Test 3
MET 335 Fluid Mechanics
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Problem 1

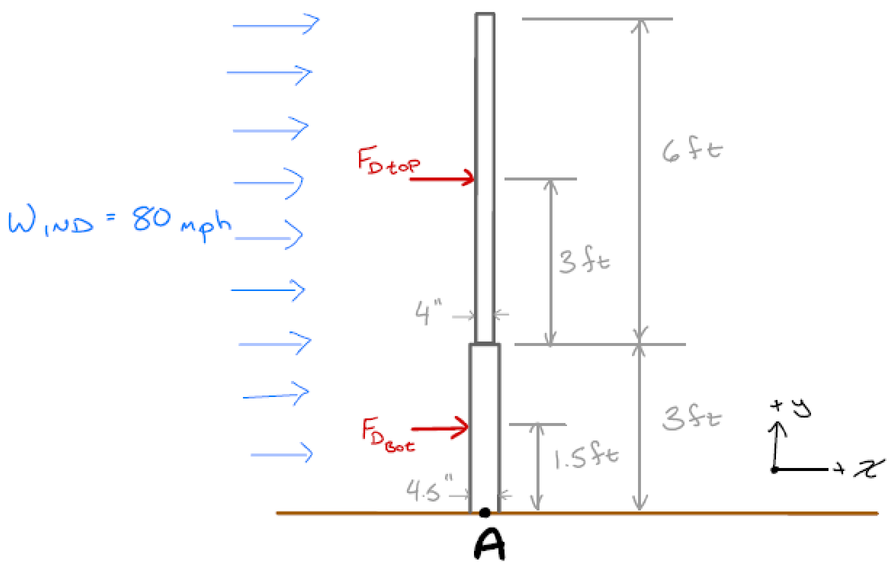
Problem Statement:

A moment is created on a pole by wind. The pole consists of two different lengths of pipe connected end to end. The lower cylinder is a 4-inch nominal schedule 40 steel pipe with a length of 3 feet, and the upper cylinder is 3.5-inch nominal schedule 40 steel pipe with a length of 6 feet. The air temperature is 71°F.

Purpose:

Calculate the moment created by the wind on the pole.

Drawing:



Design Considerations:

- Isothermal process
- Incompressible fluid
- Atmospheric pressure is assumed to be sea level
- The system has a steady state of flow

Data and Variables:

- Length dimensions of the pole are on the drawing
- Wind speed:

$$V = 80mph \cdot \left(\frac{5280ft}{3600s}\right) = 117.3 ft/s$$

- Air density at 71°F:
  - Found using Table E.2:

$$\frac{71^\circ F - 60^\circ F}{80^\circ F - 60^\circ F} = \frac{\rho_{@71^\circ F} - 2.37 \cdot 10^{-3} slugs/_{ft^3}}{2.28 \cdot 10^{-3} slugs/_{ft^3} - 2.37 \cdot 10^{-3} slugs/_{ft^3}}$$

$$\rho_{@71^\circ F} = \boxed{2.32 \cdot 10^{-3} slugs/_{ft^3}}$$

- Kinematic viscosity of air at 71°F:
  - Found using Table E.2:

$$\frac{71^\circ F - 60^\circ F}{80^\circ F - 60^\circ F} = \frac{v_{71^\circ F} - 1.58 \cdot 10^{-4} ft^2/_s}{1.69 \cdot 10^{-4} ft^2/_s - 1.58 \cdot 10^{-4} ft^2/_s}$$

$$v_{71^\circ F} = \boxed{1.64 \cdot 10^{-4} ft^2/_s}$$

- Pole Diameters (from table F.1 in textbook):

$$D_{top} = 4\ in \qquad D_{bottom} = 4.5\ in$$

Procedure:

- The purpose of the problem is to calculate the moment created where the pole meets the ground. To do that, the forces will be evaluated using a sum of moments equation:

$$\Sigma M_A = F_d \cdot d_d$$

- The force acting on the pole is the drag created by the surface of the pipes that are facing the wind. The force of drag can be calculated using:

$$F_D = C_D \left(\frac{\rho V^2}{2} \right) A$$

  - The C_D variable will require a chart to find. The chart requires that the Reynolds number for the flow surrounding the pole be calculated. It can be found with:

$$N_R = \frac{VD}{v}$$

  - Because the pipes are different diameters, the surface area creating drag will be different for both. They will be treated as two distinct forces with the drag acting at their midpoint to simplify the moment equation.
- Once the C_D is established, the force created by the wind can be calculated

