Test 2
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MET 330 Fluid Mechanics
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Problem 1

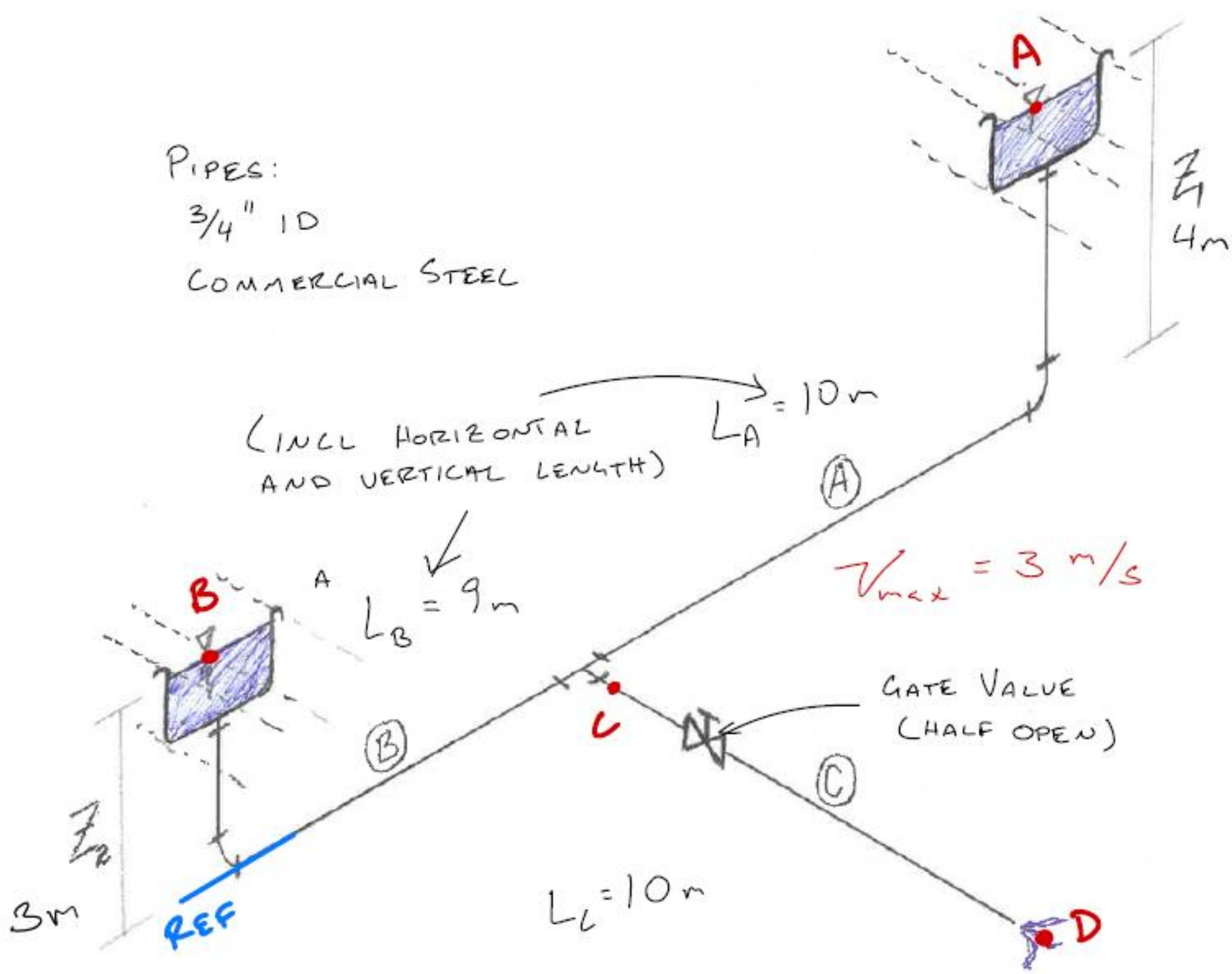
Problem Statement:

A house is designed with two elevated gutters along the sides. Each gutter drains water through pipes that are connected to bring the rainwater to ground level. In the worst-case scenario, the pipes and gutters are completely full of running water. A valve was installed in the ground level pipe. The friction factor is assumed to be only a function of the relative roughness. Determine the flow out of the system if the gate valve is half open. Do not neglect minor losses. Is the velocity criterion violated? If so, discuss potential solutions. Compute the pressure at the exit of the tee.

