MET 350	HW 2.2	ML SUNDERLAND

II

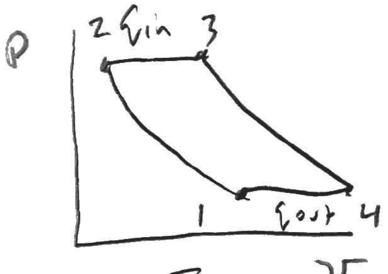
9-88

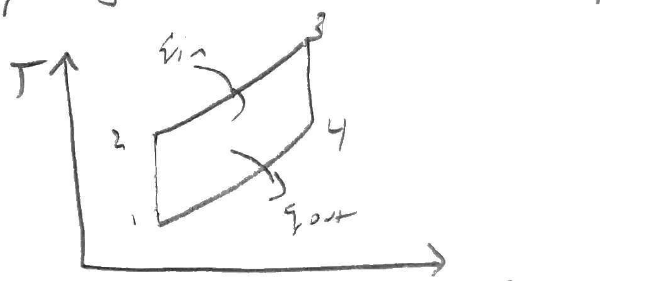
- Aircraft w/ ideal Brayton cycle engine
- $r_p = 10$
- $\dot{Q}_{in} = 500$ kW
- $\dot{m} = 1$ kg/s
- Air inlet is 70 kPa and 0°C = 273 K

$c_p = 1.005$ kJ/kg K

$k = 1.4$

- Find Power Produced by engine + Thermal efficiency





① —Isentropic, $r_p = 10$→ ② —P constant→ ③ —Isentropic, $r_p = \frac{1}{10}$→ ④ —P constant

$T_1 = 273$ K
$P_1 = 70$ kPa

$P_2 = 10P_1 = 700$ kPa
$T_2 = 527.08$ K

$P_3 = 700$ kP
$T_3 = 1024.6$ K

$P_4 = 100$ kPa
$T_4 = 530.69$ K

$T_2 = T_1(10)^{\frac{k-1}{k}}$
$= 273(10)^{\frac{1.4-1}{1.4}}$
$T_2 = 527.08$ K

$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}}$
$= \frac{240.98}{500} \times 100$
$= 48.19\%$

$T_4 = T_3\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}}$
$= 1074.6(.1)^{\frac{1.4-1}{1.4}}$
$T_4 = 530.69$ K

$q_{in} = \frac{\dot{Q}_{in}}{\dot{m}} = \frac{500 \text{ kJ}}{s} \cdot \frac{s}{1 \text{ kg}}$

$q_{in} = 500$ kJ/kg

$q_{in} = c_p(T_3 - T_2)$

$\frac{q_{in}}{c_p} + T_2 = T_3$

$T_3 = \frac{500 \text{ kJ}}{\text{kg}} \cdot \frac{\text{kg}}{1.005 \text{ kJ}} + 527.08 = 1024.6$ K

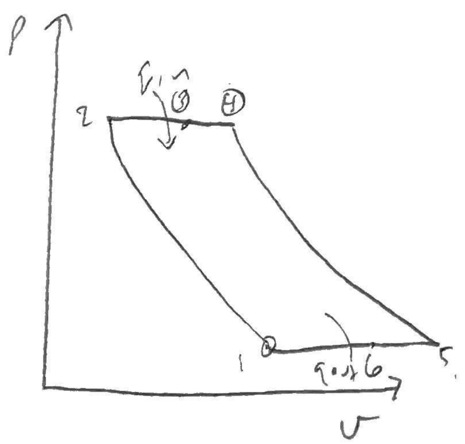
$\dot{W}_{net} = \dot{W}_{turb} - \dot{W}_{comp}$
$= \dot{m}\left(c_p(T_4 - T_3) - c_p(T_2 - T_1)\right)$

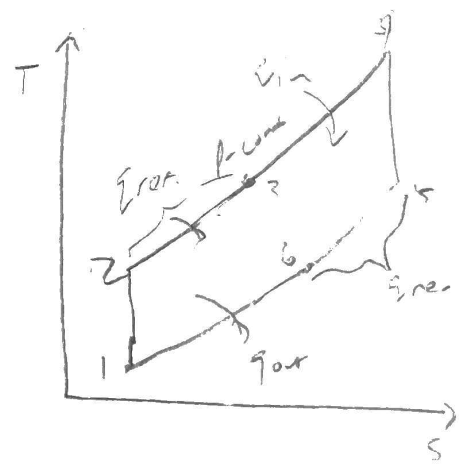
$\dot{W}_{net} = 1(1.005(1024.5 - 530.69) - 1.005(527.08 - 273))$

