Test 1

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MET 330 Fluid Mechanics

Summer 2022

Problem 1

Problem Statement:

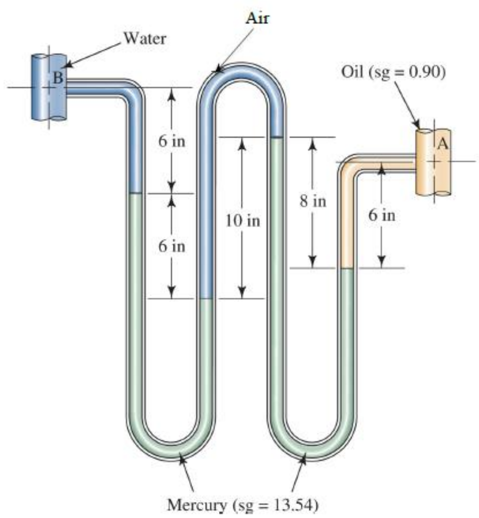
For the compound differential manometer in the figure, calculate (P_A-P_B). Keep in mind that the pressure in the space with air is the same (it does not change with elevation). At some point of the operation of the system, the pressure at A drops. That makes the oil column reduce to 5 in. and all other fluids will adjust to the change. Calculate (P_A-P_B) for the new stage of the system.

Purpose:

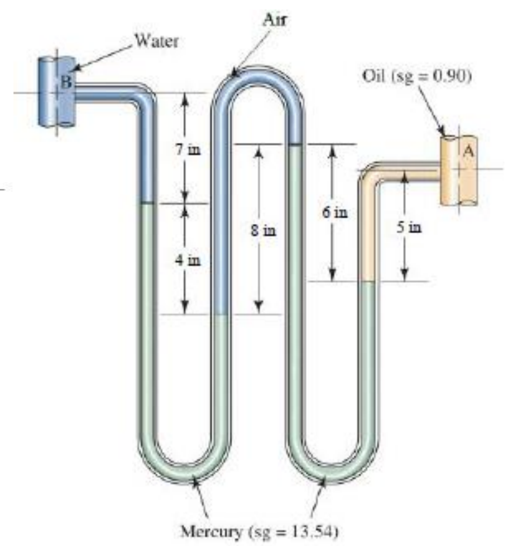
Calculate the pressure change between point A and point B before and after the pressure at A drops.

Drawing:

State 1



State 2



Design Considerations:

- Isothermal Process
- Incompressible fluids

Data and Variables:

- Measurements are given in the drawing
- Specific gravity (sg) of oil and mercury are provided in the drawing

$$sg_{oil} = 0.90$$

$$sg_{Hg} = 13.54$$

- Specific weight of water at 60°F from table A.2 in textbook:

$$\gamma_w = 62.4 \left[{}^{lb}/_{ft^3} \right]$$

Procedure:

- Manometers use increments of pressure to measure the difference in pressure between two points, in this case point A and B. The equation for calculating pressure differences in each column of fluid is:

  $$\Delta P = \gamma \cdot h \qquad \textbf{(1)}$$

  Where ΔP is the pressure difference (P_A-P_B), γ is the specific gravity of the fluid, and h is the height of the column being investigated.

- Solve (P_A-P_B) for State 1.
  - Starting from point A, calculate the ΔP for each fluid as it changes elevation within the manometer (excluding the air section, which does not have a pressure change as per the problem statement).
  - The sum of all these changes is equal to (P_A-P_B)
- Solve (P_A-P_B) for State 2:
  - Same procedure as used for State 1

Calculations

- Convert given distances to match units for specific weight (inches to feet):

$$Oil_{State\ 1} = 6''\left(\frac{1'}{12''}\right) = 0.5' \qquad Oil_{State\ 2} = 5''\left(\frac{1'}{12''}\right) = 0.417'$$

$$Hg_{State\ 1-1} = 8''\left(\frac{1'}{12''}\right) = 0.667' \qquad Hg_{State\ 2-1} = 6''\left(\frac{1'}{12''}\right) = 0.5'$$

$$Air_{State\ 1} = 10''\left(\frac{1'}{12''}\right) = 0.833' \qquad Air_{State\ 2} = 8''\left(\frac{1'}{12''}\right) = 0.667'$$

$$Hg_{State\ 1-2} = 6''\left(\frac{1'}{12''}\right) = 0.5' \qquad Hg_{State\ 2-2} = 4''\left(\frac{1'}{12''}\right) = 0.333'$$

$$Water_{State\ 1} = 6''\left(\frac{1'}{12''}\right) = 0.5' \qquad Water_{State\ 2} = 7''\left(\frac{1'}{12''}\right) = 0.583'$$

