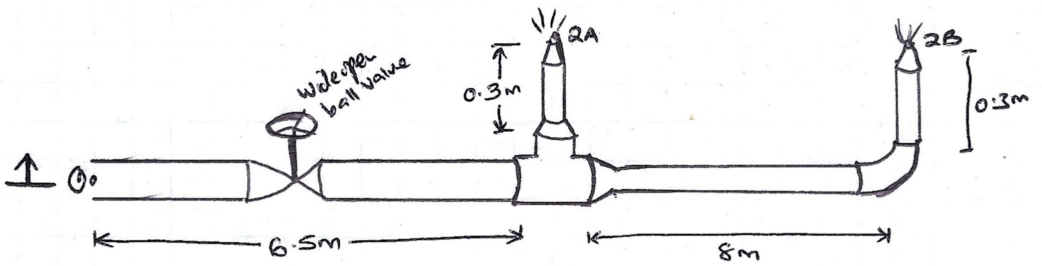
NAFTALY GICHUNGA | MET 330 | TEST 3 | I

Problem 1



$P_1 = 400\ KPa$

K of sprinkler = 50

K of ball valve = $3 f_T$

Water at 20°C (room temperature)

$\gamma = 9790\ N/m^3$

$\nu = 1.02 \times 10^{-6}$

Schedule 40 pipes

$D_1 = 0.0409\ m$ $\quad f_T = 0.020$

$D_{2A} = D_{2B} = 0.0266 m$ $\quad f_T = 0.022$

$\varepsilon = 4.6 \times 10^{-5}$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \cancel{\frac{P_{2A}}{\gamma}} + \frac{V_2^2}{2g} + Z_2 + h_{L_{1-2}}$$

$$\frac{P_1}{\gamma} - Z_2 = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} + h_{L_{1-2}}$$

For each branch

Branch 2A $\quad \frac{P_1}{\gamma} - Z_{2A} = \frac{V_{2A}^2}{2g} - \frac{V_1^2}{2g} + h_{L_{1-2A}}$

Branch 2B $\quad \frac{P_1}{\gamma} - Z_{2B} = \frac{V_{2B}^2}{2g} - \frac{V_1^2}{2g} + h_{L_{1-2B}}$

$$h_{L_{1-2A}} = h_{L pipe 1} + h_{L valve 1} + h_{Tee \downarrow 1} + h_{L contraction 2A} + h_{L pipe 2A} + h_{L sprinkler 2A}$$

$$= f_1 \frac{L_1}{D_1} \cdot \frac{V_1^2}{2g} + K_{valve} \cdot \frac{V_1^2}{2g} + K_{Tee \downarrow} \cdot \frac{V_1^2}{2g} + K_{cont} \cdot \frac{V_{2A}^2}{2g} + f_{2A} \frac{L_{2A}}{D_{2A}} \frac{V_{2A}^2}{2g} + K_{sprinkler} \cdot \frac{V_{2A}^2}{2g}$$

$$h_{L_{1-2B}} = h_{L pipe 1} + h_{L valve 1} + h_{Tee \rightarrow 1} + h_{contraction 2B} + h_{L pipe 2B} + h_{L elbow 2B} + h_{L sprinkler 2B}$$

$$= f_1 \frac{L_1}{D_1} \cdot \frac{V_1^2}{2g} + K_{valve} \cdot \frac{V_1^2}{2g} + K_{Tee \rightarrow} \cdot \frac{V_1^2}{2g} + K_{cont} \cdot \frac{V_{2B}^2}{2g} + f_{2B} \frac{L_{2B}}{D_{2B}} \cdot \frac{V_{2B}^2}{2g} + K_{elb} \cdot \frac{V_{2B}^2}{2g} + K_{sprinkler} \cdot \frac{V_{2B}^2}{2g}$$

$$h_{L_{1-2A}} = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee \downarrow}\right) \frac{V_1^2}{2g} + \left(K_{contraction} + f_{2A} \frac{L_{2A}}{D_{2A}} + K_{sprinkler}\right) \frac{V_{2A}^2}{2g}$$

$$h_{L_{1-2B}} = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee \rightarrow}\right) \frac{V_1^2}{2g} + \left(K_{cont} + f_{2B} \frac{L_{2B}}{D_{2B}} + K_{elb} + K_{sprinkler}\right) \frac{V_{2A}^2}{2g}$$