$\dot{W}_{net} = 240.98$ kW

9-94 • Gas Turb w/ regen
• Air enters @ 100 KPa and 30°C + 273 = 303 K
• $r_p = 10$
• $T_{max} = 800°C + 273 = 1073 K$
• Cold air leave regen 10°C cooler than hot stream leaves regen.
• Isentropic $W_{net} = 115$ KW

Find heat addition + rejection





$k = 1.4$

$\dot{W}_{net} = \dot{m} w_{net} = 115$ KW

$w_{net} = (h_4 - h_5) - (h_2 - h_1) = c_p (T_4 - T_5) - c_p (T_2 - T_1)$

$Q_{in} = \dot{m} c_p (T_4 - T_3)$ ⇒ heat addition $C_p = 1.005$ KJ/kg K

$Q_{out} = \dot{m} c_p (T_6 - T_1)$ ⇒ Heat rejection

(1) —Isentropic, $r_p = 10$→ (2) —P const→ (3) —P const→ (4) —Isentropic, $r_p = 10$→ (5) —P const, Regen→ (6) —P const→

$T_1 = 303\,K$ $P_1 = 100\,kPa$

$T_2 = 580\,K$

$T_3 = T_5 - 10$
$T_3 = 555.8 - 10$
$T_3 = 545.8\,K$

$T_4 = 1073\,K$

$T_5 = 555.8\,K$

$T_6 = 590\,K$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = 303(10)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 580\,K$$

$$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}}$$

$$T_5 = T_4\left(\frac{1}{10}\right)^{\frac{k-1}{k}}$$

$$= 1073(10)^{\frac{1.4-1}{1.4}}$$

$$T_5 = 555.8\,K$$

$$q_{in} = q_{out}$$

$$\cancel{mc}(T_5 - T_6) = \cancel{mc}(T_3 - T_2)$$

$$T_6 = T_5 - T_3 + T_2$$

$$= 555.8 - 545.8 + 580$$

$$T_6 = 590.0\,K$$

$$w_{net} = c_p(T_4 - T_5) - c_p(T_2 - T_1)$$

$$= 1.005(1073 - 555.8) - 1.005(580 - 303)$$

$$w_{net} = 236.4\,kJ/kg$$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{115\,kW}{236.4\,kJ/kg}$$

$$\dot{m} = .4864\,kg/s$$

$$Q_{in} = \dot{m}c(T_4 - T_3) = (.4864)(1.005)(1073 - 545.8)$$

$$\boxed{\dot{Q}_{in} = 258\,kW}$$

$$Q_{out} = \dot{m}c_p(T_6 - T_1) = \left(.4864\,\frac{kg}{s}\right)\left(1.005\,\frac{kJ}{kg\,K}\right)(590 - 303)\,K = 143\,kJ/s$$

$$\boxed{143\,kW}$$

MET 350   HW 2.2   ML Summerland

9-107

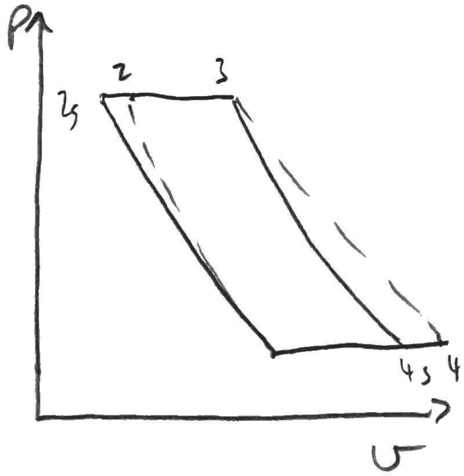
- Brayton cycle w/ regen using air
- $r_p = 7$
- $T_{max} = 1150\ K$
- $T_{min} = 310\ K$