- Calculate the specific weights of mercury and oil using the value for the specific weight of water found in the book:

$$sg = \frac{\gamma}{\gamma_{water}} \ \therefore\ \gamma = sg \cdot \gamma_{water}$$

$$\gamma_{oil} = (0.90)\left(62.4\left[{}^{lb}/_{ft^3}\right]\right) = 56.16\left[{}^{lb}/_{ft^3}\right]$$

$$\gamma_{Hg} = (13.54)\left(62.4\left[{}^{lb}/_{ft^3}\right]\right) = 844.90\left[{}^{lb}/_{ft^3}\right]$$

- Starting at point A, build an equilibrium equation for State 1 using formula (1):

$$P_B = P_A + \gamma_{oil} \cdot h_{oil,\ state\ 1} - \gamma_{Hg} \cdot h_{Hg,\ State\ 1-1} - \gamma_{Hg} \cdot h_{Hg,\ State\ 1-2} - \gamma_{water} \cdot h_{water,\ State\ 1}$$

- Insert calculated values, solve for (P_A-P_B):

$$\left(P_A - P_B\right) = -\left(56.16\left[{}^{lb}/_{ft^3}\right]\right)(0.5') + \left(844.90\left[{}^{lb}/_{ft^3}\right]\right)(0.667') + \left(844.9\left[{}^{lb}/_{ft^3}\right]\right)(0.5') + \left(62.4\left[{}^{lb}/_{ft^3}\right]\right)(0.5')$$

$$= \boxed{989.12\left[{}^{lb}/_{ft^2}\right]\ or\ 6.86\ [psi]}$$

- Perform the same steps for State 2, using the heights calculated for State 2:

$$P_B = P_A + \gamma_{oil} \cdot h_{oil,\ state\ 2} - \gamma_{Hg} \cdot h_{Hg,\ State\ 2-1} - \gamma_{Hg} \cdot h_{Hg,\ State\ 2-2} - \gamma_{water} \cdot h_{water,\ State\ 2}$$

$$\left(P_A - P_B\right) = -\left(56.16\left[{}^{lb}/_{ft^3}\right]\right)(0.417') + \left(844.90\left[{}^{lb}/_{ft^3}\right]\right)(0.5') + \left(844.9\left[{}^{lb}/_{ft^3}\right]\right)(0.333') + \left(62.4\left[{}^{lb}/_{ft^3}\right]\right)(0.583')$$

$$= \boxed{716.76\left[{}^{lb}/_{ft^3}\right]\ or\ 4.97\ [psi]}$$

Summary:

The pressure of the oil at point A is higher than the pressure of the water at point B, thus the value of (P_A-P_B) is a positive number. When the pressure of the oil is lowered, creating a change in height of the oil column, the difference between the pressure at Point A and the pressure at Point B is reduced, as they have come closer to being equal. The section of the manometer that is full of air was neglected when calculating (P_A-P_B) because it is not dense enough to experience a significant pressure change over such a small increase in height, and therefore the pressure remains the same throughout that section.

Problem 2

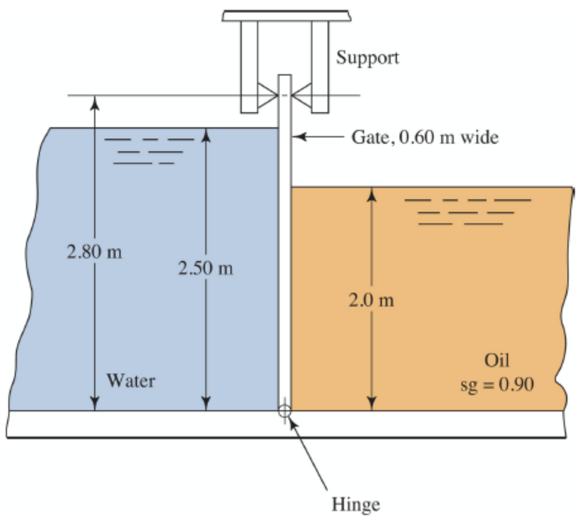
Problem Statement:

The figure shows a gate hinged at its bottom and held by a simple support at its top. The gate separates two fluids. Create an Excel spreadsheet to compute the net force on the gate, the force on the hinge, and the force on the support due to the fluid on each side. Make the spreadsheet in such a way that you can input any combination of the depth on either side of the gate and any specific gravity of the fluids. Produce a graph of the force at the hinge versus the elevation of the liquid on the right. You will do this plot for different elevation of the liquid on the right.

