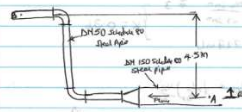
Homework 3.1

This week the topic of pipe networks was discussed. We focused primarily on a two-branch parallel pipe system. To solve this kind of problem, we must use Bernoulli's equation for each branch of pipe. This kind of problem is more complicated as there are more elbow losses involved. The relative roughness of the pipes also has a greater relevance and is used to calculate the head loss for the system. The kind of problems that involve multiple branch pipe systems often have a variable that has to be guessed. This allows for iterations of values and relies on excel to do efficiently. Excel can be used to find the percent error and once the error is below 1%, it is deemed acceptable.

Homework Problems

11.5

Problem 11.5



Oil specific weight = 8.8 KN/m³
Oil kinematic viscosity = 2.12×10^{-5} m²/s
length of DN 150 pipe = 180 m D = 146.3 mm
length of DN 50 pipe = 8 M D = 49.3mm
Elbows are long-radius type.
Pressure at B = 12.5 MPa

Calculate pressure at A   Q = 0.015 m³/s

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A}^{0} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{LA-B}$$

$$\frac{P_A}{\gamma} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{LA-B} - \frac{V_A^2}{2g}$$

$Q = 0.015$ m³/s $A_A = 1.682 \times 10^{-2}$ m² $A_B = 1.905 \times 10^{-3}$ m²
$Q = VA$ $V_A = \frac{0.015 \text{ m}^3/\text{s}}{1.682 \times 10^{-2} \text{ m}^2}$ $V_B = \frac{0.015 \text{ m}^3/\text{s}}{1.905 \times 10^{-3} \text{ m}^2}$
$V = Q/A$ $V_A = 0.8918$ m/s $V_B = 7.874$ m/s

$$h_{LAB} = h_{L_{pipe(DN150)}} + h_{L_{contraction}} + h_{L_{pipe(DN50)}} + 2h_{L\,elbow}$$

$$= f_1 \frac{L_1}{D_1} \cdot \frac{V_A^2}{2g} + K_{cont} \frac{V_B^2}{2g} + f_2 \frac{L_2}{D_2} \frac{V_B^2}{2g} + 2 \overset{K=20}{K_{elbow}} \frac{V_B^2}{2g}$$

For DN150 $N_R = \frac{vD}{\upsilon} = \frac{0.8918 \text{ m/s} \times 146.3 \times 10^{-3} \text{ m}}{2.12 \times 10^{-5} \text{ m}^2/\text{s}} = 6.15 \times 10^3$
$\varepsilon = 4.6 \times 10^{-5}$ m $D/\varepsilon = \frac{146.3 \times 10^{-3} \text{ m}}{4.6 \times 10^{-5} \text{ m}} = 318.04$ } $f_1 = 0.042$

for DN 50 $N_R = \frac{vD}{\upsilon} = \frac{7.874 \text{ m/s} \times 49.3 \times 10^{-3} \text{ m}}{2.12 \times 10^{-5} \text{ m}^2/\text{s}} = 1.83 \times 10^4$
$\varepsilon = 4.6 \times 10^{-5}$ m $D/\varepsilon = \frac{49.3 \times 10^{-3} \text{ m}}{4.6 \times 10^{-5} \text{ m}} = 1071.7$ } $f_2 = 0.0255$

For the sudden contraction $\frac{D_1}{D_2} = \frac{146.3 \text{ mm}}{49.3 \text{ mm}} \cong 3$, $V_2 = 7.87$ m/s } $K = 0.36$

$$h_{LA-B} = f_1 \frac{L_1}{D_1} \frac{V_A^2}{2g} + \left(K_{cont} + 2K_{elb} + f_2 \frac{L_2}{D_2}\right) \frac{V_B^2}{2g}$$

$$= 0.042 \times \frac{180 \text{ m}}{0.1463 \text{ m}} \times \frac{(0.8918 \text{ m/s})^2}{2 \times 10 \text{ m/s}^2} + \left[0.36 + 2(20 \times 0.019) + 0.0255\left(\frac{8 \text{ m}}{0.0493}\right)\right] \frac{(7.874 \text{ m/s})^2}{2 \times 10 \text{ m/s}^2}$$

$$= 2.05 \text{ m} + 83.64 \text{ m} = 85.69 \text{ m}$$

$$\frac{P_A - P_B}{\gamma} = \cancel{\frac{12.5 \text{ MPa}}{\gamma}} + \frac{(0.8918 \text{ m/s})^2}{2 \times 10 \text{ m/s}^2} + 4.5 \text{ m} + 85.69 \text{ m} - \frac{(7.874 \text{ m/s})^2}{2 \times 10 \text{ m/s}^2}$$

$$\frac{P_A - P_B}{\gamma} = 0.04 \text{ m} + 4.5 \text{ m} + 85.69 \text{ m} - 3.10 \text{ m} = 87.13 \text{ m}$$