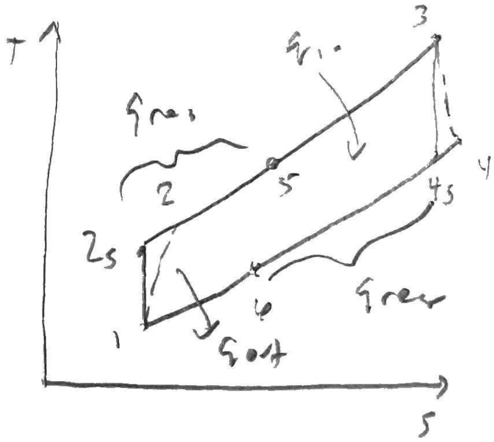
$\eta_{comp} = 75\%$

$\eta_{turb} = 82\%$

Regen effectiveness $\varepsilon = 65\%$

Determine
a) Air temp of turbine exit
b) Net work output
c) Thermal efficiency





MET 350 — HW 2.2 — ML SUNDERLAND

① Isen → ②s — P const → ② — ③ — Isent → ④s — ④ — ⑤ — ⑥

State 1: $T_1 = 310K$, $h_1 = 310.24\ kJ/kg$, $P_{r_1} = 1.5546$

State 2s: $P_2 = 7P_1$, $P_{r_2} = 10.882$, $h_{2s} = 541.256\ kJ/kg$

State 2: $h_2 = 618.26\ kJ/kg$

State 3: $T_3 = 1150\ K$, $h_3 = 1219.245\ kJ/kg$, $P_{r_3} = 200.15$

State 4s: $P_{4s} = \frac{P_7}{7}$, $P_{r_{4s}} = 28.592$, $h_{4s} = 711.789\ kJ/kg$

State 4: $h_4 = 803.131\ kJ/kg$

$$T_4 = 782.82K$$

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

$$P_{r_2} = 7(P_{r_1}) = 7(1.5546)$$

$$P_{r_2} = 10.882$$

$$\eta_c = \frac{h_{s2} - h_1}{h_2 - h_1}$$

$$\eta_c h_2 - \eta_c h_1 = h_{s2} - h_1$$

$$\eta_c h_2 = \frac{h_{s2} - h_1 + \eta_c h_1}{\eta_c}$$

$$h_2 = \frac{541.256 - 310.24 + .75(310.24)}{.75}$$

$$h_2 = 618.26\ kJ/kg$$

$$P_{r_{4s}} = \frac{P_{r_3}}{7} = \frac{200.15}{7}$$

$$P_{r_{4s}} = 28.592$$

$$\varepsilon = \frac{q_{regen,act}}{q_{regen,max}}$$

$$\varepsilon = \frac{h_3 - h_2}{h_5 - h_2}$$

$$h_5 = \frac{h_3 - h_2 + h_2\varepsilon}{\varepsilon} = \frac{1219.245 - 618.26 + 618(.65)}{.65}$$

$$h_5 = 738.425\ kJ/kg$$

$$\eta_t = \frac{h_3 - h_{4s}}{h_3 - h_4}$$

$$\cancel{\eta_t h_3}\ \cancel{\eta_t} h_4 = \frac{-h_3 + h_{4s} + \eta_t h_3}{\eta_t}$$

$$h_4 = \frac{-1219.245 + 711.789 + .82(1219.245)}{.82}$$

$$h_4 = 803.131\ kJ/kg$$

$$w_{net} = w_T - w_C = (h_3 - h_4) - (h_2 - h_1) = (1219.245 - 803.131) - (618.26 - 310.24)$$

$$w_{net} = 108.094\ kJ/kg$$

$$q_{in} = h_3 - h_5 = 1219.245 - 738.425$$

$$q_{in} = 480.82\ kJ/kg$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{108.094}{480.82} = .2248 = 22.48\%$$

9-132/ · Assume ideal process, constant specific heat

· Pure Jet

$V = 240\ m/s$

$P_{air} = 45\ kPa$

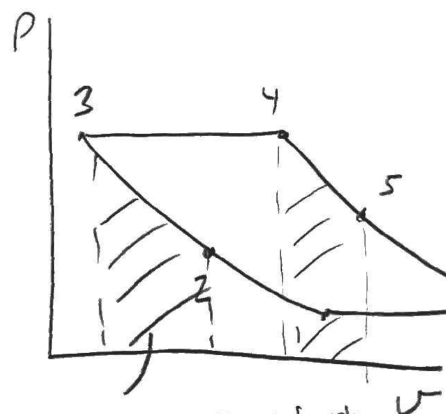
$T_{air} = -13°C + 273 =$

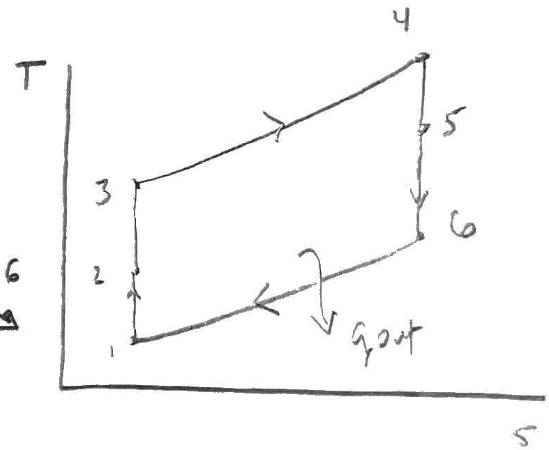
$dia_{inlet} = 1.6\ m$

$r_p = 13$

$T_{exh} = 557°C + 273$
630 K

1) Determine velocity at exit
2) Determine Thrust produced





☆ $W_{comp} \equiv W_{turb}$

P const → ① → Isent → ② → Isent → ③ → P const → ④ → Isen → ⑤ → Isent → ⑥ → P const