Purpose:

Calculate the flow in the system, the pressure at the exit of the tee, and evaluate whether the system is within the bounds of V_{max}.

Drawing:



Design Considerations:

- Isothermal Process
- Incompressible fluid
- Atmospheric pressure and temperature assumed to be at sea level and room temperature
- The system is in a steady state of flow
- Velocity at the top of the water level in the gutters is assumed to be negligible

Data and Variables:

- Dimensions are provided in the drawing
- The working fluid is water, properties found in Table A.1 of textbook

$$\gamma_w = 9.81\ {kN}/{m^3}$$

- Roughness of commercial steel pipes located in Table 8.2 of textbook

$$\varepsilon = 4.6 \cdot 10^{-5} m$$

- Friction factors for sources of minor losses in the system:
    - Entrance Losses, sudden:

    $$K = 0.5$$

    - Elbows (standard, 90°):

    $$K = 30 f_T$$

    - Tee with flow through branch:

    $$K = 60 f_T$$

    - Gate Valve, half open:

    $$K = 160 f_T$$

Procedure:

- The system is to be evaluated using Bernoulli's equation, where point A, B, C, and D have been marked on the drawing, and the reference height is the ground.

$$h_p + \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + h_L + z_2$$

    - The equation can be adjusted according to the area being investigated to calculate the flow in the system and the pressure exiting the tee.

- Looking first at the relationship between points A and B, the pipes are set up in a parallel fashion. This creates a situation where the head losses from the pipes are equal. The flow in the pipes will be different due to the difference in elevation, but because they come together at the tee, they must have the same overall energy loss.

$$h_{L,A} = h_{L,B}$$

- Head losses will be calculated using K factors for each source of friction in the system

$$Pipe\ Losses = f_T \cdot \frac{L}{D} \cdot \frac{V^2}{2g}$$

$$Minor\ Losses = K \cdot \frac{V^2}{2g}$$

  - As per the problem statement, friction will be treated as a function of relative roughness, meaning that all the flow in the system is completely turbulent. Friction factors will be determined using the relative roughness equation and the Moody diagram.

$$Relative\ Roughness = \frac{D}{\varepsilon}$$

- The flow at point D must be evaluated for each branch. Because the head losses are equal, the difference in those evaluations will be determined by the different heights of the gutters and the different velocity of the water in each branch. The flows rates have the following relationship due to conservation of mass:

$$Q_D = Q_A + Q_B$$

  - Because all the pipes are the same diameter, this equation can be further simplified:

$$Q = A \cdot V \quad \therefore \quad A \cdot V_D = A \cdot V_A + A \cdot V_B$$

$$V_D = V_A + V_B$$

- Evaluating for the pressure at the tee exit requires another iteration of Bernoulli's equation, looking at either point A and B on one side, and point C on the other and solving for the pressure at C. Because the velocity of both branches and A and B will be calculated to find the flow at D, all variables except the pressure at C will be known.

Calculations

- Convert diameter to meters to calculate the problem more easily:

$$D = 0.75'' \cdot \left(\frac{.0254m}{1''}\right) = \boxed{0.01905m}$$

- Setup head loss equation:

$$h_{L,A}=h_{L,B}$$

$$h_{L,A}=f_T\frac{L_A}{D}\cdot\frac{{V_A}^2}{2g}+K_{entrance}\cdot\frac{{V_A}^2}{2g}+K_{elbow}\cdot\frac{{V_A}^2}{2g}+K_{tee}\cdot\frac{{V_A}^2}{2g}$$

