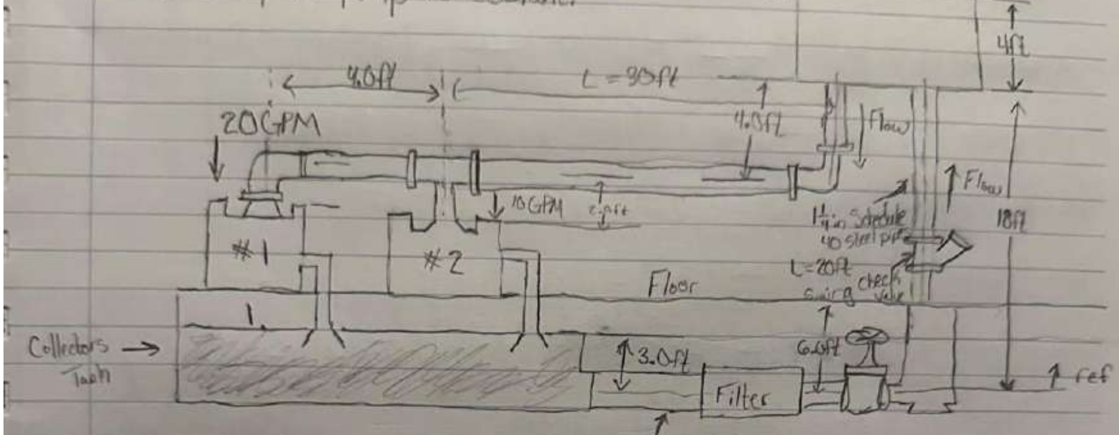
In these lessons I noticed how important organization and clear writing is for the successful completion of these problems. Solving systems of related equations is common in college level math courses. However, you must be very careful with these with parallel branches. I also noticed that as in electrical circuits the flow is inversely proportional to the energy losses which can be thought of as resistance to flow.

$P_1 = P_3 = 0$, $V_1 = V_3 = 0$

11.23 Compute the total head on the pump & the power delivered by the pump to coolant.



Given: 2-in Schedule 40 Steel pipe L = 10.0ft $g = 32.2\,ft/s^2$

Suction Pipe: 2" Schedule 40 Steel Pipe, L = 10ft

Discharge Pipe: $1\frac{1}{4}$" Schedule 40 Steel Pipe, L = 20ft

$\eta = 3.6 \times 10^{-5}\ lb\cdot s/ft^2$ Filter Coefficient: 1.85

S.G. = 0.92 $\varepsilon = 1.5 \times 10^{-4}\ ft$

$$Q = \frac{30\,gal}{min} \times \frac{1\,min}{60\,s} \times \frac{1\,ft^3}{7.48\,gal} = 0.0668\,ft^3/s$$

$$\cancel{\frac{P_1}{\gamma}} + z_1 + \cancel{\frac{V_1^2}{2g}} - h_L + h_A = \cancel{\frac{P_3}{\gamma}} + z_3 + \cancel{\frac{V_3^2}{2g}} \rightarrow h_A = (z_3 - z_1) + h_L \rightarrow h_A = 19ft + h_L$$

$$(22ft - 3ft) + h_L$$

Suction:

$$V_{suct} = \frac{Q}{A_s} = \frac{0.0668\,ft^3/s}{\pi/4\,(0.1723\,ft)^2} = 2.865\,ft/s \qquad \frac{V_s^2}{2g} = \frac{(2.865\,ft/s)^2}{2(32.2)} = 0.127\,ft$$

$$N_R = \frac{V_s D_s \rho}{\eta} = \frac{(2.865\,ft/s)(0.1723\,ft)(0.92 \times 1.94\,slug/ft^3)}{3.6 \times 10^{-5}\,lb\cdot s/ft^2} = 24{,}473.549$$

$$\frac{D}{\varepsilon} = \frac{0.1723\,ft}{(1.5 \times 10^{-4}\,ft)} = 1{,}148.67$$

Based On Moody $f_s = 0.0265$

Discharge:

$$V_{Dis} = \frac{0.0668\,ft^3/s}{\pi/4\,(0.1150\,ft)^2} = 6.431\,ft/s \qquad \frac{V_D^2}{2g} = \frac{(6.431\,ft/s)^2}{2(32.2)} = 0.642\,ft$$

$$N_R = \frac{(6.431\,ft/s)(0.115\,ft)(0.92 \times 1.94\,slug/ft^3)}{3.6 \times 10^{-5}\,lb\cdot s/ft^2} \qquad D_D = \frac{0.115\,ft}{1.5 \times 10^{-4}\,ft} = 766.667$$

$$N_R = 36{,}665.989$$

Based on Moody's $f_D = 0.0225$

<u>Losses</u>:

(entrance loss) $h_1 = k\left(\frac{V_S^2}{2g}\right) = (0.5)(0.127ft) = 0.0635ft$

(Suction friction loss) $h_2 = f_S \frac{L}{D} \frac{V_S^2}{2g} = 0.0265\left(\frac{10ft}{0.1723ft}\right)(0.127ft) = 0.195ft$

(filter losses) $h_3 = \left(\frac{V_S^2}{2g}\right)1.85 = (0.127ft)(1.85) = 0.235ft$

(Gate/Open Loss) $h_4 = f_D\left(\frac{L_e}{D}\right)\left(\frac{V_S^2}{2g}\right) = (0.0225)(8)(0.127) = 0.0223ft$

(Valve Loss) $h_5 = f_D\left(\frac{L_e}{D}\right)\left(\frac{V_D^2}{2g}\right) = (0.022)(100)(0.642ft) = 1.412ft$

(Discharge loss) $h_6 = f_D\left(\frac{L}{D}\right)\frac{V_D^2}{2g} = 0.0265\left(\frac{20ft}{0.115ft}\right)(0.642) = 2.959ft$

(Exit loss) $h_7 = 1.0\left(\frac{V_D^2}{2g}\right) = 0.642ft$

$h_L = \Delta h = 5.529\ ft$

$\gamma = S.G. \times \gamma_w$
$\gamma = 0.92 \times 62.4lb/ft^3$
$\gamma = 57.408\ lb/ft^3$

$h_A = 19ft + h_L = 24.529ft$

$Power\ (P) = h_A \gamma Q = (24.529ft)(57.408\ lb/ft^3)(0.0668ft^3/s)\left(\frac{1hp}{550\ ft \cdot lb/s}\right)$

$Power\ (P) = 0.171\ hp$

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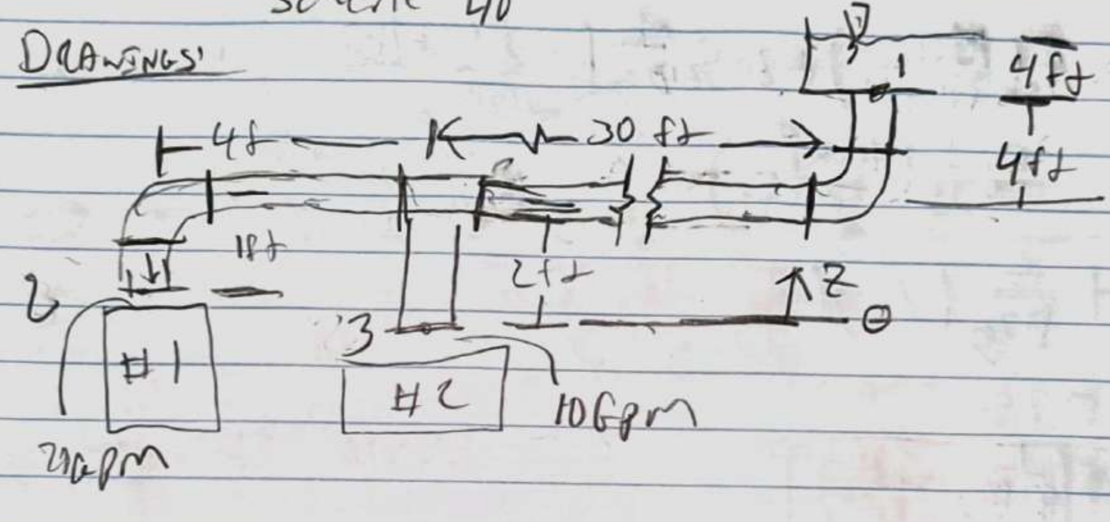
Technology Transfer Program

11.24) <u>Purpose</u>: Find the pipe size for the schedule 40

<u>Drawings:</u>



Losses → pipe 2X elbows 1X Tee

$SG = 0.92 \quad \mu = 3.6\times10^{-5}\ \text{lb s/ft}^2 \quad \gamma = 59.25\ \text{lb/ft}^3$

$Q_T = 20 + 10 = 30\ \text{GPM} \qquad V = Q/A$

$P_1 = \gamma h = 59.25\,(4\,ft) = 237\ \text{lb/ft}^2$

① $\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + z_2 + h_L$

$\frac{P_1}{\gamma} + \frac{1}{2g}\left(\frac{4Q_1}{\pi D^2}\right)^2 + z_1 = \frac{1}{2g}\left(\frac{4Q_2}{\pi D^2}\right)^2 + z_2 + h_L$