Calculations:

- Starting with the moment equation, setup how the forces are related to the moment at point A:

$$\circlearrowleft \Sigma M_A = F_{D,\ top} \cdot d_{top} + F_{D,bot} \cdot d_{bot} \qquad \textbf{(1)}$$

- The distances are already established on the drawing. To calculate the force due to drag, we will use the drag force equation mentioned in the procedure. To complete that equation, the drag coefficient (C_D) must be found for both pipe diameters, which will require the Reynolds number:

$$N_{R,top} = \frac{V_{air} \cdot D_{top}}{v_{air}} = \frac{117.3 ft/_s \cdot 0.333 ft}{1.64 \cdot 10^{-4} ft^2/_s} = \boxed{2.38 \cdot 10^5}$$

$$N_{R,bot} = \frac{V_{air} \cdot D_{bot}}{v_{bot}} = \frac{117.3 ft/_s \cdot 0.375 ft}{1.64 \cdot 10^{-4} ft^2/_s} = \boxed{2.68 \cdot 10^5}$$

- Using Figure 17.4 in the textbook, the drag coefficient for both poles:

$C_{D,top} = 1.1$ $\qquad$ $C_{D,bot} = 1$

- The drag force for both poles can now be calculated:

$$F_{D,\ top} = C_D\left(\frac{\rho V_{air}^{\ 2}}{2}\right)A_{top} = 1.1\left(\frac{2.32 \cdot 10^{-3} slugs/_{ft^3} \cdot (117.3 ft/_s)^2}{2}\right)(6ft \cdot 0.333ft)$$
$$= \boxed{35.08 lbf}$$

$$F_{D,bot} = 1\left(\frac{2.32 \cdot 10^{-3} slugs/_{ft^3} \cdot (117.3 ft/_s)^2}{2}\right)(3ft \cdot 0.375ft)$$
$$= \boxed{17.96 lbf}$$

  - *Note: the area (A) used for the drag force equation is the area facing into the fluid flow. In the case of upright cylinders, that area is a rectangle, and the area is calculated as LxW.*

- Inserting the calculated values into **(1)** solve for the moment at A:

$$\circlearrowleft \Sigma M_A = 35.08 lbf \cdot 6ft + 17.96 lbf \cdot 1.5ft$$
$$= \boxed{237.42 lbf \cdot ft}$$

Summary:

The drag force created by the top pole was 35.08 lbf, and the bottom pole created 17.96 lbf. The moment those combined forces created at point A was 237.42 lbf*ft. Despite being thinner, the top pole was twice as long as the bottom pole and created twice as much drag because of it. It was also further from point A, meaning the drag it created contributed to most of the moment that acted at point A.

Problem 2

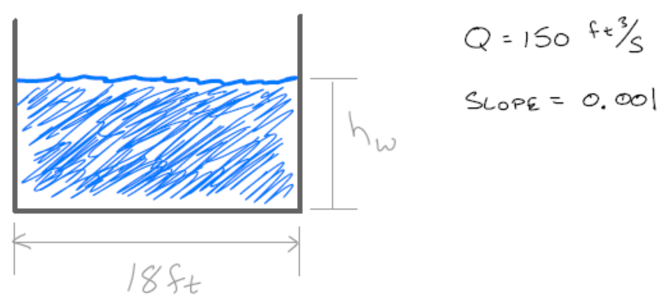
Problem Statement:

An open channel has water running through it. It is a rectangular channel made of cement with a width of 18 feet, a slope of 0.001, and the flow rate of the water is 150 ft^3/s.

Purpose:

Calculate the height of the water in the channel and whether the flow is supercritical or subcritical.

