rightarrow F_{tank} = 0.078196122\ kN$$

• Replace distributed load with singular force acting at the center of the flapper valve




to




Solve for moments about the pin of the valve

$$\Sigma M = 0 = (F_{tank} \cdot 37.5\ mm) - (F \cdot 65\ mm)$$

$$\Rightarrow F = (0.078196122\ kN \cdot 37.5\ mm) / 65\ mm$$

$$F = 0.045113473\ kN = 45.1\ Newtons$$

4.17

## Problem 4.17


If the wall is 4m long:
- Calculate the total force due to the oil pressure
- Location of the center of pressure
- Show the resultant force on the wall

slanting lengths $L = h/\sin 45°$

$L = 1.4m/\sin 45°$

$= 1.98m$

$F_R = \gamma(h/2)A = (0.86 \times 1000 kg/m^3 \times 9.81 N/kg)\left(\frac{1.4m}{2}\right)(1.98m \times 4m)$

$\underline{\underline{F_R = 46{,}772.51\ N}}$

Location of centre of pressure

$h_p = h/3 = 1.4m/3$ $\quad \underline{\underline{h_p = 0.47m}}$ from the bottom (vertical)

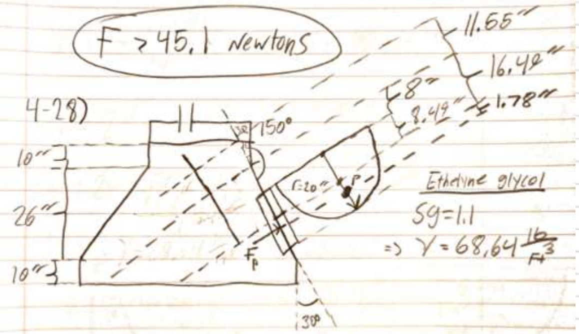
Measure along the wall, $L_p = L/3 = \frac{1.98m}{3} = 0.66M$

$\underline{\underline{L_p = 0.66m}}$ (from the bottom (along the wall))

4.28

• Since the equation is balanced when force acting to open the flapper is 45.1 Newtons, The force must be greater than that to actually open the flapper valve

$F > 45.1$ Newtons




Ethylene glycol

$sg = 1.1$

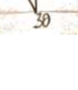
$\Rightarrow \gamma = 68.64\ \frac{lb}{ft^3}$

Centroid of a half Circle

$\left\{\bar{Y} = \frac{4 \cdot r}{3 \cdot \pi}\right\} \Rightarrow \bar{Y} = \frac{(4 \cdot 20)}{(3 \cdot \pi)} = 8.49''$

Distance to Centroid

$L_c = 16.49'' + \left(10'' \cdot \frac{\sin(90)}{\sin(60)}\right) = 28.04''$

$11.55'' =$ 

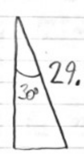

Distance to Center of Pressure

$\left\{L_p = L_c + \frac{I_c}{L_c \cdot A}\right\} \quad \left\{I_c = \frac{\pi \cdot r^4}{4}\right\}$

$\Rightarrow I_c = \frac{\pi \cdot (10^4)}{4} = 7853.98\ in^4$

$\Rightarrow L_p = 28.04'' + \left(\frac{7853.98\ in^4}{28.04'' \cdot \left(\frac{\pi \cdot 10^2}{2}\right)}\right)$

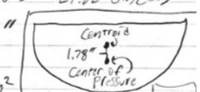
$L_p = 28.04'' + 1.78'' = 29.82''$




$h_p = L_p \sin(\theta) = 29.82'' \sin(30)$

$\Rightarrow h_p = 14.91''$

$A_{\frac{1}{2}circle} = 157.08\ in^2$




• We know P is $\boxed{1.78''}$ below the Centroid

Solving for force acting at the Center of Pressure

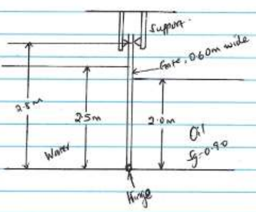
$\{P_p = h_p \cdot \gamma\}$ & $\{F = P \cdot A_{\frac{1}{2}circle}\}$

$P_p = 14.91'' \cdot 0.036\ lb/in^3 = 0.53676\ lb/in^2$ (PSI)

$F = 0.53676\ \frac{lb}{in^2} \cdot 157.08\ in^2 = \boxed{84.31\ lbs}$

4.41

Problem 4.41




Compute the net force on the gate due to the fluid on each side. Compute the force on the hinge and on the support.