$h_L(\text{ent}) = 0.5\left(\frac{1}{2g}\right)\left(\frac{Q_1 4}{\pi D^2}\right)^2$

$h_L(\text{elbow}_1) = 30 f_t \left(\frac{Q_1 4}{\pi D^2}\right)^2 \frac{1}{2g}$

$h_L(\text{elbow}_2) = 30 f_t \left(\frac{Q_2 4}{\pi D^2}\right)^2 \frac{1}{2g}$

$h_L(\text{Tee}) = 20 f_t \frac{1}{2g}\left(\frac{Q_1 4}{\pi D^2}\right)^2$

$h_L(\text{pipe}) = f\,\frac{L_1}{D}\frac{1}{2g}\left(\frac{Q_1 4}{\pi D^2}\right)^2 + f\,\frac{L_2}{D}\frac{1}{2g}\left(\frac{Q_2 4}{\pi D^2}\right)^2$

$$\frac{P_1}{\gamma} + z_1 - z_2 = \frac{1}{2g}\left(\frac{16Q_2^2 - 16Q_1^2}{\pi^2 D^4}\right) + h_L$$

$$h_L = 0.5\frac{1}{2g}\left(\frac{16Q_1^2}{\pi^2 D^4}\right) + 30 f_t \frac{1}{2g}\left(\frac{16Q_1^2 + 16Q_2^2}{\pi^2 D^4}\right) + 20 f_t \frac{1}{2g}\left(\frac{16Q_1^2}{\pi^2 D^4}\right) + \frac{f}{D2g}(L_1)\left(\frac{Q_1^2\,16}{\pi^2 D^4}\right) + \frac{f}{D2g}(L_2)\left(\frac{Q_2^2\,16}{\pi^2 D^4}\right)$$

$$f = \frac{0.25}{\left[\log\left[\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{R_e^{0.9}}\right]\right]^2} \qquad \varepsilon = 1.5\times10^{-4}\ ft$$

$$Re = \frac{vD\rho}{\mu}$$

$$h_L = \frac{1}{4g}\left(\frac{16Q_1^2}{\pi^2 D^4}\right) + \frac{f_t}{2g}\left(\frac{480Q_1^2 + 480Q_2^2}{\pi^2 D^4}\right) + \frac{f_t}{g}\left(\frac{160Q_1^2}{\pi^2 D^4}\right) + \frac{f}{2g}\frac{L_1}{D}\left(\frac{Q_1^2\,16}{\pi^2 D^4}\right) + \frac{f}{2g}\frac{L_2}{D}\left(\frac{Q_2^2\,16}{\pi^2 D^4}\right)$$

$$h_L = 11.327/D^4 + \frac{f_t(981.75)}{D^4} + \frac{311.52 f_t}{D^4} + \frac{770.29}{D^5} + \frac{50.35}{D^5}$$

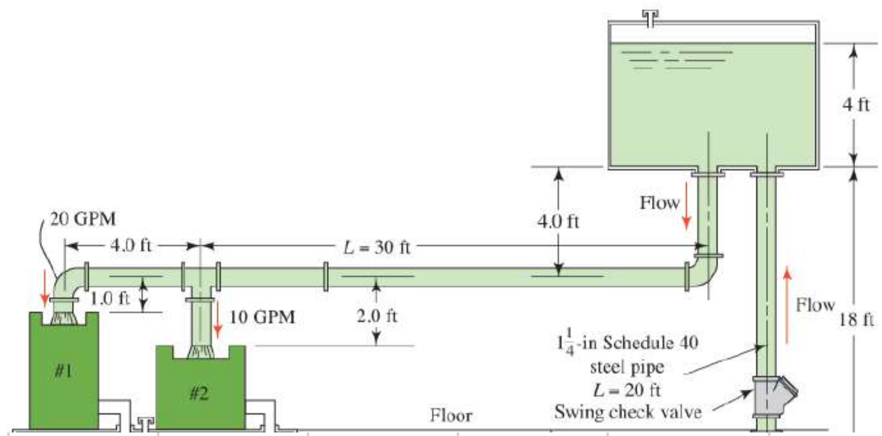
$$\frac{P_1}{\gamma} + z_1 - z_2 = \frac{-12.59}{D^4} + \frac{11.327 + 1293.27 f_t}{D^4} + \frac{820.64 f}{D^5}$$