Drawing:



Design Considerations:

- Isothermal process
- Incompressible fluid
- Atmospheric pressure and temperature are assumer to be at sea level and room temperature
- The system is in a steady state of flow
- The cement used to form the channel is finished and smooth

Data and Variables:

- Dimensions of the channel are given in the drawing
- Flow rate of water:

$$Q = 150 ft^3/s$$

- Slope of channel:

$$S = 0.001$$

- Manning's *n* value for smooth cement:

$$n = 0.013$$

Procedure:

- This is an open channel flow problem, so the open channel flow equation will be used to solve it:

$$Q=\frac{1.49}{n}AS^{1/2}R^{2/3}$$

- R in the above equation is the hydraulic radius, which is calculated using the hydraulic radius equation:

$$R=\frac{A}{WP}$$

- The purpose of this problem is to solve for the height of the water flowing in the channel. The value of the height will have an impact on the area, which appears in two places: once in the open channel flow equation and again in the hydraulic radius equation, meaning there are 2 unknown values in the open channel flow equation. This problem will require an iterative process to solve.
- Solving the open channel flow equation for the values we know results in:

$$AR^{2/3}=\frac{Qn}{1.49S^{1/2}} \qquad \textbf{(1)}$$

- Using **(1)**, an iterative process can be created which works toward a known value, the calculated value of the right-hand side (RHS)
- Excel will be used to run the process; the first cycle will be shown below manually.
- Once the height of the water has been established, an energy analysis can be made to determine whether the flow is supercritical or subcritical. This will be done by calculating and evaluating the Froude number of the flow.

$$N_F=\frac{V}{\sqrt{gyh}} \quad where \quad yh=\frac{A}{T}$$

  - *T* in the above equation represents the width of the free surface of the fluid at the top of the channel
  - *V* will also need to be calculated:

$$V=\frac{Q}{A}$$

Calculations:

- Solve the RHS of **(1)** to achieve a numerical value:

$$\frac{Qn}{1.49S^{1/2}}=\frac{150^{ft}/_{s}\cdot 0.013}{1.49\cdot .001^{1/2}}=\boxed{41.386}$$

- Simplify the LHS of **(1)** into functions of *h,* the variable which needs be found:

$$A = h \cdot W = 18h$$

$$R = \frac{A}{WP} = \frac{h \cdot W}{2h + W} = \frac{18h}{2h + 18}$$

$$AR^{2/3} = \boxed{18h \cdot \left(\frac{18h}{2h + 18}\right)^{2/3}}$$

- First assumption for h will be 1 ft.:

$$h = 1\,ft$$

$$18ft \cdot 1ft \cdot \left(\frac{18ft \cdot 1ft}{2 \cdot 1ft + 18ft}\right)^{2/3} = \boxed{16.779}$$

- Compare that answer to RHS:

$$\%\,diff = \frac{h_{old} - h_{new}}{h_{old}} \cdot 100 = \frac{41.386 - 16.779}{41.386} \cdot 100 = \boxed{59.5\%}$$

- The assumed value for *h*=1 ft is off by nearly 60%. Using Excel, new values for *h* will be attempted until a value within 1% of difference is found.
- Excel Table:

h value (ft)	New LHS value	%diff
2	49.991	-20.79
1.5	31.925	22.86
1.75	40.634	1.82
1.775	41.542	-0.38
1.77	41.360	0.06

- Final answer for *h:*

$$h = \boxed{1.77\,ft}$$

- Calculate the Froude Number:

$$yh = \frac{18ft \cdot 1.77ft}{18ft} = 1.77ft \qquad V = \frac{150ft^3/s}{18ft \cdot 1.77ft} = 4.7081\,ft/s$$

$$N_F = \frac{4.7081\,ft/s}{\sqrt{32.2ft/s^2 \cdot 1.77ft}} = \boxed{0.624 \quad SUBCRITICAL}$$

Summary:

Through iteration, the height of the water in the channel was found to be about 1.77 ft. The Froude number was calculated to be 0.624. For a Froude number under 1, the flow is deemed to be subcritical. As it is a function of depth, the shallower a channel is, the greater the Froude number will be, so this channel is deep enough to avoid being considered critical flow.

