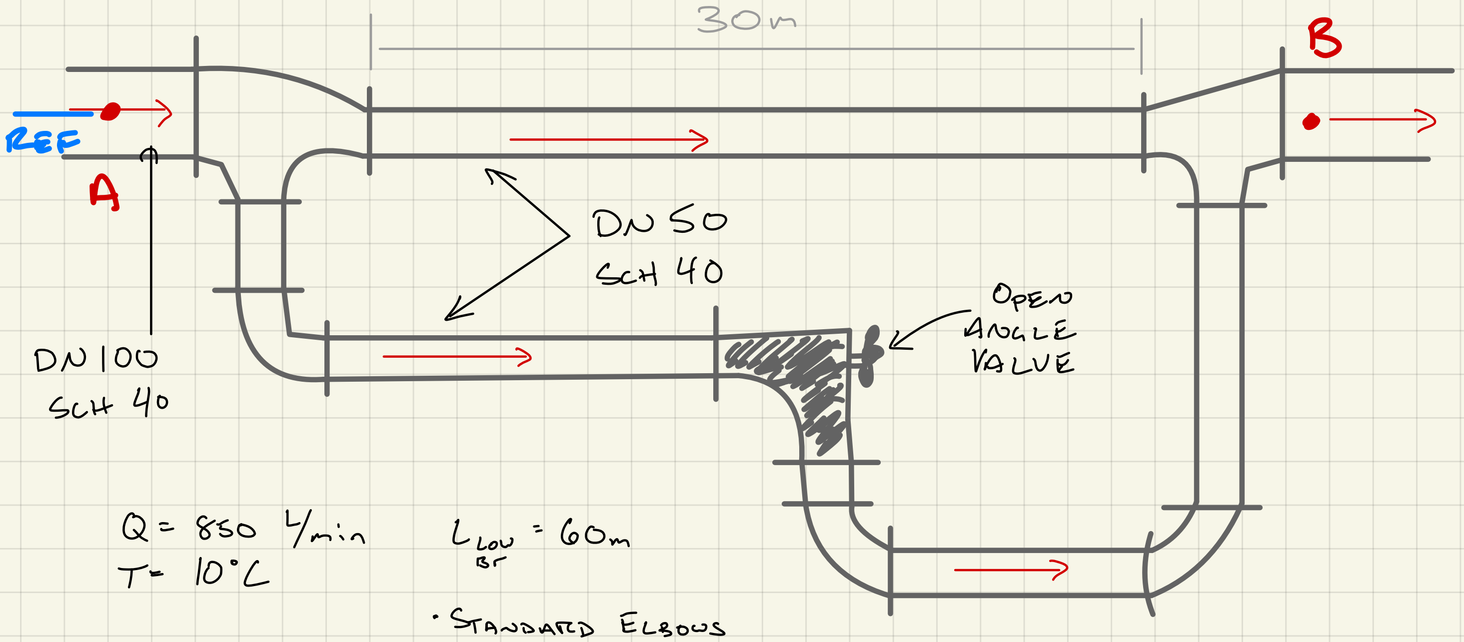
12-3 **GIVEN:**

WATER IS FLOWING IN THE SYSTEM BELOW:



$Q = 850\ L/min$

$T = 10°C$

$L_{LOW\ BR} = 60m$

- STANDARD ELBOWS

REQUIRED:

a) FIND Q IN EACH BRANCH.

b) FIND $P_A - P_B$

- INCLUDE MINOR LOSSES IN LOWER BRANCH

SOLUTION:

- START W/ BERNOULLI'S EQUATION:

$$\cancel{h_p} + \frac{P_A}{\gamma} + \cancel{\frac{V_1^2}{2g}} + \cancel{z_1} = \frac{P_B}{\gamma} + \cancel{\frac{V_2^2}{2g}} + h_L + \cancel{z_2}$$

$$\frac{P_A - P_B}{\gamma} = h_L \quad \text{OR} \quad \frac{\Delta P}{\gamma} = h_L$$

- DETERMINE LOSSES IN EACH BRANCH:

TOP:

- 2x TEE
- 30 m PIPE LOSS

BOTTOM:

- 2x TEE
- 3x ELBOW
- FULLY OPEN ANGLE VALVE
- 60 m PIPE LOSS

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- LOSS EQUATIONS:

PIPE LOSS $= f \frac{L}{D} \frac{V^2}{2g}$

ELBOWS: $f \frac{L_e}{D} \frac{v^2}{2g} = f\, 30 \frac{v^2}{2g}$

TEES: FLOW THRU: $f\, 20 \frac{v^2}{2g}$ FLOW BRANCH: $f\, 60 \frac{v^2}{2g}$

ANGLE VALVE: $f\, 150 \frac{v^2}{2g}$

- FOR UPPER BRANCH (BRANCH 1):

$$\frac{\Delta P}{\gamma} = 2\left(f_1 \cdot 20 \cdot \frac{v_1^2}{2g}\right) + f_2 \frac{L_2}{D_2} \frac{v_2^2}{2g}$$