$$=\left(f_T\frac{L_A}{D}+0.5+30f_T+60f_T\right)\cdot\frac{{V_A}^2}{2g}$$

$$=\left(f_T\frac{L_A}{D}+90f_T+0.5\right)\cdot\frac{{V_A}^2}{2g}$$

$$h_{L,B}=f_T\frac{L_B}{D}\cdot\frac{{V_B}^2}{2g}+K_{entrance}\cdot\frac{{V_B}^2}{2g}+K_{elbow}\cdot\frac{{V_B}^2}{2g}+K_{tee}\cdot\frac{{V_B}^2}{2g}$$

$$=\left(f_T\frac{L_B}{D}+0.5+30f_T+60f_T\right)\cdot\frac{{V_B}^2}{2g}$$

$$=\left(f_T\frac{L_B}{D}+90f_T+0.5\right)\cdot\frac{{V_B}^2}{2g}$$

$$f_T\left(\frac{L_A}{D}+90.5\right)\cdot\frac{{V_A}^2}{2g}=f_T\left(\frac{L_B}{D}+90.5\right)\cdot\frac{{V_B}^2}{2g}$$

- Calculate relative roughness and determine friction factor on Moody's diagram to simplify the above equation:

$$Relative\ Roughness=\frac{D}{\varepsilon}=\frac{0.01905m}{4.6\cdot10^{-5}m}=\boxed{414.13}$$

$$f_T\cong\boxed{0.025}$$

- Simplify head loss equation and solve for V_A:

$$\left((0.025)\frac{10m}{0.01905m}+(0.025)(90)+0.5\right)\cdot\frac{{V_A}^2}{2\left(9.81{}^{m}/_{s^2}\right)}=\left((0.025)\frac{9m}{0.01905m}+(0.025)(90)+0.5\right)\cdot\frac{{V_B}^2}{2\left(9.81{}^{m}/_{s^2}\right)}$$

$$0.8090{V_A}^2=0.7422{V_B}^2$$

$$V_A=\sqrt{0.9173{V_B}^2}$$

  - *Note: minor losses in pipes A and B were the same and cancelled each other out*

- Build equation for V_D using V_A for use in Bernoulli's equation:

$$V_D = V_A + V_B$$

$$V_D = \sqrt{0.9173 V_B^2} + V_B = V_B\left(\sqrt{0.9173} + 1\right)$$

- Setup Bernoulli's equation for points B and D; use equivalence defined above for V_D

$$\frac{V_B^2}{2g} + z_2 = \frac{\left(V_B \cdot \left(\sqrt{0.9173} + 1\right)\right)^2}{2g} + \left(h_{L,B} + h_{L,C}\right)$$

$$h_{L,B} = \left(f_T \frac{L_B}{D} + f_T 90 + 0.5\right) \cdot \frac{V_B^2}{2g} = 0.7422 V_B^2$$

$$h_{L,C} = f_T\left(\frac{L_c}{D} + 160\right) \cdot \frac{\left(V_B\left(\sqrt{0.9173} + 1\right)\right)^2}{2g} = 3.3451 V_B^2$$

$$\frac{V_B^2}{2g} + z_2 = \frac{\left(V_B \cdot \left(\sqrt{0.9173} + 1\right)\right)^2}{2g} + 4.0873 V_B^2$$

- Simplify and solve for V_B

$$\frac{V_B^2}{2 \cdot 9.81 m/s^2} + 3m = \frac{\left(V_B \cdot \left(\sqrt{0.9173} + 1\right)\right)^2}{2 \cdot 9.81 m/s^2} + 4.0873 V_B^2$$

$$4.2317 V_B^2 = 3m$$

$$V_B = \sqrt{\frac{3m}{4.2317}} = \boxed{0.842 m/s}$$

- Solve for V_A

$$V_A = \sqrt{0.9173 V_B^2} = \sqrt{0.9173 \cdot 0.842^2} = \boxed{0.806\ m/s}$$

- Solve for V_D

$$V_D = V_A + V_B = 0.842 m/s + 0.806 m/s = \boxed{1.648 m/s}$$

  - This does not violate the V_{max} of 3 m/s

- Calculate flow rate Q_D

$$Q_D = A_D \cdot V_D = \pi\left(\frac{D}{2}\right)^2 \cdot V_D = \pi\left(\frac{0.01905\mathrm{m}}{2}\right)^2 \cdot 1.648\mathrm{m/s} = \boxed{0.00047\mathrm{m}^3/\mathrm{s}}$$