Force due to the water $F_w = \gamma_w (h/2) A$

$= 9810 N/m^3 \times \frac{2.5m}{2} \times (2.5m \times 0.6m)$

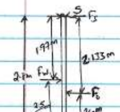
$\underline{F_w = 18,393\!.75 N}$

Force due to the oil

$F_o = \gamma_o (h/2) A$

$= (9810 N/m^3 \times 0.9)(\frac{2}{2}m)(2.0m \times 0.6m)$

$\underline{F_o = 10594.8 N}$




Moment about H $F_w(\frac{2.5}{3}m) - F_S(2.8m) - F_o(\frac{2}{3}m) = 0$

$F_S = \frac{18393.75N(\frac{2.5}{3}m) - 10594.8N \times \frac{2}{3}m)}{2.8m}$

$\underline{F_S = 2951.76N}$

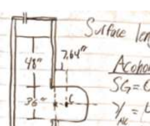
Moment about the support: $F_o(2.133m) + F_H(2.8m) - F_w(1.967m) = 0$

$F_H = \frac{18393.75N(1.967m) - 10594.8N(2.133m)}{2.8m}$

$\underline{F_H = 4850.64N}$

4.54

4-54)

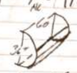



Surface length = 60" = w

Acohol

SG = 0.79

$\gamma_{alc} = 49.3\ lb/ft^3 = 0.02853 \frac{lb}{in^3}$




A = 3392.92²

Find $F_V, F_H, F_R$

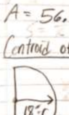
Length of Surface (S) and Area (A) of Surface

$\{A_{\frac{1}{2}circ} = S \cdot w\}$ & $\{S = \pi r\}$

$S = \pi \cdot 18" = 56.54"$

$A = 56.54" \cdot 60" = 3392.92\ in^2$

Centroid of Semi-circle and depth of Centroid




$\bar{X} = \frac{4r}{3\pi} = 7.64"$

$\Rightarrow h_c = 48" + 18" = 66"$

depth of Center of pressure

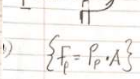
$\{h_p = (I_c/(h_c \cdot A)) + h_c\}$ & $\{I_c = \frac{w \cdot S^3}{12}\}$

$I_c = \frac{60 \cdot 56.54^3}{12} = 903727.33\ in^4$

depth of Center of Pressure & force acting at it

$\left(\frac{903727.33\ in^4}{66" \cdot 3392.92\ in^2}\right) = 4.04"$

$\Rightarrow 4.04" + 66" = 70.04" = h_p$




$\{P_p = h_p \cdot \gamma_{alc}\}$ $P_p = 70.04" \cdot 0.02853 \frac{lb}{in^3}$

$P_p = 1.9981 \frac{lb}{in^2}$

$\{F_p = P_p \cdot A\} \Rightarrow F_p = 1.9981 \frac{lb}{in^2} \cdot 3392.92\ in^2$

$\Rightarrow F_p = 6779.46\ lbs$ ← Horizontal Component

$\Rightarrow F_H = 6779.46\ lbs$

Calculating Vertical Component

$\{F_v = \gamma \cdot V\}$ & $\{V = \left(\frac{\pi \cdot r^2}{2}\right) \cdot w\}$

$V = \left(\left(\frac{(\pi \cdot 18^2)}{2}\right) \cdot 60"\right) = 30536.28\ in^3$

$\Rightarrow F_v = 0.02853 \frac{lb}{in^3} \cdot 30536.28\ in^3 = 871.2\ lbs$

Calculating resultant vector


$F_H = 6779.46\ lbs$

$F_V = 871.2\ lbs$

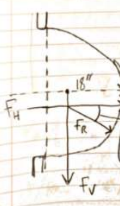
$\{F_R = \sqrt{F_H^2 + F_V^2}\}$

$\Rightarrow F_R = \sqrt{(6779.46\ lbs)^2 + (871.2\ lbs)^2}$

$F_R = 6835.21\ lbs$

$\{\theta = \tan^{-1}(F_V/F_H)\}$

$\Rightarrow \theta = \tan^{-1}(871.2/6779.46) = \boxed{7.32°}$

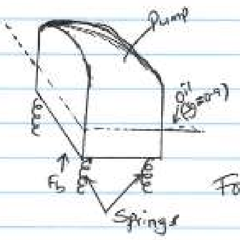



7.32°

$F_R = 6835.21\ lbs$

5.8

Problem 5.8




Pump partially submerged in oil and supported by springs. If the total weight of the pump is 14.6 lb and the submerged volume is 40 in$^3$ Calculate the supporting force exerted by the springs.