- L_2 = UPPER PIPE L
- D_2 = UPPER & LOWER D
- v_2 = v IN UPPER PIPE

- FOR LOWER BRANCH (BRANCH 2)

$$\frac{\Delta P}{\gamma} = 2\left(f_1 \cdot 60 \cdot \frac{v_1^2}{2g}\right) + 3\left(f_3 \cdot 30 \cdot \frac{v_3^2}{2g}\right) + f_3 \cdot 150 \cdot \frac{v_3^2}{2g} + f_3 \frac{L_3}{D_2} \frac{v_3^2}{2g}$$

- ADJUSTED BERNOULLI'S:

$$h_{L_1} = h_{L_2} = \frac{\Delta P}{\gamma}$$

- PROBLEM STATEMENT ASKS FOR FLOWRATE IN BRANCHES (Q_2, Q_3):

$$Q_1 = Q_2 + Q_3$$

$$Q = Av \therefore v = \frac{Q}{A}$$

- SIMPLIFY TOP AND BOTTOM EQUATIONS W/ KNOWN VALUES

$$Q_1 = 850 \frac{L}{min}\left(\frac{1\,min \cdot 1\,m^3}{60s \cdot 1000L}\right) = 0.0142\ m^3/s$$

$$V_1 = \frac{Q_1}{A_1} = \frac{(0.0142\ m^3/s)}{(8.213 \cdot 10^{-3}\ m^2)} = 1.729\ m/s$$

$$N_{R,1} = \frac{V_1 D_1}{\nu} = \frac{(1.729\ m/s)(0.1023m)}{(1.03 \cdot 10^{-6}\ Pa \cdot s)} = 1.72 \cdot 10^5$$

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$$RR = \frac{D_1}{\varepsilon} = \frac{(0.1023m)}{(4.6 \cdot 10^{-5} m)} = 2223$$

$$f_1 = \frac{0.25}{\left(\log\left(\frac{1}{3.7RR} + \frac{5.74}{N_R^{0.9}}\right)\right)^2} = \frac{0.25}{\left(\log\left(\frac{1}{3.7(2223)} + \frac{5.74}{(1.72 \cdot 10^5)^{0.9}}\right)\right)^2} = 0.0189$$

$$h_{L_1} = 2\left(0.0189 \cdot 20 \cdot \frac{1.729 m/s^2}{2(9.81 m/s^2)}\right) + f_2 \frac{(30m)}{(0.0525m)} \cdot \frac{v_2^2}{2(9.81 m/s^2)}$$

$$= 0.1152 + f_2 \cdot 29.1248\, v_2^2$$

$$h_{L_2} = 2\left(0.0189 \cdot 60 \cdot \frac{1.729 m/s^2}{2(9.81 m/s^2)}\right) + \frac{3f_3 \cdot 90 \cdot 3 v_3^2}{2(9.81 m/s^2)} + \frac{f_3 \cdot 150\, v_3^2}{2(9.81 m/s^2)} + f_3\left(\frac{60m}{0.0525m}\right)\frac{v_3^2}{2(9.81)}$$

$$= 0.3456 + f_3 \cdot v_3^2 (41.28 + 7.645 + 58.25)$$

$$= 0.3456 + f_3 \cdot 107.18\, v_3^2$$

- SET ENERGY LOSS EQUATIONS EQUAL TO EACH OTHER, SOLVE FOR v_2:

$$0.1152 + f_2 \cdot 29.1248\, v_2^2 = 0.3456 + f_3 \cdot 107.18\, v_3^2$$

$$v_2 = \sqrt{\frac{0.2304 + f_3 \cdot 107.18\, v_3^2}{29.1248 f_2}} \quad \text{——— (1)}$$

- USE CONSERVATION OF MASS RELATIONSHIP:

$$Q_1 = Q_2 + Q_3 = A_2 v_2 + A_2 v_3$$

$$= 0.002165\left(v_3 + \sqrt{\frac{0.2304 + f_3 \cdot 107.18\, v_3^2}{29.1248\, f_2}}\right) \quad \text{——— (2)}$$

- USING ITERATIVE PROCESS IN EXCEL, f_3 AND f_2 WERE FOUND TO BE:

$f_2 = 0.01849$ $\quad f_3 = 0.01988$

- ITERITIVE PROCESS ALSO USED FOR v_2 AND v_3:

$v_2 = 4.38\ m/s$ $\quad v_2 = 2.18\ m/s$

- SOLVE FOR Q_2 AND Q_3:

$$Q_2 = A_2 \cdot v_2 = (0.00216 5m^2)(4.38\ m/s) = 0.00948\ m^3/s = \boxed{569\ L/min}$$

$$Q_3 = A_2 \cdot v_3 = (0.002165 m^2)(2.18\ m/s) = 0.00472\ m^3/s = \boxed{283\ L/min}$$

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- CALCULATE ΔP:

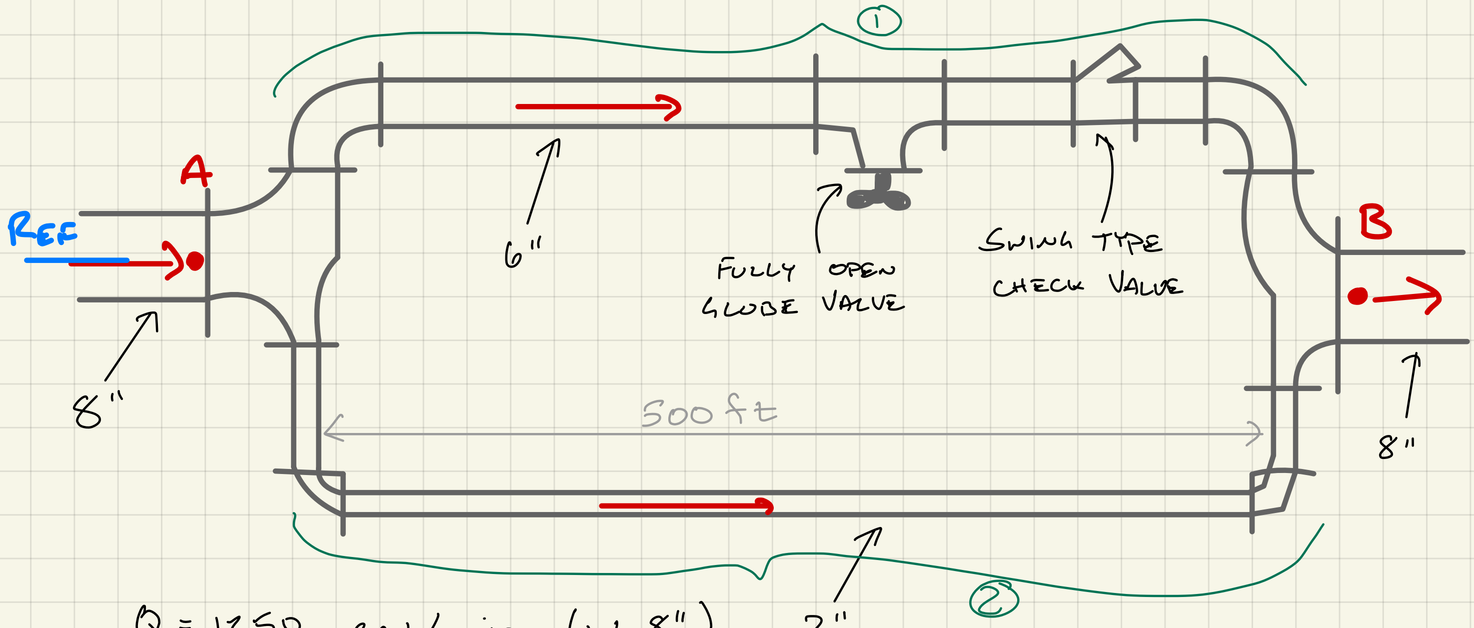
$$\frac{\Delta P}{\gamma} = h_{L1} \quad \therefore \quad \Delta P = h_{L1} \cdot \gamma$$

$$\Delta P = 0.1152 + f_2 \cdot 29.1248\, v_2^2 = 0.1152 + 0.1849 \cdot 29.1248 (4.38\ \text{m/s})^2$$

$$\boxed{= 103.4\ \text{kPa}}$$

12-4 **GIVEN:**

BENZENE IS FLOWING IN THE SYSTEM BELOW:



$Q = 1350$ gal/min (in 8")

$Sg_{Ben} = 0.87$

$T = 140°F$

ALL PIPES = SCH 40

REQUIRED:

CALCULATE THE VOLUME FLOW RATE IN 2" AND 6" PIPES

SOLUTION:

- PROBLEM WILL BE SOLVED USING BERNOULLI'S EQUATION

$$\cancel{h_p} + \frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_1} = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + h_L + \cancel{z_2}$$

- THERE IS NO PUMP HEAD, V_A AND V_B ARE EQUAL AND CANCEL EACH OTHER OUT, AND BOTH POINTS LIE ON THE REFERENCE LINE, SO z_1 AND z_2 ARE 0.

$$\frac{P_A - P_B}{\gamma_{BEN}} = h_L \quad \therefore \quad \frac{\Delta P}{\gamma_{BEN}} = h_L$$