① $V_1 = 240\ m/s$, $P_1 = 45\ kPa$, $T_1 = 260\ K$

② $V_2 = 0\ m/s$, $T_2 = 288.7\ K$

③ $P_3 = 13 P_2$, $V_3 = 0\ m/s$

④ $T_4 = 630\ K$, $P_4 = P_3$, $V_4 = 0\ m/s$

⑤ $V_5 \approx 0\ m/s$

⑥ $P_6 = P_1$

$W_{comp} = W_{turb}$

$h_3 - h_2 = h_4 - h_5$

$\cancel{c_p}(T_3 - T_2) = \cancel{c_p}(T_4 - T_5)$

$T_5 = T_4 - T_3 + T_2$

$$T_2 = T_1 + \frac{V_1^2}{2c_p}$$

$$= 260 + \frac{240^2\ m^2\ kg\ K}{s^2 \cdot 2 \cdot 1.005\ kJ} \left(\frac{\cancel{m^2}\ kg}{\cancel{s^2}\ \cancel{kJ}} \middle| \frac{1\ \cancel{kJ}}{1000\ \cancel{J}} \middle| \frac{1\ \cancel{J}\ \cancel{s^2}}{\cancel{kg}\ \cancel{m^2}} \right)$$

$T_2 = 288.7\ K$

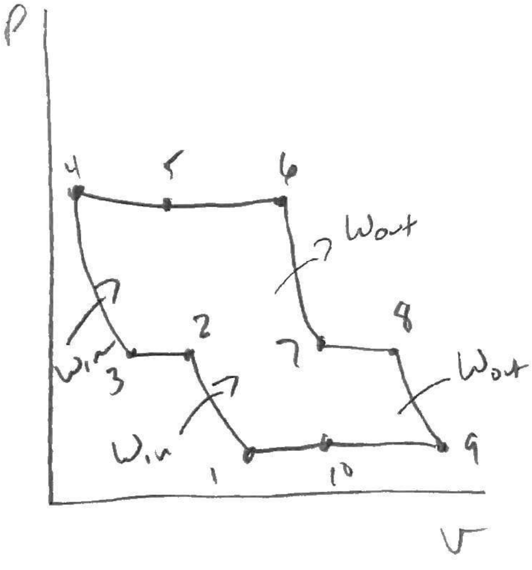
MET 350	HW 2.2	ML SUNDERLAND	6

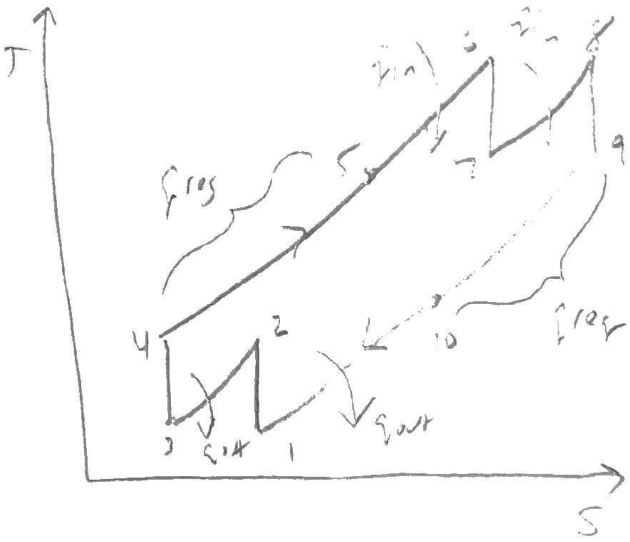
9-121

Ideal gas turbine cycle w/ 2 stages of compression & 2 expansion
$r_p = 3$ for each stage
Air enters compressor @ 300 K
Air enters each turbine @ 1200 K

Using variable specific heats

a) Determine back work ratio and thermal efficiency if 1) no regenerator used
2) regenerator w/ 75% effect





$$r_p = 3 = \frac{P_2}{P_1} = \frac{P_4}{P_3} = \frac{P_6}{P_7} = \frac{P_8}{P_9}$$