From excel

$D = 0.401\ ft = 4.8\ in$

$\boxed{\text{Closest sch 40 is 5 NPS at 5.047 in}}$

q1	30	gal/min	0.06684 ft3/s
q2	20	gal/min	0.04456 ft3/s
q3	10	gal/min	0.02228 ft3/s
P1	237	lb/ft2	
sg	0.92		
rho	1.7848	slug/ft3	
dyn vis	0.000036	lb*s/ft2	
gamma	59.25	lb/ft3	
z1	6	ft	
z2	1	ft	
ε	0.00015	ft	



v=4Q/Pi(D^2)

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)}+\frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad \varepsilon = 1.5\times10^{-4}\ ft$$

$$Re = \frac{vD\rho}{\mu}$$

$$\frac{P_1}{\gamma} + z_1 - z_2 = \frac{-12.59}{D^4} + \frac{11.320 + 1293.275f}{D^4} + \frac{820.64}{D^5}$$

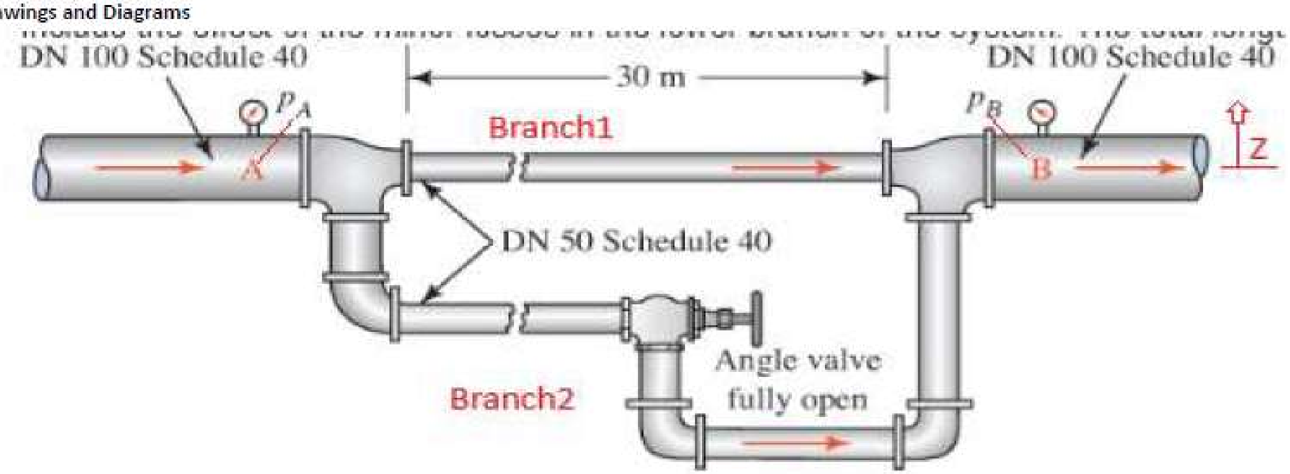
Iteration	LHS	f(est)	D (est)	RHS	D/E	Re	f	f % diff	RHS % diff
1	9	0.001	0.5	0.101797	3333.333	8438.503	0.032956	96.97%	98.87%
2	9	0.032956	0.25	86.00907	1666.667	16877.01	0.028271	-16.57%	-855.66%
3	9	0.028271	0.3	31.32574	2000	14064.17	0.029274	3.43%	-248.06%
4	9	0.029274	0.4	8.519206	2666.667	10548.13	0.031192	6.15%	5.34%
5	9	0.031192	0.401	8.973569	2673.333	10521.82	0.031211	0.06%	0.29%
6									

Problem

In the branched pipe system shown in Fig. 12.8, 850 L/min of water at 10°C is flowing in a DN 100 Schedule 40 pipe at A. The flow splits into two DN 50 Schedule 40 pipes as shown and then rejoins at B. Calculate (a) the flow rate in each of the branches and (b) the pressure difference pA - pB. Include the effect of the minor losses in the lower branch of the system. The total length of pipe in the lower branch is 60 m. The elbows are standard.

Purpose

Drawings and Diagrams



Sources

Mott, R. L., & Untener, J. A. (2015). *Applied Fluid Mechanics*. Pearson.