Purpose:

Compute the force on the hinge, the gate and its support using Excel. Create a graph showing the varying force on the gate as the elevation of the oil is changed.

Drawing:



Design Considerations:

- The system is isothermal
- The fluids are incompressible

Data and Variables:

- The measurements are provided in the drawing
- The specific gravity (sg) is provided:

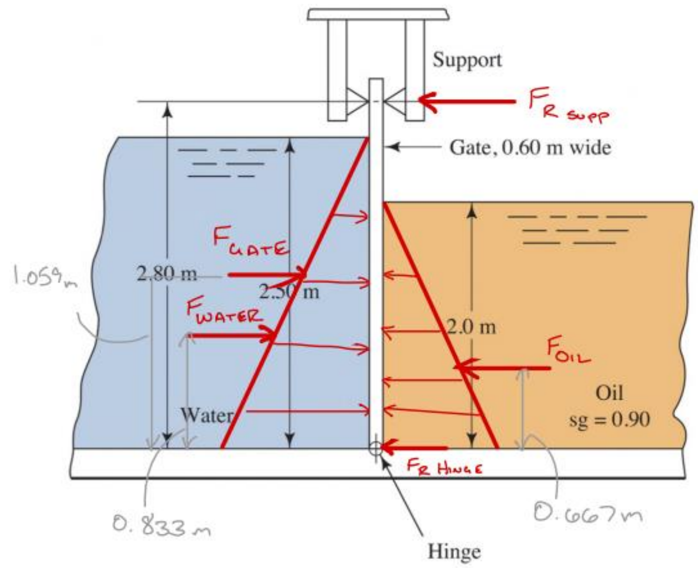
$$sg_{oil} = 0.90$$

- Specific weight of water at 60°F from table A.2 in textbook:

$$\gamma_w = 9.81 \left[{}^{kN}\!/_{m^3} \right]$$

Procedure:

- Create a Free Body Diagram (FBD) of the system to evaluate the forces acting on the gate, the hinge, and the support.



- Because the wall is vertical and the fluids are open to air at the surface, the force can be calculated using the following equation:

$$F = \frac{\gamma \cdot H_{bottom} \cdot b \cdot h}{2}$$

Where γ is the specific weight of the fluid, H_{bottom} is the distance from the bottom of the fluid to the centroid of the wall, *b* is the length of the base of the wall, and *h* is the depth of the fluid.

- The location of the forces caused by the fluids is located 1/3 of the way up the wall from the bottom of the fluid to the top of the fluid due to the triangular shape the pressure is creating on the wall.
- The location of the total force on the gate, F_{gate}, can be found by taking the moment around the hinge created by F_{gate} and setting it equal to the moment created by F_{water} and F_{oil}:

$$F_{gate} \cdot hbar = F_{water} \cdot hbar - F_{oil} \cdot hbar$$

- Using Newton's law, the forces on the support and the hinges can be calculated using the known forces created by the two fluids:

$$\Sigma F = 0 \qquad \Sigma M = 0$$

Where the sum of the forces and the sum of the moments are equal to 0 because the wall is not in motion.

- Using the data and equations above, use Excel to solve for the forces and create a dynamic system that can evaluate any given variables.

Calculations:

Sample Calculation:

Calculations done for the measurements given in the original image.

- Specific weight of oil:

$$\gamma_{oil}=\gamma_{water}\cdot sg_{oil}=9.81\left[{}^{kN}\!/_{m^3}\right]\cdot 0.90=8.83\left[{}^{kN}\!/_{m^3}\right]$$

- Net force on the gate:

$$F_{water}=\frac{9.81\left[{}^{kN}\!/_{m^3}\right]\cdot 2.5m\cdot 0.6m\cdot 2.5m}{2}=18.393\ kN$$

$$F_{oil}=\frac{8.83\left[{}^{kN}\!/_{m^3}\right]\cdot 2m\cdot 0.6m\cdot 2m}{2}=10.595\ kN$$

$$F_{gate}=F_{water}-F_{oil}=18.393\ kN-10.595\ kN=\boxed{7.799\ kN}$$

- Location of F_{gate}:

$$F_{gate}\cdot hbar=F_{water}\cdot hbar-F_{oil}\cdot hbar$$

$$\therefore$$

$$hbar=\frac{F_{water}\cdot hbar-F_{oil}\cdot hbar}{F_{gate}}=\frac{18.393\ kN\cdot 0.83m-10.595\ kN\cdot 0.67m}{7.799\ kN}$$

$$=1.05m$$

- Force on the support using the sum of the moments about the hinge:

$$\Sigma M=0=F_{support}\cdot h_{support}-F_{gate}\cdot hbar$$

$$\therefore$$

$$F_{support}=\frac{F_{gate}\cdot hbar}{h_{support}}=\frac{7.799\ kN\cdot 1.05m}{2.8m}=\boxed{2.924\ kN}$$