① Isen → ② Pconst → ③ Isen → ④ Pconst regen → ⑤ Pconst q_{in} → ⑥ Isent → ⑦ Pconst q_{in} → ⑧ Isen → ⑨ Pconst Regen → ⑩ Pconst →

9-121 Cont

$T_1 = 300K$
$h_1 = 300.19\ kJ/kg$
$P_r = 1.3860$

$P_2 = 3P_1$
$P_{r_2} = 4.158$
$h_2 = 411.16\ kJ/kg$

$T_3 = 300K$
$h_3 = h_1$
$P_{R_3} = P_{r_1}$

$P_4 = 3P_3$
$h_2 = h_4$
$h_4 = 411.16\ kJ/kg$

$P_5 = P_4$

$T_6 = 1200\ K$
$P_6 = P_5$
$P_{r_6} = 238$
$h_6 = 1277.79$

$P_7 = P_6/3$
$P_{r_7} = 79.33$
$h_7 = 946.35\ kJ/kg$

$T_8 = 1200K$
$P_8 = P_7$
$h_8 = 1277.79\ kJ/kg$
$P_{r_8} = 238$

$P_9 = P_{10}$
$h_9 = 946.35\ kJ/kg$
$P_{r_9} = 79.33$

$P_{10} = P_1$

① → ②

$$\frac{P_2}{P_1} = \frac{P_{r_2}}{P_{r_1}}$$

$$3 = \frac{P_{r_2}}{P_{r_1}}$$

$P_{r_2} = 3(1.3860)$

$P_{r_2} = 4.158$

⑥ → ⑦

$P_{r_7} = \frac{1}{3}(P_{r_6})$

$P_{r_7} = 79.33$

W/ regen

$$\eta_{th} = \frac{440.74}{796.65} = .553$$

55.3%

Regen

$q_{regen} = \varepsilon(h_9 - h_4)$

$= .75(946.35 - 411.26)$

$q_{regen} = 401.32\ kJ/kg$

$\Rightarrow q_{in} = 1197.97 - 401.32$

$q_{in} = 796.65\ kJ/kg$

Work

$w_{c1} = w_{c2}$

$w_c = w_{c_1} + w_{c_1}$

$= 2(h_2 - h_1)$

$= 2(411.16 - 300.19)$

$w_c = 222.14\ kJ/kg$

$w_{turb} = w_{turb_1} + w_{turb_2}$

$\Rightarrow 2(w_{turb})$

$= 2(h_6 - h_7)$

$= 2(1277.79 - 946.33)$

$w_{turb} = 661.88\ kJ/kg$

Back Work

$$r_{bw} = \frac{w_{c\,in}}{w_{turb}} = \frac{222.14}{667.88} = .335 =$$ **33.5%**

Heat

$q_{in} = (h_6 - h_4) + (h_8 - h_7)$

$= (1277.79 - 411.16) + (1277.79 - 946.35)$

$q_{in} = 1197.97\ kJ/kg$

$w_{net} = w_t - w_c$

$w_{net} = 661.88 - 222.14 = 440.74\ kJ/kg$

$$\eta_{th,\ no\ regen} = \frac{w_{net}}{q_{in}} = \frac{440.74}{1197.97} = .368 =$$ **36.8%**

MET 350

HW 7.2

ML SUNDERLAND

9-129E)