Design Considerations

Data and Variables

Description	Symbol	Qty	Unit	Source
Flow	Q	850	L/min	given in task statement
Pipe Length B1	L1	30	m	given in task statement
Pipe Length B2	L2	60	m	given in task statement
H2O Temp	T	10	C	given in task statement
Specific Weight	gamma	9.810	kN/m^3	Table A.1 (Mott 2015)
Flow	Qt	0.0142	m^3/s	Q * (1m/s^2)/(60000 L/min)
ID DN 100	Dm	0.102	m	Table F.1 (Mott 2015)
Flow Area DN 100	Am	8.213E-03	m^2	Table F.1 (Mott 2015)
ID DN 50	Db	0.053	m	Table F.1 (Mott 2015)

Flow Area DN 50	Ab	2.168E-03	m^2	Table F.1 (Mott 2015)
Roughness	e	4.600E-05	m	Table 8.2 (Mott 2015)
Rel Rough(branch)	D/e	1141.304		Db/e
kine viscosity	v	1.30E-06	m^2/s	Table A.1 (Mott 2015)
friction fact of turb	fT	0.020		Moody chart
				Table 8.2 (Mott 2015)

Procedure

I will start with what I know about flow rate to develop equation 1. Then I will apply bernouilli's to each branch to get the relationship of thier energy losses to the DeltaP

My solving technique will require guessing the branch flow rate, Q_total is given so Q_lower will be calculated from the relationship. Next I will use the Q of each branch to calculate the friction losses. Due to the relationships of pressure drop the hL of each branch mush be equal so I will compare the percentage diffreance and iterate. After the error is minimized I will adjust the f values and iterate again. Upon completion of this process I will use the total energy losses to determine the pressure diffrence.

Calculations

① $Q_T = Q_{B1} + Q_{B2}$

UPPER BRANCH (B1)

$\cancel{z_A} + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{h_A} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{h_R} + h_L + \cancel{z_B}$

$\frac{P_A}{\gamma} - \frac{P_B}{\gamma} = \cancel{\frac{V_A^2}{2g}} + \cancel{\frac{V_B^2}{2g}} + h_L \quad (V_A = V_B)$

$\frac{\Delta P}{\gamma} = h_L \qquad h_L = f \times \frac{L}{D} \times \frac{V_{B1}^2}{2g} + 2K_r \frac{V_{B1}^2}{2g}$

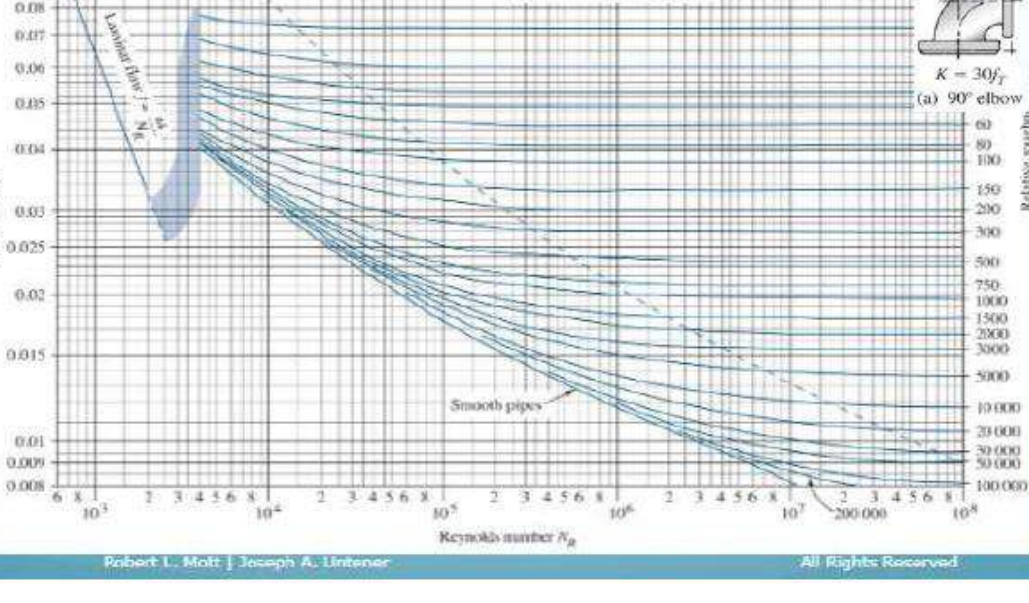
LOWER BRANCH (B2)

$\cancel{z_A} + \frac{P_A}{\gamma} + \frac{V_A^2}{2g} \cancel{h_A} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{h_R} + h_L + \cancel{z_B}$

$\frac{\Delta P}{\gamma} = h_L \qquad h_L = f \times \frac{L}{D} \times \frac{V_{B2}^2}{2g} + 3 \cdot K_{90} \frac{V_{B2}^2}{2g} + 2K_r \frac{V_{B2}^2}{2g}$