- ENERGY LOSS CAN BE FOUND FROM EITHER PATH:

$$\frac{\Delta P}{\gamma_{BEN}} = h_{L,1} \qquad \frac{\Delta P}{\gamma_{BEN}} = h_{L,2}$$

12.4 cont

- Losses for branch 1:

$$h_{L1} = 2\left(K_{Tee} f_{T_A} \frac{V_A^2}{2g}\right) + f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g} + 2\left(K_{elb} f_{T_1} \frac{V_1^2}{2g}\right) + K_{globe} f_{T_1} \frac{V_1^2}{2g} + K_{valve} f_{T_1} \frac{V_1^2}{2g}$$

$$= 2\left(K_{Tee} f_T \frac{V_A^2}{2g}\right) + \left(f_1 \frac{L_1}{D_1} + f_T\left(2K_{elb} + K_{globe} + K_{valve}\right)\right)\frac{V_1^2}{2g}$$

- Substitute V for Q values:

$$Q = AV \therefore V = \frac{Q}{A} = \frac{4Q}{\pi D^2}$$

$$h_{L,1} = 2\left(K_{Tee} f_{T_A} \frac{16 Q_A^2}{\pi^2 D_A^2 \cdot 2g}\right) + \left(f_1 \frac{L_1}{D_1} + f_{T_1}\left(2K_{elb} + K_{globe} + K_{valve}\right)\right)\frac{16 Q_1^2}{\pi^2 D_1^4 \cdot 2g} \quad (1)$$

- Losses for branch 2:

$$h_{L,2} = 2\left(K_{Tee} f_{TA} \frac{V_A^2}{2g}\right) + f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} + 2\left(K_{elb} f_{T,2} \frac{V_2^2}{2g}\right)$$

$$= 2\left(K_{Tee} f_{TA} \frac{V_A^2}{2g}\right) + \left(f_2 \frac{L_2}{D_2} + 2K_{elb} f_{T,2}\right)\frac{V_2^2}{2g}$$

$$= 2\left(K_{Tee} f_{TA} \frac{16 Q^2}{\pi^2 D^4 \cdot 2g}\right) + \left(f_2 \frac{L_2}{D_2} + 2K_{elb} f_{T,2}\right)\frac{16 Q^2}{\pi^2 D^4 \cdot 2g} \quad (2)$$

- Energy loss is the same for both branches, so:

$$h_{L,1} = h_{L,2}$$

- Due to conservation of mass, Q_A is:

$$Q_A = Q_1 + Q_2$$

- Q_A is known, solving $h_{L,1} = h_{L,2}$ for Q_1 will allow for the creation of a system of equations:

$$\cancel{2\left(K_{Tee} f_{TA} \frac{16 Q_A^2}{\pi^2 D_A^4 \cdot 2g}\right)} + \left(f_1 \frac{L_1}{D_1} + f_{T,1}\left(2K_{elb} + K_{globe} + K_{valve}\right)\right)\frac{\cancel{16} Q_1^2}{\cancel{\pi^2} D_1^4 \cdot \cancel{2g}} =$$

$$\cancel{2\left(K_{Tee} f_{TA} \frac{16 Q_A^2}{\pi^2 D_A^4 \cdot 2g}\right)} + \left(f_2 \frac{L_2}{D_2} + 2K_{elb} f_{T,2}\right)\frac{\cancel{16} Q_2^2}{\cancel{\pi^2} D_2^4 \cdot \cancel{2g}}$$

$$Q_1 = \sqrt{\frac{\left(f_2 \frac{L_2}{D_2} + 2K_{elb} f_{T,2}\right) Q_2^2 \cdot D_1^4}{f_1 \frac{L_1}{D_1} + f_{T,1}\left(2K_{elb} + K_{globe} + K_{valve}\right)}}$$

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$$Q_A = \sqrt{\frac{\left(f_2 \frac{L_2}{D_2} + 2K_{elb} f_{T,2}\right) Q_2^2 \cdot D_1^4}{f_1 \frac{L_1}{D_1} + f_{T,1}(2K_{elb} + K_{SWING} + K_{VALVE})}} + Q_2$$

- USING ITERATIONS IN EXCEL, THE FRICTION FACTORS f_1 AND f_2 WERE FOUND TO BE:

$$f_1 = 0.01726 \qquad f_2 = 0.01922$$

- USING THE ABOVE FRICTION FACTORS, Q_2 WAS DETERMINED TO BE:

$$Q_2 = \boxed{2.166 \ ft^3/s}$$

- USING CONSERVATION OF MASS, Q_1 IS:

$$Q_A = Q_1 + Q_2 \quad \therefore \quad Q_1 = Q_1 - Q_A$$

$$Q_1 = (3.0078 \ ft^3/s) - (2.166 \ ft^3/s) = \boxed{0.842 \ ft^3/s}$$