$$V = \frac{4Q}{\pi D^2} \Rightarrow V^2 = \frac{16Q^2}{\pi^2 D^4}$$

The equations become

$$h_{L1-2A} = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee}\right) \cdot \frac{8}{g\pi^2 D_1^4} Q_1^2 + \left(K_{cont} + f_{2A}\frac{L_{2A}}{D_{2A}} + K_{sprinkler}\right) \cdot \frac{8}{g\pi^2 D_{2A}^4} Q_{2A}^2$$

$$h_{L1-2B} = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee}\right) \frac{8}{g\pi^2 D_1^4} Q_1^2 + \left(K_{cont} + f_{2B}\frac{L_{2B}}{D_{2B}} + K_{elbow} + K_{sprinkler}\right) \cdot \frac{8}{g\pi^2 D_{2B}^4} Q_{2B}^2$$

The original equation was

$$\left(\frac{P_1}{\gamma} - Z_{2A}\right) = \frac{V_{2A}^2}{2g} - \frac{V_1^2}{2g} + h_{L1-2A} = \frac{8}{g\pi^2 D_{2A}^4} Q_{2A}^2 - \frac{8}{g\pi^2 D_1^4} Q_1^2 + h_{L1-2A}.$$

Which can be simplified as

$$\left(\frac{P_1}{\gamma} - Z_{2A}\right) = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee} - 1\right) \frac{8Q_1^2}{g\pi^2 D_1^4} + \left(K_{cont} + f_{2A}\frac{L_{2A}}{D_{2A}} + K_{sprin} + 1\right) \frac{8}{g\pi^2 D_{2A}^4} Q_{2A}^2$$

$$Q_{2A} = \sqrt{\frac{\left(\frac{P_1}{\gamma} - Z_{2A}\right) - \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee} - 1\right) \frac{8}{g\pi^2 D_1^4} Q_1^2}{\left(K_{cont} + f_{2A}\frac{L_{2A}}{D_{2A}} + K_{sprinkler} + 1\right) \frac{8}{g\pi^2 D_{2A}^4}}}$$

Similarly

$$\left(\frac{P_1}{\gamma} - Z_{2B}\right) = \frac{V_{2B}^2}{2g} - \frac{V_1^2}{2g} + h_{L1-2B} = \frac{8}{g\pi^2 D_{2B}^4} Q_{2B}^2 - \frac{8}{g\pi^2 D_1^4} Q^2 + h_{L1-2B}$$

Simplified to

$$\left(\frac{P_1}{\gamma} - Z_{2B}\right) = \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee} - 1\right) \frac{8}{g\pi^2 D_1^4} Q_1^2 + \left(K_{cont} + f_{2B}\frac{L_{2B}}{D_{2B}} + K_{sprinkler} + K_{elbow}\right) \frac{8}{g\pi^2 D_{2B}^4} Q_{2B}^2$$

$$Q_{2B} = \sqrt{\frac{\left(\frac{P_1}{\gamma} + Z_{2B}\right) - \left(f_1 \frac{L_1}{D_1} + K_{valve} + K_{Tee} - 1\right) \frac{8}{g\pi^2 D_1^4} Q_1^2}{\left(K_{cont} + f_{2B}\frac{L_{2B}}{D_{2B}} + K_{elb} + K_{sprinkler}\right) \frac{8}{g\pi^2 D_{2B}^4}}}$$

But $Q_1 = Q_{2A} + Q_{2B}$.

Iteration procedure:

* Guess f_{2A} and f_{2B}
* Guess $Q_1 = Q_T$
* Compute V_T, Re_T and f_T
* Compute Q_{2A} and Q_{2B}
* Compute RHS
* Compare RHS with LHS (Guessed Q_T) (% diff)
* Compute Re_{2A} and Re_{2B}
* Compute f_{2A} and f_{2B}.
* Compare f_{2A} and f_{2B} to the Guessed values (% diff)

* Repeat the iteration using the computed f_{2A}, f_{2B} and QT(RHS) untill the % diff is acceptable.