Problem 3

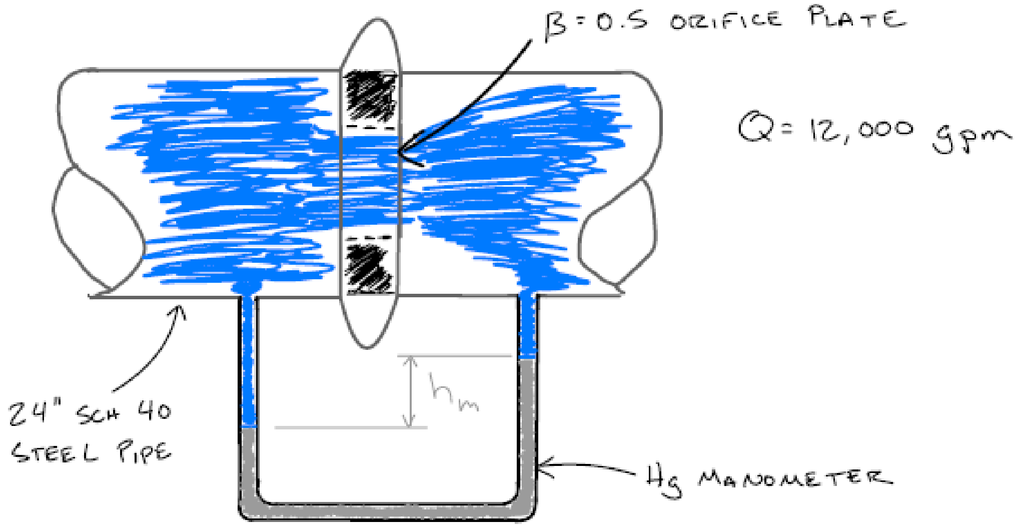
Problem Statement:

A mercury manometer is connected to a pipe on either side of an orifice plate. The orifice plate has a β value of 0.5 and is used to measure the flow rate of 12,000 gpm of water running through a 24-inch schedule 40 steel pipe.

Purpose:

Calculate the reading given by the mercury manometer.

Drawing:



Design Considerations:

- Process is isothermal
- Fluids are incompressible
- Temperature is assumed to be room temperature
- System has a steady rate of flow

Data and Variables:

- Internal diameter of pipe:

$$D_p = 1.886\, ft$$

- Diameter of orifice in orifice plate:

$$D_o = D_p \cdot \beta = 1.886\, ft \cdot 0.5 = 0.943\, ft$$

- Velocity of the water:

$$Q=12,000gpm\cdot\left(\frac{1ft^3\cdot 1min}{7.481gal\cdot 60s}\right)=26.73\,ft^3/_s$$

- Specific weights of water and mercury:

$$\gamma_{water}=62.3\,lb/_{ft^3}\qquad \gamma_{Hg}=844.9\,lb/_{ft^3}$$

- Flow area of the pipe and the orifice plate:

$$A_p=\pi\left(\frac{D}{2}\right)^2=\pi\left(\frac{1.886\text{ft}}{2}\right)^2=2.794\text{ft}^2$$

$$A_o=\pi\left(\frac{0.943\text{ft}}{2}\right)^2=0.698\text{ft}^2$$

- Dynamic viscosity of water:

$$v_w=1.05\cdot 10^{-5}\,ft^2/_s$$

Procedure:

- For this problem, the derived equation for dealing with orifice plate flow meters will be used:

$$Q=A_p\cdot C_o\cdot\sqrt{\frac{2gh\left(\frac{\gamma_m}{\gamma_w}-1\right)}{\left(\frac{A_p}{A_o}\right)^2-1}}$$

- There are two unknowns in the above equation: *h,* which is what we are trying to solve for, and C_o which can be found using the Reynolds number for the flow through the pipe and Figure 15.7 in the textbook.

$$N_R=\frac{VD_p}{v}$$

- Solving the orifice plate flow meter equation for *h* will yield the answer.