- Calculate the pressure at the end of the tee, point C using Bernoulli's equation between point A and point C:

$$z_1 = \frac{P_c}{\gamma} + \frac{V_D}{2g} + h_{L,A}$$

  - *Note: point A has no velocity and is at atmospheric pressure, so the velocity and pressure terms have been eliminated. Point C is at the reference height and has no z value. The resulting answer will be a gauge pressure with atmospheric pressure being the reference.*

- Solve for P_C

$$P_c = \left(z_1 - \frac{V_D}{2g} - h_{L,A} \right) \gamma$$

$$h_{L,A} = 0.8090 \cdot V_A{}^2 = 0.7842 \cdot \left(0.806 \,{}^{m}\!/\!_{s} \right)^2 = 0.526m$$

$$\therefore$$

$$P_c = \left(4m - \frac{\left(1.648 \,{}^{m}\!/\!_{s} \right)^2}{2 \cdot 9.81 \,{}^{m}\!/\!_{s^2}} - 0.526m \right) 9.81 \,{}^{kN}\!/\!_{m^3} = \boxed{32.72kPa}$$

Summary

The velocity of the fluid at the end of pipe C is 1.648 m/s, which is less than the stated V_{max} of 3 m/s, meaning that the system is not operating above its critical velocity. The flow rate at the end of the pipe is 0.00047 m^3/s. The pressure at the end of the tee is 32.72 kPa. Opening the valve further would alleviate this pressure by increasing the flow rate of the entire system and the velocity at point D.

Problem 2

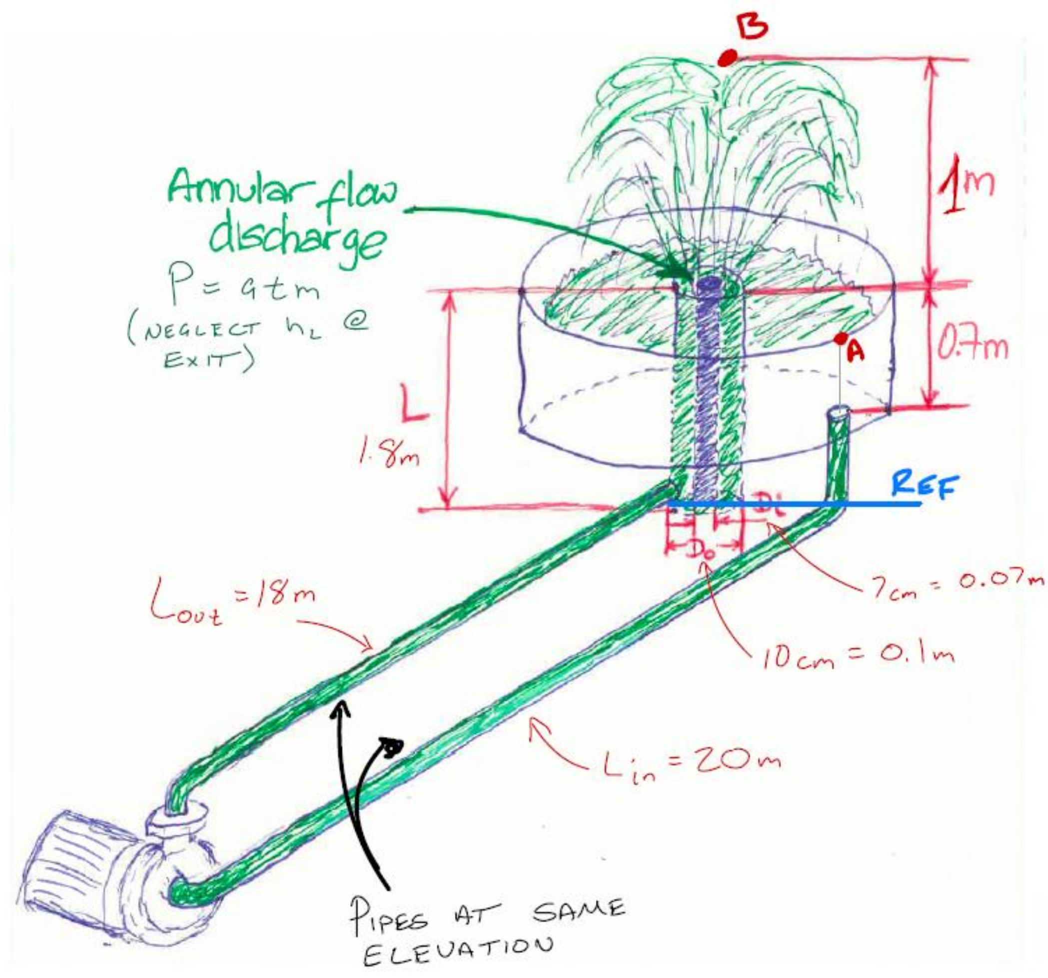
Problem Statement:

You oversee designing a new fountain at ODU. It consists of a water reservoir and piping to and from the pump as shown in the figure. The pipes shall be PVC. The outlet line leads to the bottom of an annular flow line (use the hydraulic radius for the energy loss calculations of such an annular flow passage). There is a negligible loss at the exit of the annulus, which is exposed to the atmosphere. Consider all other minor losses. What is the pump power required for the flow configuration shown (the water reaches 1m above the annular exit)? If the pump-motor combination has an efficiency of 92% determine the electrical power requirements.

Purpose:

Determine the design requirements for the fountain, including the diameter of the PVC pipes, the power of the pump necessary to drive the fountain, and the electrical power required if the pump-motor system operates at 92% efficiency.

Drawing:



Design Considerations:

- Isothermal Process
- Incompressible fluid
- Atmospheric pressure and temperature assumed to be at sea level and room temperature
- The system is in a steady state of flow

Data and Variables:

- Dimensions are provided in the drawing
- Friction coefficients necessary:

$K_{elbow} = 30 f_T$

$K_{entrance,\ \mathrm{tank}} = 0.5 f_T$

$K_{annular\ flow\ entrance} = 2$

- Properties of water were found in Table A.1 in the textbook

$\gamma_w = 9.81\ {kN}/{m^3}$

$\nu = 1.3 \cdot 10^{-6}\ {m^2}/{s}$

- Properties of PVC pipes were found in Table G.3 and Table 8.2 in the textbook

$\varepsilon = 3.0 \cdot 10^{-7} m$

Procedure:

- To start this project, the required velocity to create the fountain display desired and the pipe size to accommodate it must be decided upon.
    - To determine the velocity necessary to propel the water 1m above the annulus, the conservation of kinetic energy principle will be used:

      $PE = KE$

      $$\gamma g h = \frac{1}{2} \gamma V^2 \quad \therefore \quad V = \sqrt{2gh}$$

- Using this calculated velocity, the necessary flow rate can be determined. The flow rate within the system will be constant. Calculating the flow rate will provide the information necessary to choose the correct pipe diameters and calculate the pump characteristics.

- Bernoulli's equation, applied between the points A and B on the drawing, with the level of the underground pipes as the reference, will be used to calculate the properties of the pump.

$$h_p + \frac{P_A}{\gamma} + \frac{{V_A}^2}{2g} + z_A = \frac{P_B}{\gamma} + \frac{{V_B}^2}{2g} + h_L + z_B$$

- Both points are exposed to atmosphere and therefore P_A and P_B cancel each other out. Both points are also considered to have a velocity of 0 as the fluid level in the tank/pond will change very slowly and the water shooting up to point B is at its apex and therefore has a velocity of 0. The V_A and V_B terms also cancel. The system has a pump adding power, so h_P remains, there are major and minor losses, so h_L remains, and both points are above the reference level so z_A and z_B remain.