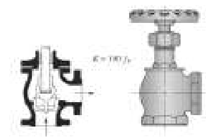
$+ K_{valv} \cdot \frac{V_{B2}^2}{2g}$

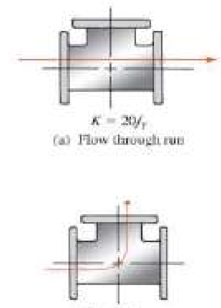
$L = h + h$



State	Q_b1	Q_b2	V_b1	V_b2	VD/v Re_b1	Re_b2	f_b1	f_b2	hL_b1	hL_mi_b1	hL_b2	hL_mi_b2	hL_b1	hL_b2	error
1	0.010000	0.004167	4.61	1.92	1.86E+05	7.76E+04	0.0225	0.0235	13.94206	0.845818	5.056148	1.321591	14.78788	6.377739	56.9%
2	0.008000	0.006167	3.69	2.84	1.49E+05	1.15E+05	0.0225	0.0235	8.922918	0.541324	11.07499	2.894813	9.464241	13.9698	-47.6%
3	0.009000	0.005167	4.15	2.38	1.68E+05	9.62E+04	0.0225	0.0235	11.29307	0.685113	7.774333	2.032078	11.97818	9.806411	18.1%
4	0.008700	0.005467	4.01	2.52	1.62E+05	1.02E+05	0.0225	0.0235	10.55274	0.6402	8.70337	2.274913	11.19294	10.97828	1.9%
5	0.008650	0.005517	3.99	2.54	1.61E+05	1.03E+05	0.0225	0.0235	10.4318	0.632862	8.863306	2.316717	11.06466	11.18002	-1.0%
6	0.008670	0.005497	4.00	2.54	1.62E+05	1.02E+05	0.0225	0.0235	10.48009	0.635792	8.799157	2.29995	11.11588	11.09911	0.2%
7	0.008668	0.005499	4.00	2.54	1.61E+05	1.02E+05	0.0225	0.0235	10.47526	0.635499	8.805561	2.301624	11.11076	11.10718	0.0%
8	0.008668	0.005499	4.00	2.54	1.61E+05	1.02E+05	0.0215	0.022	10.00969	0.635499	8.243504	2.301624	10.64519	10.54513	0.9%
9	0.008652	0.005515	3.99	2.54	1.61E+05	1.03E+05	0.0215	0.022	9.972772	0.633155	8.291548	2.315038	10.60593	10.60659	0.0%

$h_L = f \times \frac{L}{D} \times \frac{v^2}{2g}$ $h_L = K\frac{v^2}{2g}$





$K = 20f_T$
(a) Flow through run

$K = 60f_T$
(b) Flow through branch

$h_{LT} = h_{LB1} + h_{LB2}$ 2g

$h_{LT} = 10.61\text{ m} + 10.61\text{ m} = 21.21\text{ m}$

$\frac{\Delta P}{\gamma} = h_{LT}$ $\Delta P = 21.21\text{ m} \cdot 9.81\ \frac{kN}{m^3} = 208.07\ \frac{kN}{m^2}$

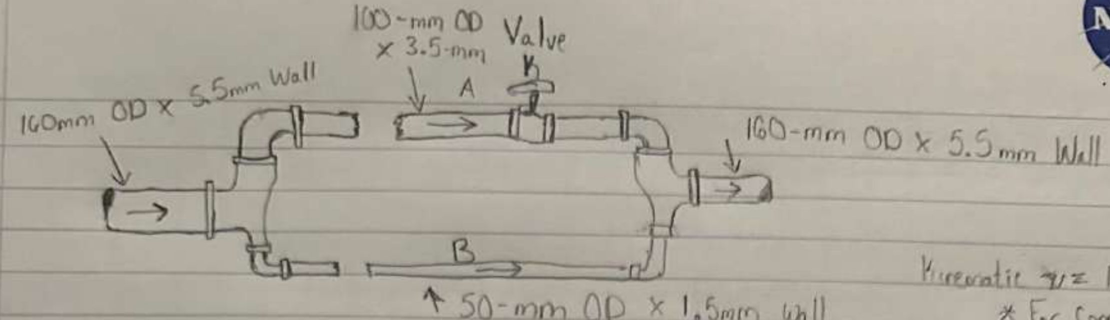
$\boxed{\Delta P = 208.1\text{ kPA}}$

Summary

The volume flow rate of the upper branch is .0087 m^3/s and the lower branch is .0055 m^3/s. The pressure difference from A to B is 208.1 kPA.