Calculations:

- Solve the orifice plate flow meter equation for *h:*

$$h=\frac{\left(\frac{Q}{A_pC_o}\right)^2\cdot\left(\left(\frac{A_p}{A_o}\right)^2-1\right)}{\left(\frac{\gamma_m}{\gamma_w}-1\right)\cdot 2g}\qquad \textbf{(1)}$$

- Find C_o by first calculating the Reynolds number in the pipe and then using Figure 15.7 in the textbook. Must first convert the flow rate into velocity before calculating the Reynolds number:

$$V = \frac{Q}{A_p} = \frac{26.73 ft^3/s}{2.794 ft^2} = \boxed{9.567 ft/s}$$

$$N_{R,p} = \frac{9.567 ft/s \cdot 1.886 ft}{1.05 \cdot 10^{-5} ft^2/s} = \boxed{1.72 \cdot 10^6}$$

$$C_o = \boxed{0.601}$$

- Insert C_o into **(1)** and solve:

$$h = \frac{\left(\frac{26.73 ft^3/s}{2.794 ft^2 \cdot 0.601}\right)^2 \cdot \left(\left(\frac{2.794 ft^2}{0.698 ft^2}\right)^2 - 1\right)}{\left(\frac{844.9 lb/ft^3}{62.3 lb/ft^3} - 1\right) \cdot 2\left(32.2 ft/s^2\right)} = \boxed{4.71 ft}$$

Summary:

The height of the manometer in the system described in the problem statement would be 4.71 ft.

Problem 4

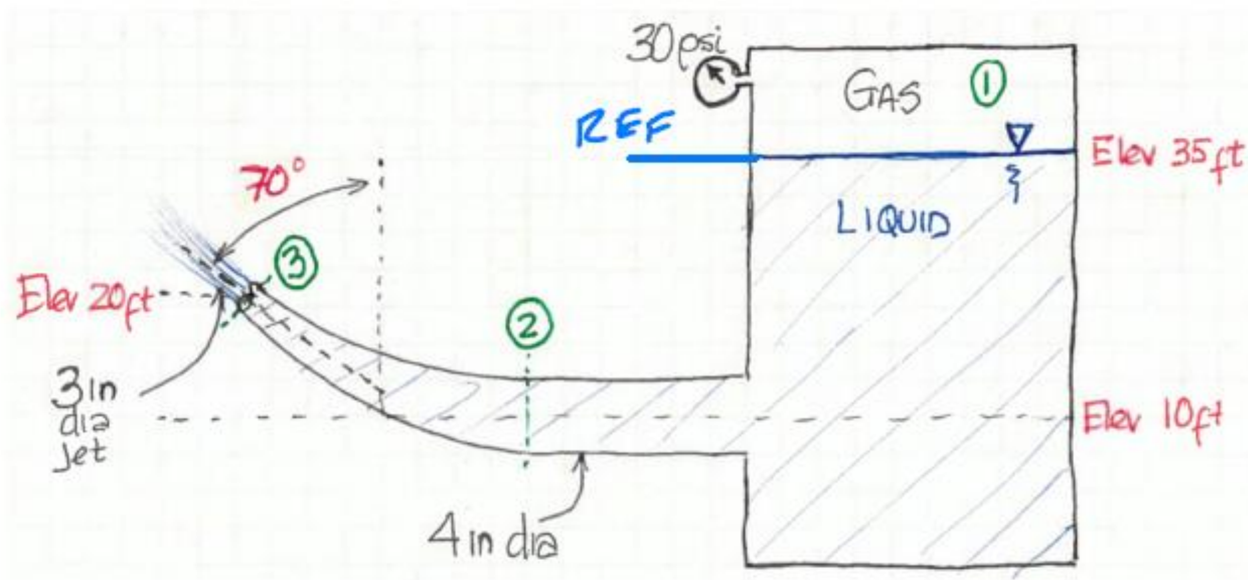
Problem Statement:

An apparatus has a tapered and curved pipe attached to a water tank. The pressure in the tank is 30psi and causes a jet of water to be ejected from the curved tube. The length of the curved tube is 10 ft.

Purpose:

Calculate the force on the curved tube created by the liquid flowing through it ignoring any energy losses.