$$h_p + z_A = h_L + z_B$$

- This system is set up as a series, so the losses will be additive. With point B placed at the peak of the shooting water, the calculated pump head will incorporate the power necessary for the fountain to function correctly.
    - This step will require the calculation of the friction coefficient for each stage of the system: inlet, outlet, and annulus. This will require Reynold's number, the relative roughness, and the Moody diagram.
- Reynolds number:

$$N_R = \frac{VD}{v} \quad (for\ circular\ pipes) \qquad N_R = \frac{V(4R)}{v} \quad (for\ the\ annulus)$$

- Relative Roughness:

$$RR = \frac{D}{\varepsilon} \quad (for\ circular\ pipes) \qquad RR = \frac{4R}{\varepsilon} \quad (for\ the\ annulus)$$

    - *Note: R for the annulus is calculated by:*

$$R = \frac{A}{Wet\ Perimeter}$$

- To calculate the electrical power needed to power the pump if it runs at 92% efficiency, the power equation will be used:

$$P = \gamma Q h_p$$

$$\eta = \frac{P_{out}}{P_{in}} \quad \therefore \quad P_{in} = \frac{P_{out}}{\eta}$$

Calculations:

- To select the pipe diameter, Figure 6.3 in the textbook will be utilized. Selecting the diameter requires first calculating the necessary velocity as the water exits the annulus, and then calculating the flow rate necessary to achieve that velocity. The water must gain 2.8m in height, so:

$$V_{annulus} = \sqrt{2 \cdot 9.81 m/s^2 \cdot 2.8m} = \boxed{7.41 m/s}$$

$$Q = A_{annulus} \cdot V_{annulus}$$

$$A_{annulus} = \frac{\pi}{4}\left(D_o^{\,2} - D_i^{\,2}\right) = \frac{\pi}{4}\left((0.1m)^2 - (0.07m)^2\right) = 0.004006m^2$$

$$Q = \left(0.004006m^2\right) \cdot \left(7.41 m/s\right) = \boxed{0.02968\ m^3/s} \cdot \left(\frac{3600s}{1hr}\right) = 106.8\ m^3/h$$

- Using the flow rate, the proper pipe diameter can be selected from Figure 6.3

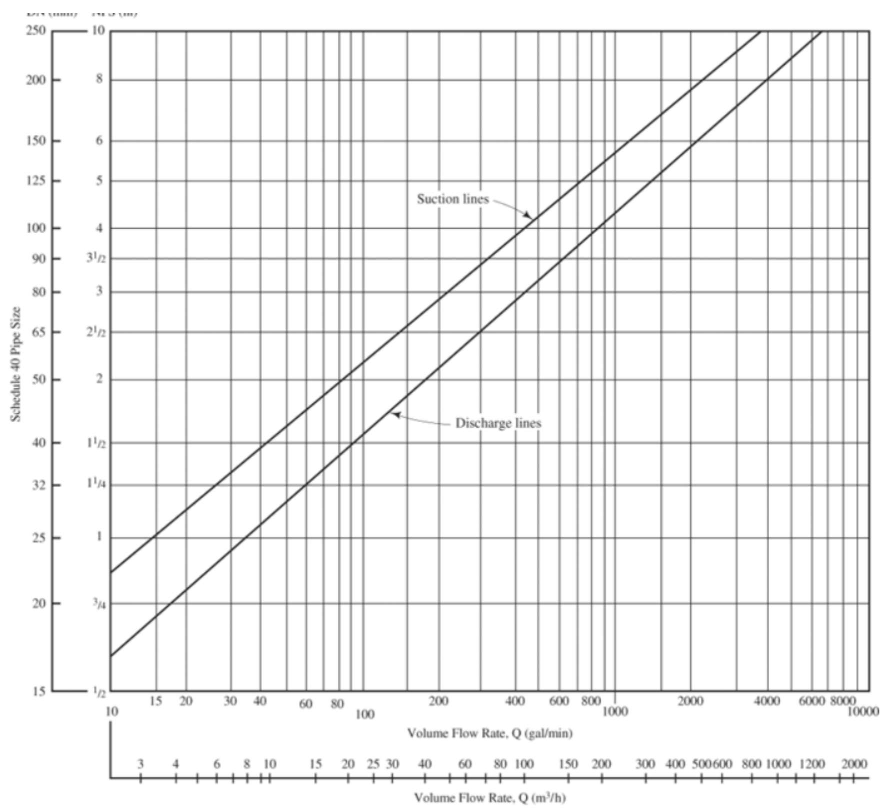


Figure 6.3
Pipe-size selection aid.