- Turbojet engine

$V = 900 \text{ ft/s}$ @ altitude of 20,000 ft

$P = 7 \text{ psia}$ $T = 10F + 460 = 470R$

$r_p = 13$

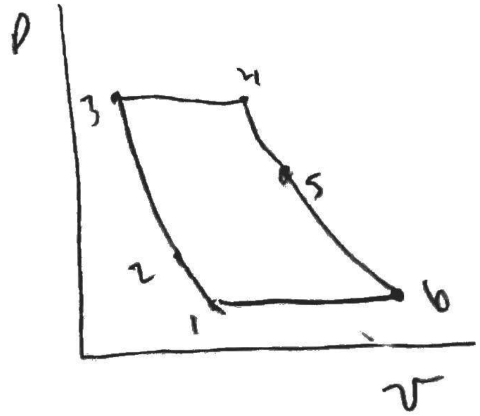
$T_{Turb, inlet} = 2400 R$

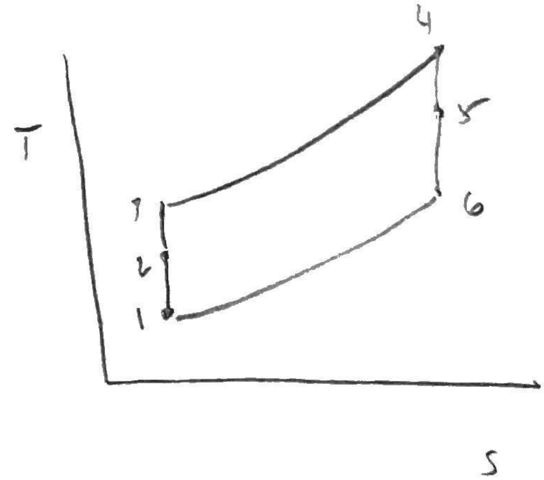
- Assume ideal + constant specific heats for air @ room temp

a) Determine pressure @ turbine exit (P_5)

b) velocity of exhaust gasses (V_6)

c) propulsive efficiency





Constants

$C_p = .240 \text{ BTU/lbm·R}$

$C_v = 0.171 \text{ Btu/lbm·R}$

$k = 1.4$

$1 \text{ Btu/lbm} = 25,037 \text{ ft}^2/\text{s}^2$

$R = F + 460$

Isen → Isent → Pconst → Isen ($W_C = W_T$) → Isen → Pcond →

① ② ③ ④ ⑤ ⑥

①	②	③	④	⑤	⑥
$V_1 = 900$ ft/s	$V_1 = 0$ ft/s	$V_3 = 0$ ft/s	$V_4 = 0$ ft/s	$V_5 = 0$ ft/s	$P_6 = P_1$
$P_1 = 7$ psia	$T_2 = 537.4$ R	$P_3 = 13 P_2$	$T_4 = 2400$ R	$T_5 = 1819.1$ R	$T_6 = 1008.4$ R
$T_1 = 470$ R	$P_2 = 11.2$ psia	$P_3 = 145.6$ psia	$P_4 = P_3$	$P_5 = 55.2$ psia	$P_6 = 7$ psia
		$T_3 = 1118.3$ R	$P_4 = 145.6$ psia		

9-129E cont

MET 350

HW 2.2

MCSUNDERLAND

For T_2

$$T_2 = T_1 + \frac{V_1^2}{2C_p}$$

$$T_2 = 470 + \frac{900\,ft^2}{s^2}\Big|\frac{}{2}\Big|\frac{lbm\ R}{.240\ Btu}\Big|\frac{1\ Btu\ s^2}{25{,}037\ ft^2\ lbm}$$

$$T_2 = 537.4\ R$$

For P_2

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}$$

$$= 7\left(\frac{537.4}{470}\right)^{\frac{1.4}{.4}}$$

$$P_2 = 11.2\ psia$$

For T_3

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}}$$

$$T_3 = T_2 (r_p)^{\frac{k-1}{k}}$$

$$= 537.4(13)^{\frac{.4}{1.4}}$$

$$T_3 = 1118.3\ R$$

For T_5

$$W_{comp} = W_{turb}$$

$$h_3 - h_2 = h_4 - h_5$$

$$c_p(T_3 - T_2) = c_p(T_4 - T_5)$$

$$T_5 = T_4 - T_3 + T_2$$

$$T_5 = 2400 - 1118.3 + 537.4$$

$$T_5 = 1819.1$$

For P_5

$$P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}}$$

$$P_5 = 145.6\left(\frac{1819.1}{2400}\right)^{\frac{1.4}{.4}}$$

$$P_5 = 55.2\ psia$$

For T_6

$$T_6 = T_5\left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}}$$

$$= 1819.1\left(\frac{7}{55.2}\right)^{.4/1.4}$$

$$T_6 = 1008.4\ R$$

For V_6

$$h_5 + \frac{V_5^2}{2} = h_6 + \frac{V_6^2}{2}$$

$$0 = h_6 - h_5 + \frac{V_6^2}{2}$$

$$= c_p(T_6 - T_5) + \frac{V_6^2}{2}$$

$$\frac{V_6^2}{2} = -c_p(T_6 - T_5)$$

$$V_6^2 = -2c_p(T_6 - T_5)$$

$$V_6^2 = 2c_p(T_5 - T_6)$$

$$V_6 = \sqrt{2(.240)(1819 - 1008.4)\,25{,}037}$$

$$V_6 = 3121\ ft/s$$

c

9-124E cont

$$\dot{W}_p = V_{aircraft}\left(V_{exit} - V_{inlet}\right)\dot{m}$$

$$w_p = V_{aircraft}\left(V_{exit} - V_{inlet}\right)$$

$$= \frac{900\ ft}{s}\left(3121\ ft/s - 900\ ft/s\right) \frac{1\ BTU}{25{,}037\ ft^2/s^2\ lbm}$$