Results
With a RHS-LHS % diff of 0.00%, $Q_{2A} = 0.00214\ m^3/s$ $\quad V_{2A} = 3.86\ m/s$

$Q_{2B} = 0.00202\ m^3/s$ $\quad V_{2B} = 3.63\ m/s$

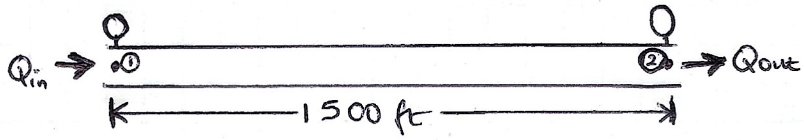
$Q_T = 0.00416\ m^3/s$ $\quad V_T = 3.166\ m/s$

- From the iterations, the flows through each sprinkler are relatively the same.

- The fluid velocity through the main pipe and through the ~~sprin~~branches are very close to the critical velocity (3 m/s)

If the flow rates and/or the velocity were to be change, the most applicable way of altering them would be to use valves. A different kind of valve can be installed in the main pipe that can be adjusted to affect the flow rate.

Problem 2 (GRADE THIS PROBLEM FOR TEST 3)



$Q = 65\,gpm = 0.1448\,ft^3/s$

Water at $70°F$ (room temperature)

~~Dynamic~~ $\gamma = 62.3\,lb/ft^3$

$\nu = 1.05\times10^{-5}$

$D = 2in = 0.1558\,ft$

$\varepsilon = 1.5\times10^{-4}\,ft.$

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + h_{L_{1-2}}$$

$$\frac{P_1 - P_2}{\gamma} = h_{L_{1-2}} = h_{L\,pipe\,1-2}$$

$$h_{L_{1-2}} = f\frac{L}{D}\frac{V^2}{2g} \qquad \text{but } V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$h_{L_{1-2}} = f\frac{L}{D}\cdot\frac{8}{g\pi^2 D^4}Q^2$$

$$V = \frac{4Q}{\pi D^2} = \frac{4\times0.1448}{\pi(0.1558)^2}$$

$$\underline{V = 7.5953\,ft/s}$$

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2}$$

$$D/\varepsilon = \frac{0.1558\,ft}{1.5\times10^{-4}} = \underline{1038.67}$$

$$N_R = \frac{VD}{\nu} = \frac{7.5953\times0.1558}{1.05\times10^{-5}}$$

$$= \underline{1.127\times10^5}$$

$$= \frac{0.25}{\left[\log\left(\frac{1}{3.7(1038.67)} + \frac{5.74}{(1.127\times10^5)^{0.9}}\right)\right]^2}$$

$$= 0.022.$$

$$h_{L_{1-2}} = 0.022\times\frac{1500\,ft}{0.1558\,ft}\times\frac{8}{9.\pi^2(0.1558)^4}\times0.1448^2$$

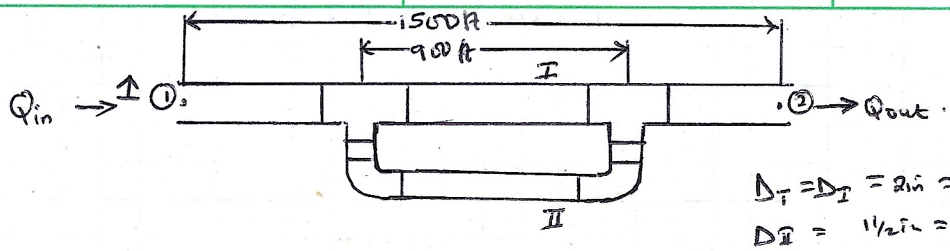
$$h_{L_{1-2}} = 189.46\,ft.$$

$$\frac{\Delta P}{\gamma} = 189.46\,ft.$$

$$\Delta P = 189.46\,ft\times62.3\,lb/ft^3$$

$$= 11803.358\,lb/ft^2 \times \frac{ft^2}{144\,in^2}$$

$$\underline{\Delta P = 81.97\,psi}$$



$D_I = D_{①} = 2in = 0.1558ft$

$D_{II} = 1\tfrac{1}{2} in = 0.1142ft.$

$$\frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{Z_1} = \frac{P_2}{\gamma} + \cancel{\frac{V_2^2}{2g}} + \cancel{Z_2} + h_{L_{1-2}}$$