- Force on the hinge using the sum of forces on the gate:

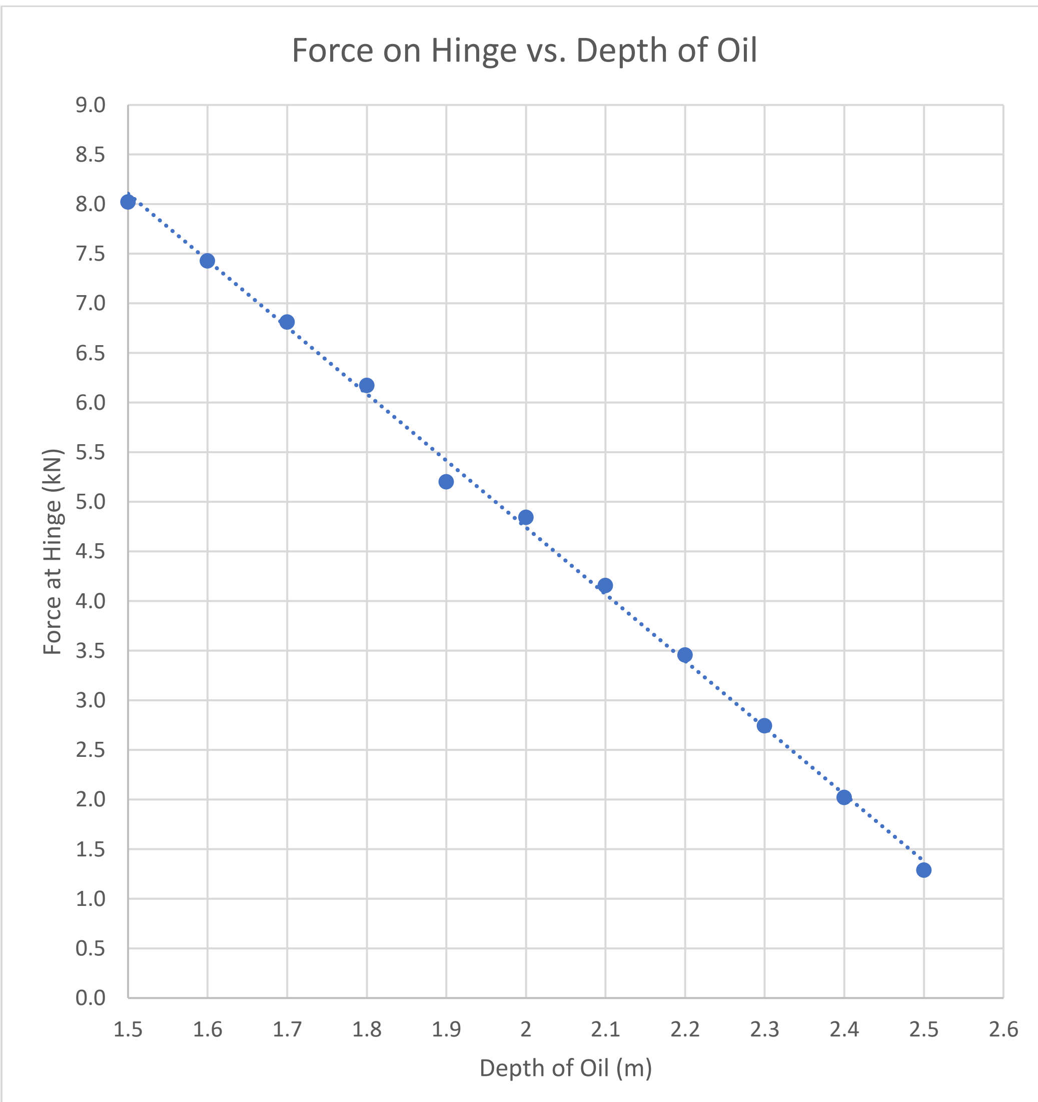
$$\Sigma F=0=F_{gate}-F_{support}-F_{hinge}$$

$$\therefore$$

$$F_{hinge}=F_{gate}-F_{support}=7.799\ kN-2.924\ kN=\boxed{4.875\ kN}$$

- Table and graph for adjusted elevation of oil created using the Excel spreadsheet:

Elevation of Oil (m)	Force at Hinge (kN)
1.5	8.02
1.6	7.43
1.7	6.81
1.8	6.18
1.9	5.20
2	4.85
2.1	4.16
2.2	3.46
2.3	2.74
2.4	2.02



Summary:

The force on the gate is impacted by the amount of fluid on each side, with the oil having a smaller impact because it has a smaller specific weight than the water. The table shows that as the elevation of the oil rises, the force on the hinge lowers. This is because the oil is acting as a bracing force against the water. When the oil reaches the same elevation as the water, the water is still pushing the gate to the right, but at a much smaller value than when the oil was at a lower elevation. If the oil were to rise enough above the water, the force on the hinge and the support would reverse, creating an overall force on the gate toward the left, and creating reaction forces at the support and the hinge toward the right.

Problem 3

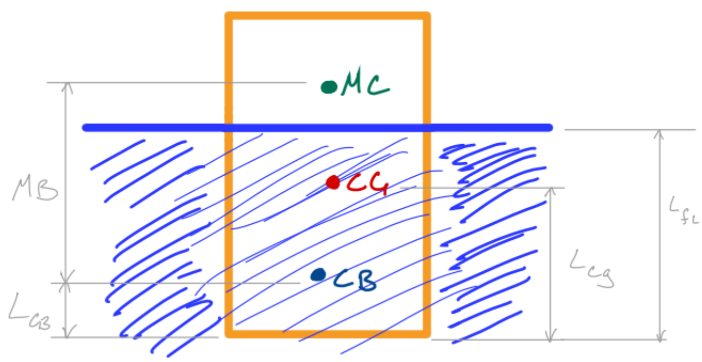
Problem Statement:

Create an Excel spreadsheet to evaluate the stability of a circular cylinder placed in a fluid with its axis vertical. Make the spreadsheet in such a way that you can input any combination of diameter, length, and weight (or specific weight) of the cylinder, and the specific weight of the fluid. The spreadsheet should provide the position of the cylinder when it is floating, the location of the center of buoyancy, and the metacenter location. Compare the location of the metacenter with the center of gravity to evaluate. Produce a graph of the metacenter location and the center of buoyancy location versus the cylinder length. You will do this plot for different cylinder diameters for fixed specific weights of the cylinder and fluid.

Purpose:

Create a spreadsheet that defines the buoyancy characteristics of a cylinder in fluid with adjustable variables for the physical properties of the cylinder and the fluid.

Drawing:



Design Considerations:

- Fluid is incompressible
- System is isothermal

Data and Variables:

- All data will vary based on the dimensions, material, and fluid chosen in the spreadsheet

Procedure:

- For a cylinder to float, its weight must be equal to the buoyancy force acting on it. The metacenter of the cylinder must also be higher than the center of gravity of the cylinder.

$$F_b = W_{cylinder} \qquad\qquad L_{CB} + MB > L_{CG}$$

- *MB* is the distance from the center of buoyancy (which is half the distance from the bottom of the cylinder to the surface of the water) to the metacenter. *MB* is defined as:

$$MB=\frac{I}{V_{d\ cylinder}}$$

Where *I* is the moment of inertia about the plane represented by the intersection of the fluid with the cylinder and $V_{d\ cylinder}$ is the volume displaced by the cylinder below the fluid. For circular planes, the moment of inertia is:

$$I=\frac{\pi D^4}{64}$$

- With these limits in place, any combination of variables regarding the dimensions of the cylinder and the specific weights of the cylinder and the water that fall within those requirements will produce a cylinder that floats with stability.

Calculations:

Sample calculations consist of the variables used to determine the physical properties of various cylinders used in the Excel Spreadsheet. See Spreadsheet or tables for specific values.

- There are four variables that must be adjustable in Excel: cylinder diameter (*D*), cylinder length (*L*), specific weight of the cylinder ($\gamma_{cylinder}$), and specific weight of the fluid (γ_{fluid})
  - The above equations need to be solved in terms of these four variables for the desired table values consisting of: center of buoyancy (L_{CB}), metacenter location (L_{MC}), center of gravity (L_{CG}), and position while floating (L_{fl})
- L_{fl} is determined by the relationship between weight and cylinder length, so:

$$F_b=V_{fluid}\cdot\gamma_{fluid}=L_{fl}\cdot A\cdot\gamma_{fluid}$$

$$W_{cylinder}=V_{cylinder}\cdot\gamma_{cylinder}=L\cdot A\cdot\gamma_{cylinder}$$

$$\therefore$$

$$L_{fl}=\frac{L\cdot A\cdot\gamma_{cylinder}}{A\cdot\gamma_{fluid}}=\frac{L\cdot\gamma_{cylinder}}{\gamma_{fluid}}$$