Drawing:



Design Considerations:

- Isothermal process
- The liquid is incompressible
- The atmospheric pressure is sea-level, and the temperature is room temperature
- The surface of the liquid in the tank has a negligible velocity

Data and Variables:

- Dimensions for the tank and liquid levels are given in the drawing
- Length of the curved tube:

$$L_{ct} = 10\,ft$$

- Specific weight of the liquid:

$$\gamma = 55 {lb}/{ft^3}$$

- Pressure and velocity at point 1:

$$P_1 = 30\ psi = 4320 {lb}/{ft^2} \qquad V_1 = 0 {ft}/{s}$$

- Pressure at point 3:

$$P_3 = 0\ psi$$

  - Open to atmosphere

Procedure:

- The purpose of this problem is to calculate the force on the section of tubing between points 2 and 3. The impulse theorem will provide the external forces created in this area:

$$\Sigma F_{ct} = \rho Q \left(V_3 - V_2 \right)$$

- None of the variables in the impulse theorem are known directly for this problem, so they must be calculated using other known variables
- The other forces acting on the pipe are the pressure created at point 2 which will be additive to the pressure acting in a horizontal direction:

$$F_{P,\,x} = P_2 A_2$$

  - There is no pressure acting at point 3 due to being open to atmosphere
- The density of the liquid can be found using the specific weight given:

$$\gamma = \rho g \quad \therefore \quad \rho = \frac{\gamma}{g}$$

- To calculate the velocities and the flow rate, Bernoulli's equation will be used, first between points 1 and 3 to find the velocity at point 3, and then between points 2 and 3 to find the velocity at point 2. Points 1 and 3 will be first because the pressure at both of those points is given, making it possible to calculate the velocity directly:

$$\frac{P_1}{\gamma} + \frac{V_1^{\,2}}{2g} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^{\,2}}{2g} + z_3$$

  - There is no pump, so the head gain for pumps has been left out; the problem statement instructs us to neglect head loss, so that has also been left out.

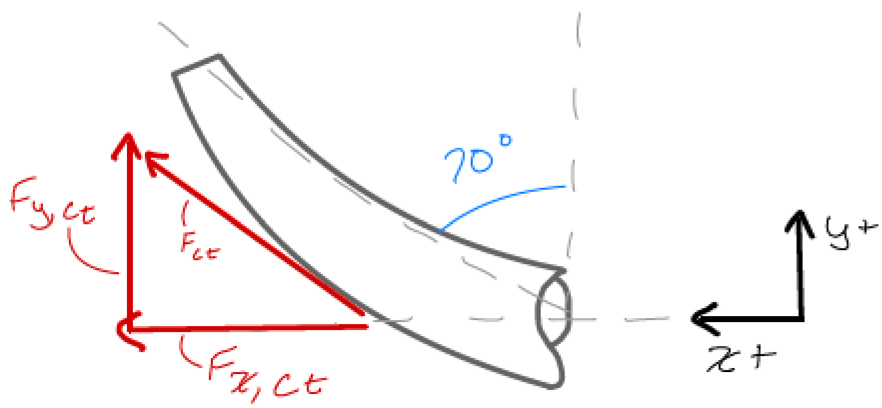
  - Additionally, $P3$, V_1 and z_1 will be cancelled in the calculations as they are equal to 0.
- The flow rate Q for the system can be found using either V_3 as the pipe diameter is known at that location:

$$Q = AV$$

- Using the value for V_3 calculated above, P_2 can be found by applying Bernoulli's equation between points 2 and 3. The velocity at point 2 can be expressed as:

$$V_2 = \frac{Q}{A_2}$$

- With all the variables calculated, returning to the impulse equation will yield the forces. However, because the pipe is curved, these forces will have an x and y value, so the impulse equation will be used for both the horizontal and vertical components of the force.
- Free Body Diagram:



- Force component equations:

$$F_{x,\,ct} = \rho Q\left(\sin(70°)\cdot V_3 - V_2\right) + P_2A_2$$
$$F_{y,\,ct} = \rho Q\left(\cos(70°)\cdot V_3 - 0\right)$$

  - There is no vertical component to V_2 which is why it is 0 in the $F_{y,\,ct}$ equation
- The total force on the pipe and the direction of that force can be found with trigonometry using the component forces:

$$F_{ct} = \sqrt{F_{x,\,ct}^{\,2} + F_{y,\,ct}^{\,2}}$$
$$\theta_{ct} = \tan^{-1}\left(\frac{F_{x,\,ct}}{F_{y,\,ct}}\right)$$