  - Suction pipe size (line in): 118.8mm ID, 125mm OD PVC
  - Discharge pipe size (line out): 84.4mm ID, 90mm OD PVC
  - *Note: sizes selected err on the large side and are determined from Table G.3 using the DN sizes on Figure 6.3 as a guide*

- Having calculated the flow rate and chosen a pipe diameter for the suction and discharge lines, Bernoulli's equation can be used to calculate the pump head.

$$h_p = h_L + (z_B - z_A) \qquad \textbf{(1)}$$

- The z values are given, but h_L must be calculated:

$$h_L = K_{\text{tank } entrance} \cdot \frac{V_{in}^2}{2g} + 2\left(K_{elbow} \cdot \frac{V_{in}^2}{2g}\right) + K_{elbow} \cdot \frac{V_{out}^2}{2g} + K_{ann.\ entrance} \cdot \frac{V_{out}^2}{2g}$$
$$+ f_{in}\left(\frac{L_{in}}{D_{in}} \cdot \frac{V_{in}^2}{2g}\right) + f_{out}\left(\frac{L_{out}}{D_{out}} \cdot \frac{V_{out}^2}{2g}\right) + f_{ann.}\left(\frac{L_{ann.}}{\frac{4R}{\varepsilon}} \cdot \frac{V_{ann}^2}{2g}\right)$$

- Simplified, V replaced with equivalent Q:

$$h_L = \left(K_{\text{tank}} + 2K_{elb} + f_{in} \cdot \frac{L_{in}}{D_{in}}\right) \cdot \frac{Q^2}{A_{in}^2 \cdot 2g} + \left(K_{elb} + K_{ann.\ ent.} + f_{out} \cdot \frac{L_{out}}{D_{out}}\right) \cdot \frac{Q^2}{A_{out}^2 \cdot 2g}$$
$$+ f_{ann.}\left(\frac{L_{ann}}{\frac{4R}{\varepsilon}}\right) \cdot \frac{Q^2}{A_{ann.}^2 \cdot 2g} \qquad \textbf{(2)}$$

- Calculate the area of the inlet, outlet and annulus pipes:

$$A_{in} = \pi\left(\frac{\text{D}}{2}\right)^2 = \pi\left(\frac{0.1188\text{m}}{2}\right)^2 = 0.01109\text{m}^2$$

$$\text{A}_{out} = \pi\left(\frac{0.0844m}{2}\right)^2 = 0.005595\text{m}^2$$

$$\text{A}_{\text{ann.}} = 0.004006\text{m}^2$$

- Calculate V_{in} and V_{out} using Q and areas:

$$V_{in} = \frac{Q}{A_{in}} = \frac{0.02968 m^3/s}{0.01109 m^2} = 2.676\ m/s$$

$$V_{out} = \frac{Q}{A_{out}} = \frac{0.02968 m^3/s}{0.005595 m^2} = 5.305 m/s$$

- The friction factor must be found for each stage of the system using Reynolds number and the relative roughness of the pipes.
    - For the circular pipes (inlet, outlet)

$$RR_{in} = \frac{D_{in}}{\varepsilon} = \frac{0.1188m}{3 \cdot 10^{-7}m} = 3.96 \cdot 10^5$$

$$RR_{out} = \frac{D_{out}}{\varepsilon} = \frac{0.0844m}{3 \cdot 10^{-7}m} = 2.81 \cdot 10^5$$