Force (upwards), exerted by the displaced fluid

$$F_b = \gamma_f V_f$$

$$= \left(0.9 \times 62.4 \, lb/ft^3\right)\left(40 in^3\right)\left(\frac{1 ft^3}{1728 in^3}\right)$$

$$= 1.3 \, lb$$

Supporting force = Weight of the pump - Buoyancy force

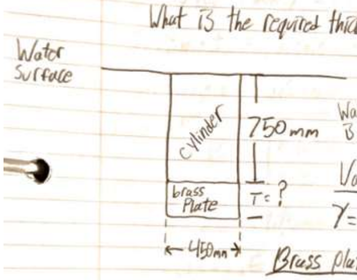
$$= 14.6 \, lb - 1.3 \, lb$$

$$\underline{= 13.3 \, lb}$$

5.24

5-24) A brass weight is to be attached of the cylinder as seen below. The cylinder will be completely submerged and neutrally bouyant in water at 95C. The brass is to be a cylinder with the same diameter as the original cylinder shown below.

What is the required thickness of the brass?




Water @ 95°C

$\gamma = 9.44 \, kN/m^3 = 9.44 \cdot 10^{-6} \, N/mm^3$

Brass plate

$\gamma_{brass} = 84.0 \, kN/m^3 = 8.4 \cdot 10^{-5} \, N/mm^3$

$A = \pi \cdot r^2 = \pi \cdot (225^2) = 159043.13 \, mm^2$

$V = A \cdot T \Rightarrow V = (159043.13 \cdot T) \, mm^3$

$W_{brass} = \gamma_{brass} \cdot V_{brass} = (8.4 \cdot 10^{-5}) \cdot (159043.13 \cdot T)$

$\{W_{brass} = F_b\} \, \& \, \{F_b = \gamma_w \cdot V_{displaced}\}$

$\Rightarrow (\gamma_{brass} \cdot V_{brass}) = (\gamma_{water} \cdot V_{displaced})$

$8.4 \cdot 10^{-5} \frac{N}{mm^3} \cdot (159043.13 \, mm^2 \cdot t)$

$= 9.44 \cdot 10^{-6} \frac{N}{mm^3} \cdot (159043.13 \, mm^2 \cdot (750 + t))$

$\Rightarrow \left(13.36 \frac{N}{mm} \cdot t \, mm\right) = \left(1.501 \frac{N}{mm} \cdot (750 + t)\right)$

$13.36 \cdot t = 1126.03 + (1.501 \cdot t)$

$\Rightarrow 11.859 \cdot t = 1126.03$

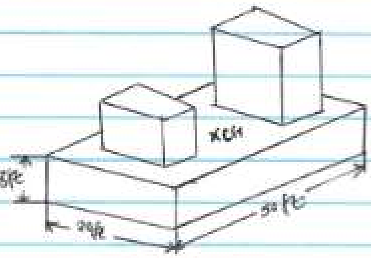
$\Rightarrow t = \frac{1126.03}{11.859} = \boxed{94.95 \, mm}$

$13.36 \frac{N}{mm} \cdot 94.95 mm = 1.501 \frac{N}{mm} \cdot (844.95 mm)$

$1268.6 = 1268.3$

5.41

Problem 5.41




Total weight = 450 000 lb

CG is on top of platform 8 ft from the bottom

Will the platform be stable in seawater?

$sg = 1.030$

$F_b = \gamma_w V_f = W$

$$(62.4\ lb/ft^3 \times 1.03) \times (20ft \times 50ft \times h) = 450000\ lb$$

$$h = 7.00\ ft.$$


L = 50 ft

B = 20 ft

X = 7 ft.

$MB = I/V_d$

$V_d = (50 \times 20 \times 7) ft^3 = 7000 ft^3$

$I = \frac{LB^3}{12} = \frac{50ft \times (20ft)^3}{12} = \frac{100000}{3} ft^4$

$MB = \frac{100000}{3} ft^4 \div 7000 ft^3$

$= 4.76 ft.$

$y_{mc} = y_{cb} + MB$

$= 3.5ft + 4.76 ft$

$y_{mc} = 8.26 ft.$

$y_{mc} = 8.26 ft$

$y_{cg} = 8 ft$

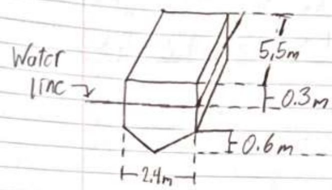
In this case $y_{mc} > y_{cg}$

The platform is stable

5.61

5-61) A boat is shown below. It's geometry at the water line is the same as the top surface. The hull is solid

Is the boat stable?