$$w_p = 79.8\ BTU/lbm$$

$$q_{in} = h_4 - h_3$$

$$= c_p\left(T_4 - T_3\right)$$

$$= \frac{.24\ Btu}{lbm\ R}\left(2400 - 1118.3\right) R$$

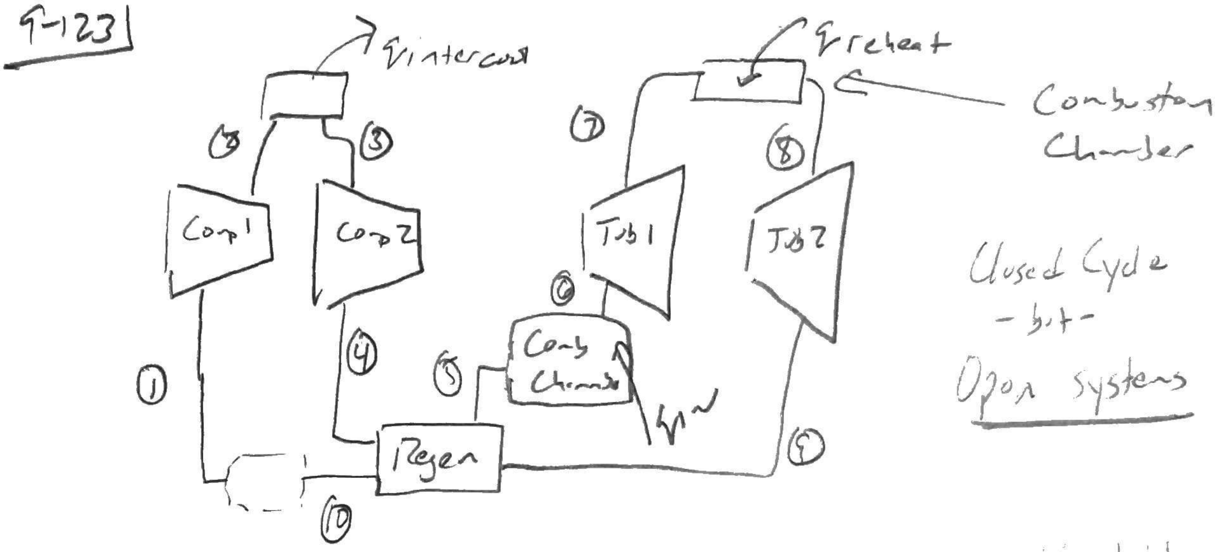
$$q_{in} = 307.6\ Btu/lbm$$

$$\eta_p = \frac{w_p}{q_{in}} = \frac{79.8}{307.6} = .259 = \boxed{25.9\ \%}$$

MET 350	HW 22	ML SUNDERLAND

13

9-123



Closed Cycle
- but -
Open systems

Air enters Expansion @ 100 kPa @ 17°C ⇒ k = 1.4

$c_p = 1.005$ kJ/kg·K

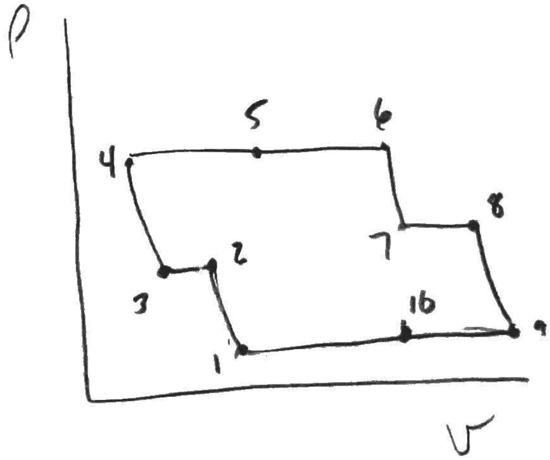
Compression ratios = 4

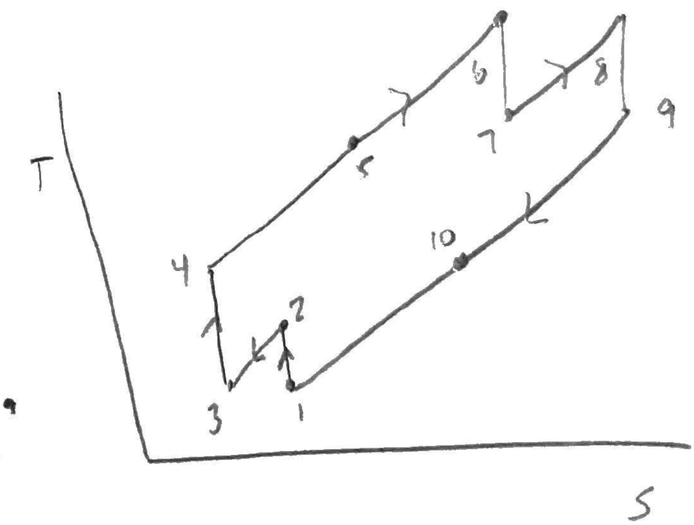
300 kJ/kg heat added in combustion chamber

Regenerator increases cold air by 20°C

Isentropic for all turb + compressors

Constant specific heats





- Determine Thermal Efficiency $\eta_{th} = \frac{W_{net}}{q_{in}}$

$W_{net} = W_{out} - W_{in}$

$W_{out} = W_{6\text{-}7} + W_{8\text{-}9}$ $\quad W_{in} = W_{1\text{-}2} + W_{3\text{-}4}$

Isent → Pcont → Isent → Pconst → Pconst → Isent → Pcont → Isent → Pcont → Pconst

(1) $T_1 = 290K$, $P_1 = 100 kPa$

(2) $P_2 = 4P_1$, $P_2 = 400 kPa$, $T_2 = 432.9 K$

(3) $P_2 = P_3$, $T_3 = T_1$, $T_3 = 290K$, $P_3 = 400 kPa$

(4) $P_4 = 4P_3$, $P_4 = 1600 kPa$, $T_4 = 430.9 K$

(5) q_{in}: $P_5 = P_4$, $T_5 = 20K + T_4$, $T_5 = 450.9K$, $P_5 = 1600 kPa$

(6) $P_6 = P_5$, $P_6 = 1600 kPa$, $T_6 = 749.4 K$

(7) q_{in}: $P = 400 kPa$, $T_7 = 504.3$

(8) $P = 400 kPa$, $T_8 = 802.8$