- Center of gravity (L_{CG}) is determined by the cylinder length:

$$L_{CG}=\frac{L}{2}$$

- Center of Buoyancy (L_{CB}) is determined by the depth of the cylinder in the water:

$$L_{CB} = \frac{L_{fl}}{2} = \frac{L \cdot \gamma_{cylinder}}{2 \cdot \gamma_{fluid}}$$

- Distance from the bottom to the metacenter (L_{MC}) is calculated by adding MB and L_{CB}:

$$L_{MC} = MB + L_{CB}$$

- MB is determined using I and $V_{d\ cylinder}$ which must be solved in terms of the four variables listed above:

$$I = \frac{\pi D^4}{64} \qquad V_{d\ cylinder} = V_{fluid} = L_{fl} \cdot A = L_{fl} \cdot \frac{\pi D^2}{4}$$

$$\therefore$$

$$MB = \frac{\frac{\pi D^4}{64}}{L_{fl} \cdot \frac{\pi D^2}{4}} = \frac{D^2}{16 L_{fl}}$$

- Replacing the terms for L_{MC} gives:

$$L_{MC} = \frac{D^2}{16 L_{fl}} + \frac{L \cdot \gamma_{cyl}}{2 \cdot \gamma_{fluid}} = \frac{D^2 \cdot \gamma_{fluid}}{16 \cdot L \cdot \gamma_{cyl}} + \frac{L \cdot \gamma_{cyl}}{2 \cdot \gamma_{fluid}}$$

- To set the bounds for stability in the cylinder L_{MC} must satisfy the condition that it be above the center of gravity of the cylinder, so:

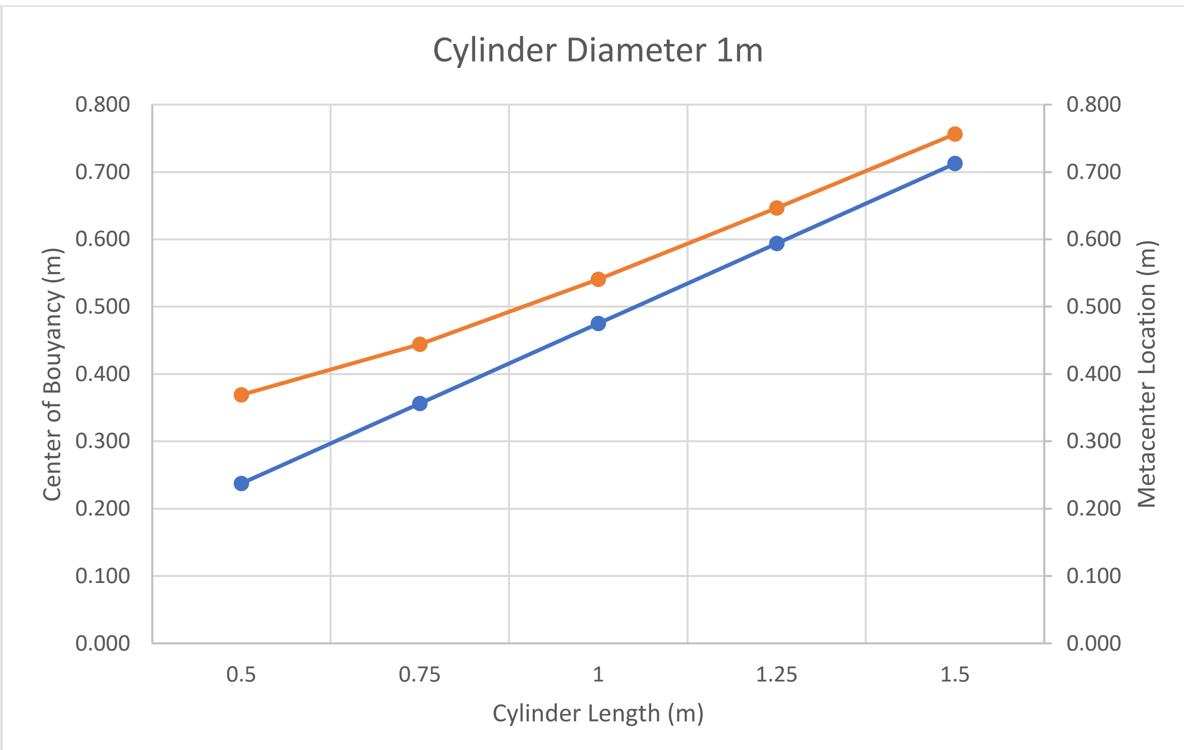
$$L_{MC} > L_{CG} \quad \therefore \quad L_{MC} - L_{CG} > 0$$