• A floating body is stable if metacenter is above the center of gravity, and can be quantified using MB

$\{MB = I/V_d\}$

Volume displaced

0.9m, 2.4m $\Rightarrow A_{rectangle} = 2.16 m^2$

0.6m, 2.4 $= A_{triangle} = \frac{0.6 \cdot 2.4}{2} = 0.72 m^2$

$\Rightarrow$ Cross Sectional Area Submerged $= A_\square + A_\Delta = 2.16 + 0.72 = 2.88 m^2$

$\Rightarrow$ Volume displaced $= 2.88 m^2 \cdot 5.5 m = 15.84 m^3$

buoyancy force acting on the boat

$\{F_b = \gamma_{water} \cdot V_d\} \Rightarrow F_b = 9.44 \frac{kN}{m^3} \cdot 15.84 m^3$

$\Rightarrow F_b = 149.53 kN \Rightarrow W = 149.53 kN$

Calculating for Center of buoyancy

0.9m, 2.4m $\Rightarrow \bar{x} = 1.2m \quad \bar{y} = 0.45m$

Area $= 2.16 m^2$

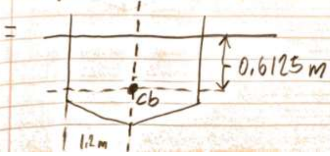
0.6m, 2.4m $\Rightarrow \bar{x} = 1.2m \quad \bar{y} = \frac{h}{3} = \frac{0.6}{3} = 0.2m + 0.9$

Area $= 0.72 m^2$

| | $\bar{x}$ | $\bar{y}$ | $A$ | $\bar{x}A$ | $\bar{y}A$ |
|---|---|---|---|---|---|
| rectangle | 1.2m | 0.45m | 2.16m² | 2.592m³ | 0.972m³ |
| triangle | 1.2m | 1.1m | 0.72m² | 0.864m³ | 0.792m³ |
| Composite | | | 2.88m² | 3.456m³ | 1.764m³ |

$\bar{X}_{Composite} = \frac{3.456 m^3}{2.88 m^2} = 1.2 m$

$\bar{Y}_{Composite} = \frac{1.764 m^3}{2.88 m^2} = 0.6125 m$




Calculating Center of gravity

1.2m, 2.4m $\Rightarrow \bar{x} = 1.2m \quad \bar{y} = 0.6m$

$A = 2.88 m^2$

0.6m, 2.4m $\bar{x} = 1.2m \quad \bar{y} = 0.2m + 1.2 = 1.4m$

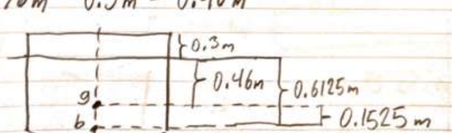
$A = 0.72 m^2$

| | $\bar{x}$ | $\bar{y}$ | $A$ | $\bar{x}A$ | $\bar{y}A$ |
|---|---|---|---|---|---|
| rectangle | 1.2m | 0.6m | 2.88m² | 3.456m³ | 1.728m³ |
| triangle | 1.2m | 1.4m | 0.72m² | 0.864m³ | 1.008m³ |
| Composite | | | 3.6m² | 4.32m³ | 2.736m³ |

$\Rightarrow \bar{X} = \frac{4.32 m^3}{3.6 m^2} = 1.2m \quad \bar{Y} = \frac{2.736 m^3}{3.6 m^2} = 0.76m$

depth from water level to center of gravity

$0.76m - 0.3m = 0.46m$




Calculating Composite moment of inertia

$\{I = I_{rec} + I_{tri}\} \quad \{I_{rec} = \frac{L \cdot W^3}{12}\} \quad \{I_{tri} = \frac{L \cdot (\frac{W}{2})^3}{12}\}$

$L = 5.5m \quad W = 2.4m$

$I_{rec} = \frac{(5.5) \cdot 2.4m^3}{12} = 6.336 m^4$

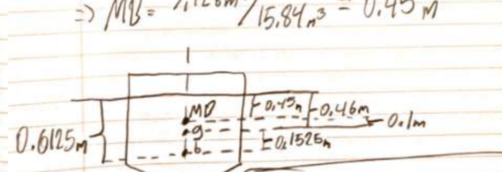
$I_{tri} = \frac{5.5 \cdot (1.2)^3}{12} = 0.792 m^4$

$\Rightarrow I = 7.128 m^4$

Metacenter depth

$\{MB = I/V_d\} \quad I = 7.128 m^4 \quad V_d = 15.84 m^3$

$\Rightarrow MB = 7.128 m^4 / 15.84 m^3 = 0.45 m$




• The metacenter is 0.1m above the center of gravity therefore the boat is stable