Calculations:

- Calculate the area for point 2 and 3:

$$A_2=\pi\left(\frac{D_2}{2}\right)^2=\pi\left(\frac{0.333\text{ft}}{2}\right)^2=\boxed{0.0871\text{ft}^2} \qquad A_3=\pi\left(\frac{0.25\text{ft}}{2}\right)^2=\boxed{0.0491\text{ft}^2}$$

- Calculate the density of the fluid:

$$\rho=\frac{\gamma}{g}=\frac{55^{lb}/_{ft^3}}{32.2^{ft}/_{s^2}}=\boxed{1.708^{slugs}/_{ft^3}}$$

- Calculate V_3 using Bernoulli's equation between point 1 and point 3:

$$V_3=\sqrt{2g\left(\frac{P_1}{\gamma}-z_3\right)}=\sqrt{2\cdot\left(32.2^{ft}/_{s^2}\right)\cdot\left(\frac{4320^{lb}/_{ft^2}}{55^{lb}/_{ft^3}}-(-15ft)\right)}$$
$$=\boxed{77.62^{ft}/_s}$$

- Calculate flow rate:

$$Q=A_3V_3=0.0491ft^2\cdot 77.62^{ft}/_s=\boxed{3.81^{ft^3}/_s}$$

- Calculate P_2 using Bernoulli's equation between points 1 and 2:

$$P_2=P_1+\gamma\left(-\frac{Q^2}{A^2 2g}-z_2\right)$$
$$=4320^{lb}/_{ft^2}+55^{lb}/_{ft^3}\left(-\frac{\left(3.81^{ft^3}/_s\right)^2}{(0.0871ft^2)^2\cdot 2\cdot 32.2^{ft}/_{s^2}}-(-25ft)\right)$$
$$=\boxed{4061^{lb}/_{ft^2}}$$

- Calculate velocity at point 2:

$$V_2=\frac{Q}{A_2}=\frac{3.81^{ft^3}/_s}{0.0871ft^2}=\boxed{43.74^{ft}/_s}$$

- Calculate force components:

$$F_{x,\,ct}=1.708^{slugs}/_{ft^3}\cdot 3.81^{ft^3}/_s\left(\sin(70°)\cdot 77.62^{ft}/_s-43.74^{ft}/_s\right)$$
$$+4061^{lb}/_{ft^2}\cdot 0.0491ft^2$$
$$=\boxed{389.41\ lbf}$$

$$F_{y,\,ct}=1.708^{slugs}/_{ft^3}\cdot 3.81^{ft^2}/_s\left(\cos(70°)\cdot 77.62^{ft}/_s\right)$$
$$=\boxed{172.76\ lbf}$$

- Calculate total force and the direction of the force:

$$F_{ct} = \sqrt{(389.41 lbf)^2 + (172.76 lbf)^2}$$
$$= \boxed{426.01\ lbf}$$
$$\theta = \tan^{-1}\left(\frac{172.76 lbf}{389.41 lbf}\right)$$
$$= \boxed{23.92°}$$

Summary:

The net force acting on the curved pipe is 426.01 lbf at 23.92° above the horizontal plane in the FBD. Interestingly, the length of the curved tube was not necessary for this calculation because it instructed us to neglect any energy losses, so the length of the tube is irrelevant to the force if the angle and the height of the nozzle remain the same. If energy losses were accounted for, the length of the tube would have an impact due to the friction lost while traversing the length of the tube.