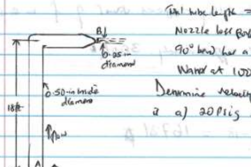
$$P_A - P_B = 87.13 \text{ m} \times 8800 \text{ N/m}^3 = 766744 \text{ N/m}^3 = 0.77 \text{ MPa}$$

$$P_A = P_B + 0.77 \text{ MPa} = 12.5 \text{ MPa} + 0.77.$$

$$P_A = 13.25 \text{ MPa}$$

11.13

Problem 11.13



Total tube length = 20.0 ft
Nozzle loss coefficient K = 0.15
90° bend has a radius of 6.0 in
Water at 100°F $\gamma = 62.0$ lb/ft³ $\nu = 7.37 \times 10^{-6}$ ft²/s
Determine velocity at the nozzle if the pressure at the bottom is a) 20 psig and b) 80 psig

$Q_A = Q_B$
$A_A v_A = A_B v_B$
$v_A = \frac{A_B}{A_A} v_B = \frac{D_B^2}{D_A^2} v_B = \frac{0.25^2}{0.5^2} v_B$
$v_A = \frac{1}{4} v_B$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A}^{=0} = \cancel{\frac{P_B}{\gamma}}^{=0} + \frac{V_B^2}{2g} + \overset{=18ft}{z_B} + h_{LA-B}$$

$$\frac{P_A}{\gamma} - z_B = \frac{V_B^2}{2g} - \frac{1}{16}\frac{V_B^2}{2g} + h_{LAB}$$

for the bend
$r/D = \frac{6 \text{ in}}{0.5 \text{ in}} = 12$
$\Rightarrow \frac{L_e}{D} = 12$
$D/\varepsilon = \frac{0.5/12}{5.0 \times 10^{-6}} = 8333$
$f_T = 0.016$
$K = 0.016 \times 12 = 0.192$

$$h_{LA-B} = h_{L_{pipe}} + h_{L_{bend}} + h_{L_{nozzle}}$$

$$= f\frac{L}{D}\left(\frac{1}{16} \cdot \frac{V_B^2}{2g}\right) + 0.192\left(\frac{1}{16}\frac{V_B^2}{2g}\right) + 0.15 \frac{V_B^2}{2g}$$

$$= \left[f\left(\frac{20}{0.5/12}\right)\left(\frac{1}{16}\right) + 0.192\left(\frac{1}{16}\right) + 0.15\right]\frac{V_B^2}{2g} = (30f + 0.162)\frac{V_B^2}{2g}$$

$$\frac{P_A}{\gamma} - z_B = (1 - 0.0625 + 30f + 0.162)\frac{V_B^2}{2g}$$

$$= \underbrace{(1.0995 + 30f)\frac{V_B^2}{2g}}_{RHS}$$

$$\left(\frac{20 \times 144}{62.0}\right) - 18 = 28.45$$

$$28.45 \times 2g = (1.0995 + 30f)\frac{V_B^2}{2g} \times 2g$$

$$V_B = \sqrt{\frac{1832.28}{1.0995 + 30f}}$$

$N_R = \frac{VD}{\nu} = \frac{\frac{1}{4} V_B \times 0.5/12 \text{ ft/s}}{7.37 \times 10^{-6} \text{ ft}^2/\text{s}} = 1431 V_B$
$D/\varepsilon = \frac{0.5/12}{5.0 \times 10^{-6}} = 8333$
Range of $f = 0.013 \rightarrow 0.034$
using $f = 0.015$,
$V_B = 34.38$ ft/s $N_R = 1431 \times 34.38$ ft/s $= 4.9 \times 10^4$
This value of N_R gives an f value of 0.022.

~~Using a f value of 0.017~~ Using different values of f in a spreadsheet

V_B for $P_A = 20$ psig is approximately 30 ft/s

for $P_A = 80$ psig,
$$LH = \left(\frac{80 \times 144}{62.0}\right) - 18 = 167.81 \text{ ft}$$

$$167.81 \times 2g = (1.0995 + 30f)\, V_B^2$$

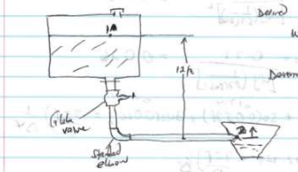
$$V_B = \sqrt{\frac{10806}{1.0995 + 30f}}$$

$N_R = 1431.35 V_B$
$D/\varepsilon = 8333$

By iteration, $V_B \cong 75$ ft/s

11.20

Problem 11.20



Schedule 40 steel pipe $L = 75\,ft$.