$$\frac{P_1 - P_2}{\gamma} = h_{L_{1-2}} \Rightarrow \frac{\Delta P}{\gamma} = h_{L_{1-2(I)}}$$

$$\Rightarrow \frac{\Delta P}{\gamma} = h_{L_{1-2(II)}}$$

$$h_{L_{1-2(I)}} = h_{L\,pipe①} + 2h_{\underset{\rightarrow}{Tee}①} + h_{L\,pipe\,I}$$

$$= f_{①}\frac{L_{①}}{D_{①}}\cdot\frac{V_{①}^2}{2g} + 2K_{\underset{\rightarrow}{Tee}}\cdot\frac{V_{①}^2}{2g} + f_I\frac{L_I}{D_I}\cdot\frac{V_I^2}{2g}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$h_{L_{1-2I}} = \left(f_{①}\frac{L_{①}}{D_{①}} + 2K_{\underset{\rightarrow}{Tee}}\right)\frac{8}{g\pi^2 D_1^4}Q_1^2 + \left(f_I\frac{L_I}{D_I}\right)\cdot\frac{8}{g\pi^2 D_I^4}Q_I^2 = \frac{\Delta P}{\gamma}$$

$$Q_I = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_{①}\frac{L_{①}}{D_{①}} + 2K_{\underset{\rightarrow}{Tee}}\right)\frac{8}{g\pi^2 D_{①}^4}Q_{①}^2}{f_I\frac{L_I}{D_I}\cdot\frac{8}{g\pi^2 D_I^4}}}$$

$$h_{L_{1-2II}} = h_{L\,pipe①} + 2h_{\underset{\downarrow}{Tee}①} + h_{L\,contraction\,II} + h_{L\,enlargement\,II} + h_{elbow\,II} + h_{pipe\,II}$$

$$= f_1\frac{L_1}{D_1}\frac{V_1^2}{2g} + 2K_{\underset{\downarrow}{Tee}}\cdot\frac{V_1^2}{2g} + K_{cont}\cdot\frac{V_{II}^2}{2g} + K_{enlarg}\frac{V_{II}^2}{2g} + K_{elb}\cdot\frac{V_{II}^2}{2g} + f_{II}\frac{L_{II}}{D_{II}}\cdot\frac{V_{II}^2}{2g}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$h_{L_{1-2II}} = \left(f_1\frac{L_1}{D_1} + 2K_{\underset{\downarrow}{Tee}}\right)\frac{8}{g\pi^2 D_1^4}Q_1^2 + \left(K_{cont} + K_{enlarg} + K_{elb} + f_{II}\frac{L_{II}}{D_{II}}\right)\frac{8}{g\pi^2 D_{II}^4}Q_{II}^2 = \frac{\Delta P}{\gamma}$$

$$Q_{II} = \sqrt{\frac{\frac{\Delta P}{\gamma} - \left(f_1\frac{L_1}{D_1} + 2K_{\underset{\downarrow}{Tee}}\right)\frac{8}{g\pi^2 D_1^4}Q^2}{\left(K_{cont} + K_{enlarg} + K_{elb} + f_{II}\frac{L_{II}}{D_{II}}\right)\frac{8}{g\pi^2 D_{II}^4}}}$$

Also $Q_1 = Q_I + Q_{II}$

Iteration procedure
- Guess f_I and f_{II}
- Guess $Q_1 = Q_T$
- Compute V_T, Re_T and f_T
- Compute Q_I and Q_{II}
- Compute RHS.
- Compare RHS with LHS (Guessed Q_T)
- Compute Re_I and Re_{II}
- Compute f_I and f_{II}
- Compare f_I and f_{II} with the guessed values.

- Repeat the procedure using the computed f_I, f_{II} and Q_T until % diff is acceptable.

Results
With RHS-LHS % diff of 0.36%, $Q_I = 0.1218\ ft^3/s$

$Q_{II} = 0.05\ ft^3/s$

$Q_T = 0.1712\ ft^3/s$.

$\Delta Q_T = (0.1712 - 0.1448)\ ft^3/s = 0.0264\ ft^3/s$

$\therefore$ Adding a branch increase the flow rate by $0.0264\ ft^3/s$