Problem 5

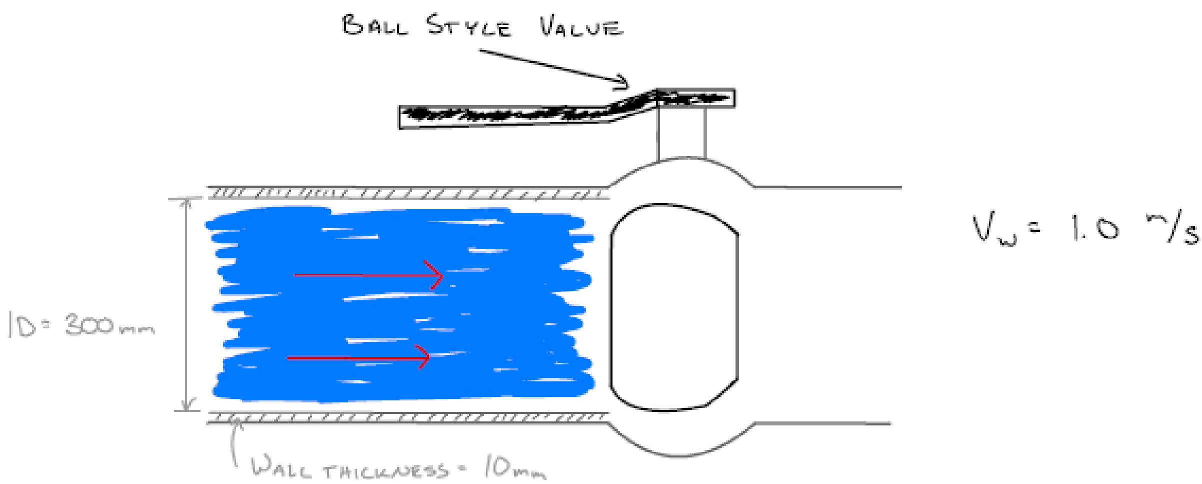
Problem Statement:

At the end of a steel pipe with a diameter of 300mm and a thickness of δ=10mm there is a valve. The velocity of the water in the pipe is 1.0m/s. The valve is closed suddenly.

Purpose:

Calculate the pressure increment when the valve is closed

Drawing:



Design Considerations:

- Isothermal process
- The temperature is room temperature
- The system was in a steady flow state prior to the closure of the valve

Data and Variables:

- Elasticity of the pipe:

$$E = 2 \cdot 10^7 N/cm^2 \cdot \left(\frac{10,000cm^2}{1m^2}\right) = 2 \cdot 10^{11} N/m^2$$

- Bulk modulus of the water:

$$E_o = 2.03 \cdot 10^5 N/cm^2 \cdot \left(\frac{10,000cm^2}{1m^2}\right) = 2.03 \cdot 10^9 N/m^2$$

- Internal diameter and wall thickness of the pipe:

$$D_p = 300mm = 0.3m \qquad \delta = 10mm = 0.01m$$

- Velocity of the water when the valve is closed:

$$V_w = 1 \, {}^{m}\!/_{s}$$

- Density of water at 20°C:

$$\rho = 998 \, {}^{kg}\!/_{m^3}$$

Procedure:

- The pressure increment created by the water hammer phenomenon is governed by the equation:

$$\Delta P = \rho C V$$

Where *C* is a multiplier which can be found with the equation:

$$C = \frac{\sqrt{\dfrac{E_o}{\rho}}}{\sqrt{1 + \dfrac{E_o D}{E \delta}}}$$

- All these variables have been given in the problem statement. Utilize the equations to solve for *ΔP*

Calculations:

- Solve for *C*:

$$C = \frac{\sqrt{\dfrac{2.03 \cdot 10^9 \, {}^{N}\!/_{m^2}}{998 \, {}^{kg}\!/_{m^3}}}}{\sqrt{1 + \dfrac{\left(2.03 \cdot 10^9 \, {}^{N}\!/_{m^2}\right) \cdot 0.3m}{\left(2 \cdot 10^{11} \, {}^{N}\!/_{m^2}\right) \cdot 0.01m}}} = \boxed{1248.71 \, {}^{m}\!/_{s}}$$

- Solve for *ΔP*:

$$\Delta P = 998 \, {}^{kg}\!/_{m^3} \cdot 1248.71 \, {}^{m}\!/_{s} \cdot 1 \, {}^{m}\!/_{s} = 1246210 \; Pa = \boxed{1246.21 \; kPa}$$

Summary:

The pressure increment created by instantly closing the valve would be 1246.21 kPa. This was a straightforward answer once all the units had been converted to the correct format to run through the formulas.