(9) $P_9 = P_{10} = 100 kPa$, $T_9 = 540.2K$

(10) $P_{10} = P_1 = 100 kPa$, $T_{10} = T_9 - 20K$, $T_{10} = 520.2K$

$$T_8 = \frac{300}{1.005} + T_7$$

$$T_8 = 802.8K$$

$$T_9 = T_8\left(\frac{1}{r_p}\right)^{k-1/k}$$

$$= 802.8K\left(\frac{1}{4}\right)^{1.4-1/4}$$

$$T_9 = 540.2K$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} =$$

$$T_2 = T_1 (4)^{\frac{1.4-1}{1.4}} = T_4$$

$$T_2 = 430.9 K$$

$$q_{in} = c_p(T_6 - T_5)$$

$$T_6 = \frac{q_{in}}{c_p} + T_5$$

$$T_6 = \frac{300}{1.005} + 450.9$$

$$T_6 = 749.4 K$$

$$\frac{T_7}{T_6} = \left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}}$$

$$r_p = \frac{P_7}{P_6} = \sqrt{\frac{P_6}{P_5}} = \sqrt{\frac{1600}{1000}}$$

$$r_p = 4$$

$$r_p = \frac{P_7}{1600}$$

$$4 = \frac{P_7}{1600}$$

$$P_7 = 400$$

$$T_7 = T_6\left(\frac{1}{4}\right)^{\frac{1.4-1}{1.4}}$$

$$T_7 = 504.3K$$

$$W_{net} = W_{out} - W_{in} = 509.64 - 283.2 = 226.43\ kJ/kg$$

$W_{out} = W_{6-7} + W_{8-9}$

$= c_p(T_6 - T_7) + c_p(T_8 - T_9)$

$= 1.005(749.4 - 504.3) + 1.005(802.8 - 540.8)$

$W_{out} = 509.64$

$W_{in} = W_{1-2} + W_{3-4}$

$= c_p(T_2 - T_1) + c_p(T_4 - T_3)$

$= 1.005(430.9 - 290) + 1.005(430.9 - 290)$

$W_{in} = 283.2$

$W_{net} = 226.43\ kJ/kg$

$q_{in} = q_{comb} + q_{reheat}$

$= 300 + 300$

$q_{in} = 600\ kJ/kg$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{226.43}{600} \times 100$$

$$\eta_{th} = 37.7\%$$