Desired $Q = 400\,gal/min = 0.891\,ft^3/s$

Water @ 80°F

Determine the size of the pipe

$$\cancel{\frac{p_1}{\gamma}}^{=0} + \cancel{\frac{V_1^2}{2g}}^{=0} + z_1 = \cancel{\frac{p_2}{\gamma}}^{=0} + \frac{V_2^2}{2g} + \cancel{z_2}^{=0} + h_L$$

$$V = \frac{4Q}{\pi D^2}$$

$$z_1 = \frac{V_2^2}{2g} + h_{L_{1-2}}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$h_{L_{1-2}} = h_{L_{entrance}} + h_{L_{valve}} + h_{L_{elbow}} + h_{L_{pipe}}$$

$$= K_{ent}\frac{V_2^2}{2g} + K_{elbow}\frac{V_2^2}{2g} + K_{valve}\frac{V_2^2}{2g} + f\frac{L}{D}\frac{V_2^2}{2g}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$= \left(0.5 + 30 f_T + 340 f_T + f\frac{L}{D}\right)\frac{8Q^2}{\pi^2 D^4 g}$$

$$z_1 = \frac{8Q^2}{\pi^2 D^4 g} + \left(0.5 + 30 f_T + 340 f_T + f\frac{75}{D}\right)\frac{8Q^2}{\pi^2 D^4 g}$$

$$\frac{32.2\,\pi^2}{8(0.891\,ft^3/s)} \times 12 = \left(1 + 0.5 + 30 f_{T_{elbow}} + 340 f_{T_{valve}} + \frac{75f}{D}\right)\frac{8(0.891\,ft^3/s)^2}{\pi^2 D^4 g} \times \frac{\pi^2 \times 32.2}{8(0.891\,ft^3/s)^2}$$

$$600.47 = \left(1.5 + 30 f_{T_{elb}} + 340 f_{T_{valve}} + \frac{75f}{D}\right)\frac{1}{D^4}$$

for the elbow $f_T = \dfrac{0.25}{\left[\log\left(\frac{1}{3.7(300)}\right)\right]^2} = 0.0598$

for the valve $f_T = \dfrac{0.25}{\left[\log\left(\frac{1}{3.7(3000)}\right)\right]^2} = 0.026$

$$600.47 = \left(1.5 + \underbrace{30(0.0598)}_{=1.794} + \underbrace{340(0.026)}_{=8.84} + \frac{75f}{D}\right)\frac{1}{D^4}$$

$$600.47 = \left(12.134 + \frac{75f}{D}\right)\frac{1}{D^4}$$

$$600.47\,D^4 = 12.134 + \frac{75f}{D}$$

$\varepsilon = 4.6 \times 10^{-5}$

$\nu = 9.15 \times 10^{-6}\,ft^2/s$

$V = \dfrac{4Q}{\pi D^2}$

Using iteration on spreadsheet.

Compute N_R and D/ε for any given value of D

$N_R = \dfrac{VD}{\nu}$

Iteration gives a pipe size of ~~…~~ 0.40 ft

pi	3.1425	
e	1.50E-04	ft
vis	9.15E-06	ft2/s
Q	0.891	ft3/s

d (ft)	v(ft/s)	d/e	nr	den1	den2	den3	f		LHS	RHS		% diff
0.1000	113.4128	6.67E+02	1.24E+06	4.05E-04	1.88E-05	1.14E+01	2.20E-02		0.060047	2.86E+01		-4.76E+04
0.2000	28.35322	1.33E+03	6.20E+05	2.03E-04	3.51E-05	1.31E+01	1.90E-02		0.960752	1.93E+01		-1.91E+03
0.3000	12.60143	2.00E+03	4.13E+05	1.35E-04	5.06E-05	1.39E+01	1.80E-02		4.863807	1.66E+01		-2.42E+02
0.4000	7.088305	2.67E+03	3.10E+05	1.01E-04	6.56E-05	1.43E+01	1.75E-02		15.37203	1.54E+01		-3.06E-01
0.4050	6.914365	2.70E+03	3.06E+05	1.00E-04	6.63E-05	1.43E+01	1.75E-02		16.15516	1.54E+01		4.82E+00
0.4100	6.746751	2.73E+03	3.02E+05	9.89E-05	6.71E-05	1.43E+01	1.75E-02		16.96784	1.53E+01		9.63E+00
0.5000	4.536515	3.33E+03	2.48E+05	8.11E-05	8.02E-05	1.44E+01	1.74E-02		37.52937	1.47E+01		6.07E+01