Materials
Water at 60F
New Schedule 40 Iron Pipe

12.5



Kinematic $\nu = 1.307 \times 10^{-6}\ m^2/s$

* For Copper Tubing $\varepsilon = 1.5 \times 10^{-6}\ m$

Given: Hydraulic Copper Tubing $L = 30m$ 1.5mm Wall

H_2O at $10°C$ $g = 9.81 m/s^2$

$Q = \frac{500L}{min} \times \frac{1min}{60s} \times \frac{1m^3}{1000L} = 0.00833 m^3/s = Q_A = Q_B$ $Q = AV$

$V_A = \frac{Q}{A_A} = \frac{(0.00833 m^3/s)}{\pi/4\,(0.100m)^2} = 1.061 m/s$ $N_{Ra} = \frac{V_A D_A}{\nu} = \frac{(1.061m/s)(0.100m)}{(1.307 \times 10^{-6} m)} = 81,178.27$

$\frac{D_A}{\varepsilon} = \frac{0.100m}{1.5 \times 10^{-6}m} = 66,666.67$ $f_A \approx 0.0186$

$V_B = \frac{Q}{A_B} = \frac{(0.00833 m^3/s)}{\pi/4\,(0.05m)^2} = 4.242 m/s$ $N_{RB} = \frac{V_B D_B}{\nu} = \frac{(4.242m/s)(0.05m)}{(1.307 \times 10^{-6}m)} = 162,280.03$

$\frac{D_B}{\varepsilon} = \frac{0.05m}{1.5 \times 10^{-6}m} = 33,333.33$ $f_B \approx 0.0163$

Elbow $f = 0.010$

$h_L(A)$ Losses:

(friction) $h = f_A \frac{L}{D}\frac{V_A^2}{2g} = f_A \frac{(30m)}{0.100m}\frac{V_A^2}{2g} = 300 f_A \frac{V_A^2}{2g}$

(Elbows)
$h = 2 \times (30)\frac{V_A^2}{2g} = 0.6\frac{V_A^2}{2g}$

(Valve)
$h = K\frac{V_A^2}{2g}$

$h_L(A) = (300 f_A + 0.60 + K)\frac{V_A^2}{2g}$

$h_L(A) = (300(0.0186) + 0.6 + K)$

$h_L(A) = (6.18 + K)\frac{V_A^2}{2g}$

$h_L(B)$ Losses:

(friction) $h = f_B \frac{L}{D}\frac{V_B^2}{2g} = f_B \left(\frac{(30m)}{(0.05m)}\right)\frac{V_B^2}{2g} = 600 f_B \frac{V_B^2}{2g}$

(Elbows)
$h = 2 \times (0.30)\frac{V_B^2}{2g}$

$h_L(B) = (600 f_B + 0.60)\frac{V_B^2}{2g}$

$h_L(B) = (600(0.0163) + 0.60)\frac{V_B^2}{2g}$

$h_L(B) = 10.38\frac{V_B^2}{2g}$

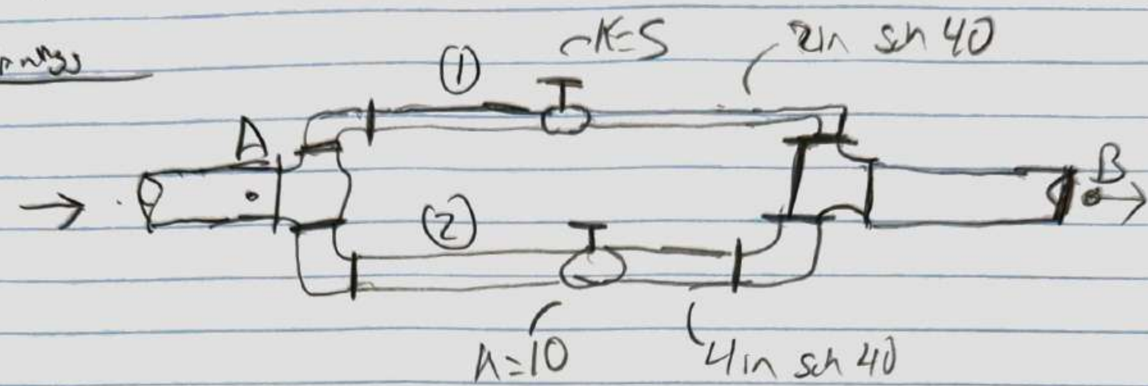
$A_A V_A = A_B V_B$ $V_B = V_A\left(\frac{D_A}{D_B}\right)^2 = 4V_A \Rightarrow V_B = 16 V_A^2$ * $h_L(A) = h_L(B)$