$$N_{R,\ in} = \frac{V_{in} D_{in}}{v} = \frac{\left(2.676 {}^{m}\!/_{s}\right)(0.1188m)}{1.3 \cdot 10^{-6} {m^2}\!/_{s}} = 2.44 \cdot 10^5$$

$$N_{R,\ out} = \frac{V_{out} D_{out}}{v} = \frac{\left(5.305 {}^{m}\!/_{s}\right)(0.0844m)}{1.3 \cdot 10^{-6} {m^2}\!/_{s}} = 1.74 \cdot 10^5$$

$$f_{in} = \boxed{0.0147}$$

$$f_{out} = \boxed{0.016}$$

- For the annulus:

$$R = \frac{A}{WP} = \frac{0.004006m^2}{\pi(0.1\text{m} + 0.07\text{m})} = 0.007501m$$

$$RR_{ann.} = \frac{4R}{\varepsilon} = \frac{4 \cdot (0.007501m)}{3 \cdot 10^{-7}m} = 1.00 \cdot 10^5$$

$$N_{R,\ ann.} = \frac{V_{ann.}(4R)}{v} = \frac{\left(7.41 {}^{m}\!/_{s}\right) \cdot 4 \cdot (0.007501m)}{1.3 \cdot 10^{-6} {m^2}\!/_{s}} = 1.71 \cdot 10^5$$

$$f_{ann.} = \boxed{0.0158}$$

- *Note: for PVC pipes, the relative roughness is very high, and the pipes are considered smooth. At the values calculated, the friction factor is only a function of the Reynolds number. The K values for the minor losses will be found with the equivalent length method instead of the f_T method due to this.*

- Solve equation **(2)**:

$$h_L = \left(0.5 + 0.0147 \cdot 2 \cdot 30 + 0.0147 \cdot \frac{20m}{0.1188m}\right) \cdot \frac{\left(0.02968 \, {}^{m^3}\!/_{s}\right)^2}{\left(0.01109m^2\right)^2 \cdot 2\left(9.81 \, {}^{m}\!/_{s^2}\right)}$$

$$+ \left(0.016 \cdot 30 + 2 + 0.016 \cdot \frac{18m}{0.0844m}\right) \cdot \frac{\left(0.02968 \, {}^{m^3}\!/_{s}\right)^2}{\left(0.005595m^2\right)^2 \cdot 2\left(9.81 \, {}^{m}\!/_{s^2}\right)}$$

$$+ (0.0158)\left(\frac{1.8m}{4 \cdot 0.007501m}\right) \cdot \frac{\left(0.02968 \, {}^{m^3}\!/_{s}\right)^2}{\left(0.004006m^2\right)^2 \cdot 2\left(9.81 \, {}^{m}\!/_{s^2}\right)}$$

$$= \boxed{12.511m}$$

- Solve equation **(1)**:

$$h_p = h_L + \left(z_B - z_A\right) = (12.511m) + (2.8m - 1.8m) = \boxed{13.511m}$$

- Find the power needed from the pump:

$$P = \gamma Q h_p = \left(9810 \, {}^{N}\!/_{m^3}\right) \cdot \left(0.02968 \, {}^{m^3}\!/_{s}\right) \cdot (13.511m) = 3934 \; W = \boxed{3.93 \; kW}$$

- Find the energy needed to run the pump if it has an efficiency of 92%:

$$P_{in} = \frac{P_{out}}{\eta} = \frac{3.93 \; kW}{0.92} = \boxed{4.27 \; kW}$$

Summary

For the water to reach 1m above the annulus, the velocity at the exit of the annulus must be at least 7.41 m/s, which equates to a flow rate of 0.02968 m^3/s. Using PVC pipes with sufficient wall thickness and internal diameter to contain the pressure in the system and maintain the required flow rate meant the suction pipe should have an internal diameter of 118mm while the discharge pipe should have an internal diameter of 84.4mm (as found using Figure 6.3 and Table G.3). The pump will need to output 3.93 kW of power to maintain the necessary pump head to reach the annular exit at the required velocity, and if it is 92% efficient, it will require 4.27 kW of power to run at that level.