$$\therefore$$

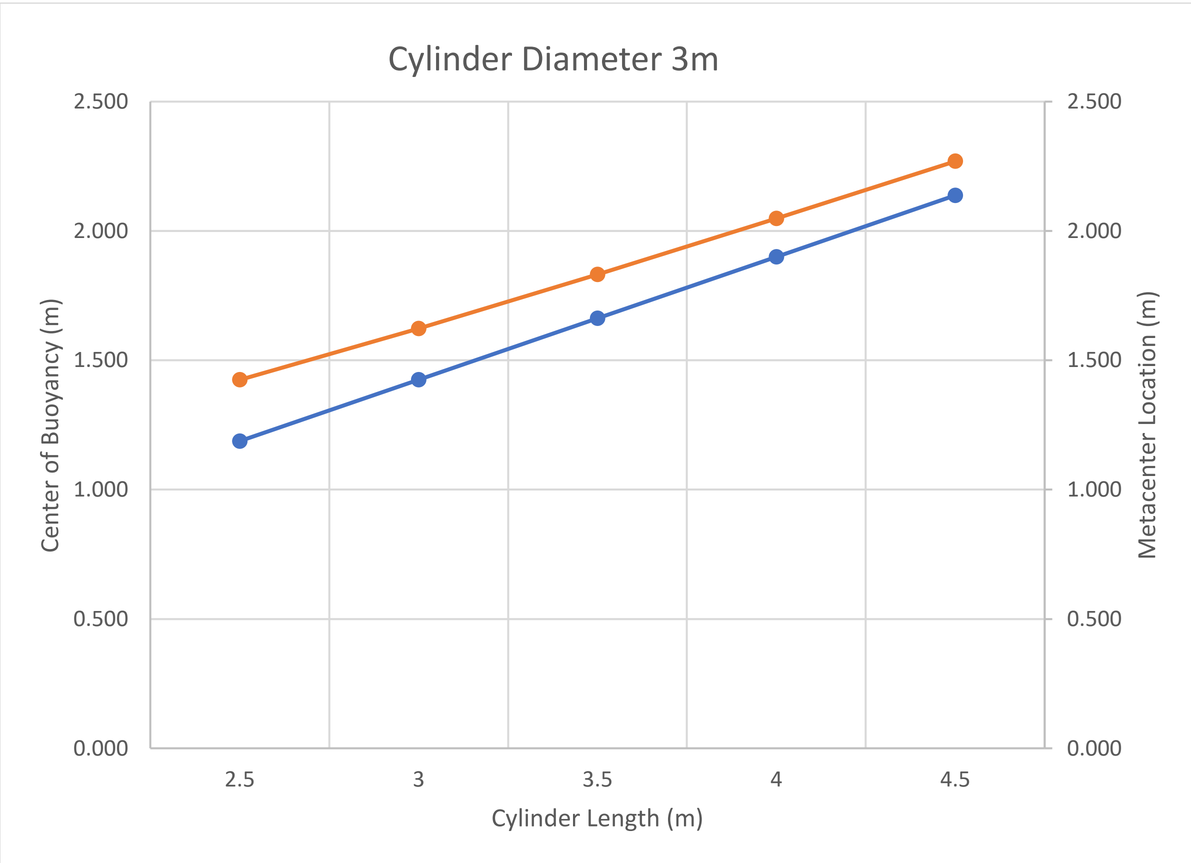
$$\left(\frac{D^2 \cdot \gamma_{fluid}}{16 \cdot L \cdot \gamma_{cyl}} + \frac{L \cdot \gamma_{cyl}}{2 \cdot \gamma_{fluid}} \right) - \frac{L}{2} > 0$$

- Graphs and tables developed for 3 different cylinder diameters at a variety of lengths:

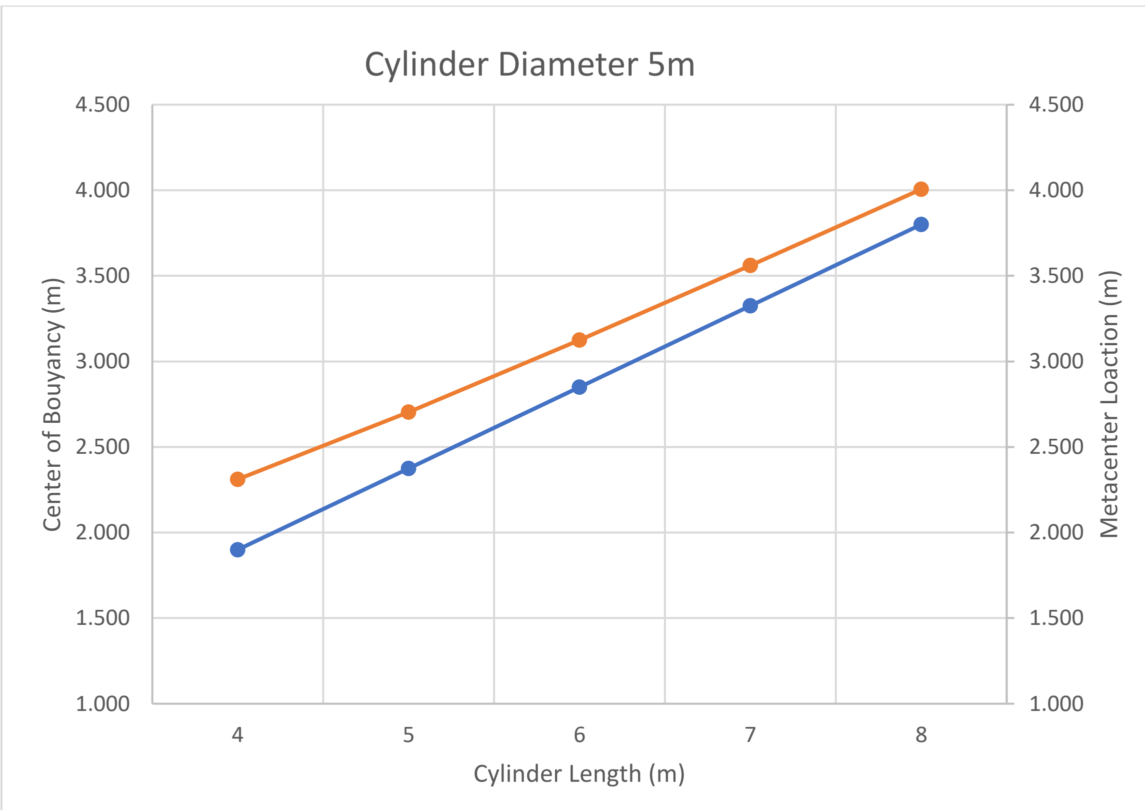
Cylinder Diameter: 1m		
Cylinder Length (m)	**Center of Bouyancy Location (m)**	**Metacenter Location (m)**
0.5	0.238	0.369
0.75	0.356	0.444
1	0.475	0.541
1.25	0.594	0.646
1.5	0.713	0.756



Cylinder Diameter: 3m		
Cylinder Length (m)	**Center of Bouyancy Location (m)**	**Metacenter Location (m)**
2.5	1.188	1.424
3	1.425	1.622
3.5	1.663	1.832
4	1.900	2.048
4.5	2.138	2.269



Cylinder Diameter: 3m		
Cylinder Length (m)	Center of Bouyancy Location (m)	Metacenter Location (m)
4	1.900	2.311
5	2.375	2.704
6	2.850	3.124
7	3.325	3.560
8	3.800	4.006



Summary:

The stability of cylinders floating in a liquid depends on many factors, the most important of which are the specific weight of the cylinders and the fluid. Small changes in those values results in instability, and if the cylinder has a greater specific weight than the liquid, it will sink below the surface, potentially finding a neutral buoyancy, but not floating. If the cylinder has a significantly smaller specific weight than the fluid, it can not reach stability on its vertical axis, resulting in tipping. When those two values are in the right window, changes in the dimensions of the cylinder can affect its stability, with more range for variance as the diameter grows, as can be seen on the graphs. At small diameters, only a small range of lengths will remain stable. At larger diameters, that range grows. This is because a change in diameter creates a bigger difference in the volume of the cylinder than changes to its length.

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2. When following the Honor Code a student may not share information about any aspect of the exam with other members of the class, other faculty members, or other people who has not already taken the exam this year, or its equivalent in future years.

3. When following the Honor Code a student must direct all questions concerning the exam or homework assignment to the course instructor or teaching assistant.

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Student Name: Alexander Higgins

Student Signature: ____________

Date: 06/05/2022