$(6.18 + K)\left(\cancel{\frac{V_A^2}{2g}}\right) = 10.38\left(\frac{16\cancel{V_A^2}}{\cancel{2g}}\right) \Rightarrow 166.08 = 6.18 + K$

$K = 159.9$

12.6) Purpose Calculate the total volumetric flow rate

Drawings



$K_{elbow} = 0.9$

$P_A = 20\ psig = 2880\ lb/ft^2$ $\quad P_B = 0$ $\quad \gamma = 62.3\ lb/ft^3$

$\mu = 2.04\times10^{-5}\ lb\cdot s/ft^2$ $\quad \rho = 1.94\ slugs/ft^3$

③ $Q_1 + Q_2 = Q_T$ $\quad D_1 = 0.1723\ ft$ $\quad D_2 = 0.3355\ ft$

$A_1 = 0.02333\ ft^2$ $\quad A_2 = 0.0884\ ft^2$

①

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{z_A} = \cancel{\frac{P_B}{\gamma}} + \frac{V_B^2}{2g} + \cancel{z_B} + h_L$$

$$h_L = 5f_t\left(\frac{v_1^2}{2g}\right) + 2(0.9)f_t\frac{v_1^2}{2g} = \frac{v_1^2}{2g}(6.8f_t)$$

$$h_L = \frac{6.8}{2g}\frac{Q^2}{A_1^2}f_t$$

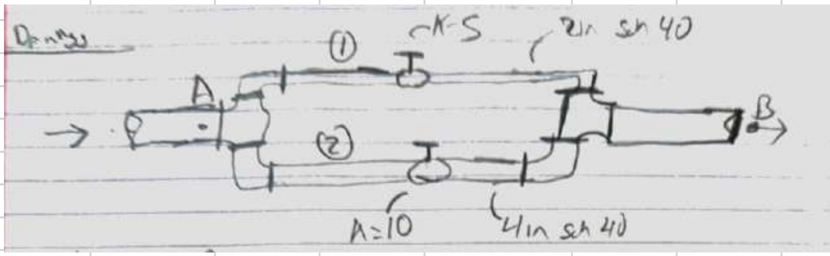
$$\frac{P_A}{\gamma} = \frac{Q_1^2}{2gA_1^2} + \frac{6.8Q^2}{2gA_1^2} \qquad Q_1 = \sqrt{\frac{P_A\cdot\cancel{2}\cdot g\cdot A_1^2}{\gamma\cdot 3.9\, f_t}}$$

②
$$\frac{P_A}{\gamma} = \frac{Q_2^2}{2gA_2^2} + \frac{11.8Q_2^2}{2gA_2^2} \qquad Q_2 = \sqrt{\frac{P_A\cdot g\cdot A_2^2}{\gamma\cdot 6.4\, f_t}}$$

$$Q_T = \sqrt{\frac{P_A\cdot g\cdot A_1^2}{\gamma\cdot 3.9\cdot f_t}} + \sqrt{\frac{P_A\cdot g\cdot A_2^2}{\gamma\cdot 6.4\cdot f_t}}$$

Excel

$$\boxed{Q_T = 1.065\ ft^3/s}$$



$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad \varepsilon = 1.5\times10^{-4}\ ft$$

$$Re = \frac{V D \rho}{\mu}$$

$$Q_1 = \sqrt{\frac{P_A \cdot g \cdot A_1^2}{\gamma \cdot 39 \cdot f_t}} + \sqrt{\frac{P_A \cdot g \cdot A_2^2}{\gamma \cdot 64 \cdot f_t}}$$

PA	240	lg/ft2
gamma	62.3	lb/ft3
dy vis	0.0000204	lb*s/ft2
Rho	1.94	slug/ft3
D1	0.1723	ft
D2	0.3355	ft
A1	0.02333	ft2
A2	0.0884	ft2
ε	0.00015	ft
g	32.2	ft/s2

	ft	Q1num	Q1den	Q1	Q2num	Q2den	Q2	Qt	Re	ft	%diff
1	0.001	4.206265	0.24297	4.160753	60.39092	0.39872	0.262482	4.423236	2922228	0.019155604	94.78%
2	0.019155604	4.206265	4.654237	0.950657	60.39092	7.637722	0.125466	1.076123	667676.6	0.019598064	2.26%
3	0.019598064	4.206265	4.761742	0.939865	60.39092	7.81414	0.124752	1.064617	660096.6	0.019604096	0.03%