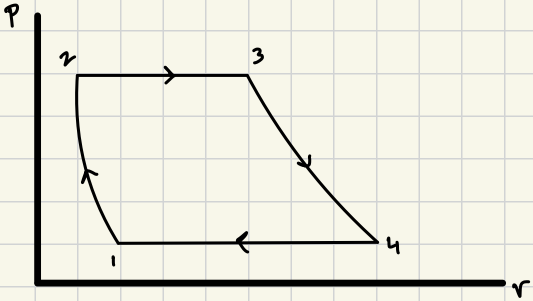
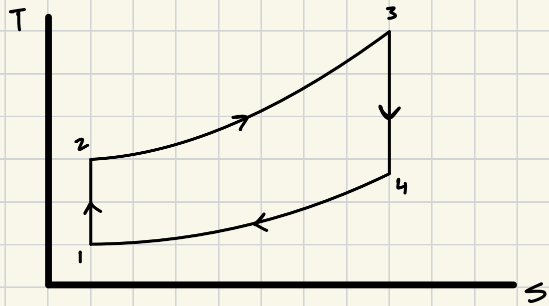
HW 1.4 Ch 9- 88, 99, 107, 119, 121

9–88 An aircraft engine operates on a simple ideal Brayton cycle with a pressure ratio of 10. Heat is added to the cycle at a rate of 500 kW; air passes through the engine at a rate of 1 kg/s; and the air at the beginning of the compression is at 70 kPa and 0°C. Determine the power produced by this engine and its thermal efficiency. Use constant specific heats at room temperature.

$$\frac{P_2}{P_1} = 10$$





$$\rightarrow (1) \xrightarrow{\text{ISENT}} (2) \xrightarrow{\text{P CONST}} (3) \xrightarrow{\text{ISENT}} (4) \xrightarrow{\text{P CONST}}$$

(1)
$P_1 = 70\,kPa$
$T_1 = 273\,K$
$v_1 = 1.12\ m^3/kg$

(2)
$P_2 = 700\,kPa$
$T_2 = 527.1\,K$
$v_2 = .216\ m^3/kg$

$T_2 = T_1 (10)^{.4/1.4}$

(3)
$P_3 = 700\,kPa$
$T_3 = 1024.6\,K$

$Q_{sup} = \dot{m} c_p (T_3 - T_2)$
$\rightarrow 500 = 1 \cdot 1.005 (T_3 - 527.1)$
$T_3 =$

(4)
$T_4 = 530.7$
$P_4 = 70\,kPa$

$T_4 = \frac{1024.6}{10^{.4/1.4}}$

$\frac{P_3}{P_4} = 10$ a $P_4 = P_1$

$$\text{Power}(P) = \dot{m}(w_{TURB} - w_{comp}) = \dot{m}\left[c_p(T_3 - T_4) - (T_2 - T_1)\right]$$

$$\Rightarrow 1.005(1024.6 - 530.7) - (527.1 - 273) \Rightarrow$$

$$\boxed{P = 242.3\ kW}$$

$$\eta_{TH} = \frac{P}{Q_{sup}} = \frac{242.3}{500} \Rightarrow \boxed{\eta_{TH} = 48.45\%}$$

9–99 A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 30°C. The compressor pressure ratio is 10; the maximum cycle temperature is 800°C; and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and rejection for this cycle when it produces 115 kW. Use constant specific heats at room temperature. *Answers: 258 kW, 143 kW*

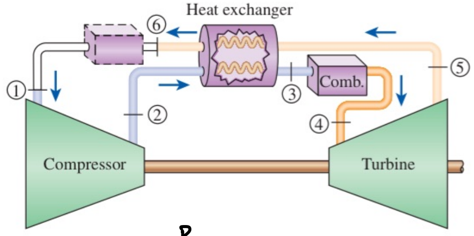
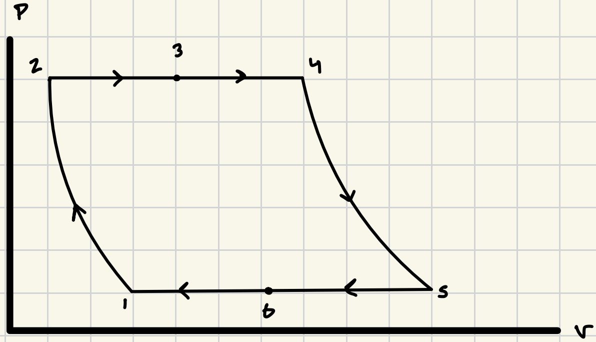
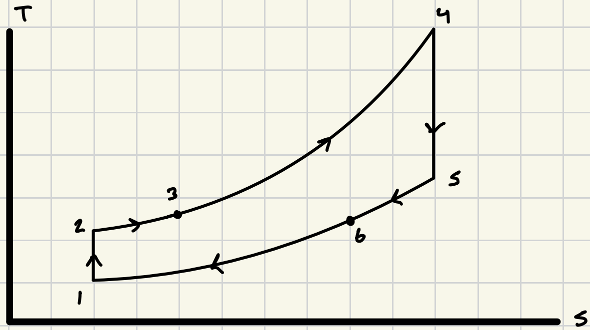


FIGURE P9–99





$$\frac{P_2}{P_1} = 10$$

$$T_3 = T_5 - 10$$

① $\xrightarrow{\text{ISENT}}$ ② $\xrightarrow{\text{P CONST}}$ ③ $\xrightarrow{\text{P CONST}}$ ④ $\xrightarrow{\text{ISENT}}$ ⑤ $\xrightarrow{\text{P CONST}}$ ⑥

1	2	3	4	5	6
$T_1 = 303K$	$T_2 = 585K$	$T_3 = 545.8K$	$T_4 = 1073K$	$T_5 = 555.8K$	$T_6 = 595K$
$P_1 = 100kPa$	$P_2 = 1000kPa$	$P_3 = 1000KPa$	$P_4 = 1000kPa$	$P_5 = 100kPa$	$P_6 = 100kPa$
$v_1 = .869\ m^3/kg$	$v_2 = .168\ m^3/kg$				

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{.4/1.4}$$

$$T_3 = T_5 - 10$$

$$\frac{T_5}{T_4} = \frac{1}{10}^{.4/1.4}$$

$$\frac{T_5}{T_4} = \frac{P_5}{P_4}^{\frac{k-1}{k}} = \frac{P_1}{P_2}^{\frac{k-1}{k}}$$

$$T_5 - T_6 = T_3 - T_2$$

$$w_{net} = w_T - w_c = c_p(T_4 - T_5) - c_p(T_2 - T_1)$$

$$= 1.005(1073 - 555.8) - 1.005(585 - 303) \Rightarrow \underline{w_{net} = 236.38\ kJ/kg}$$

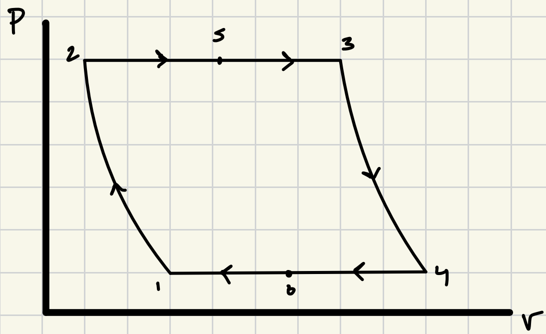
$$\dot{m} = \frac{\dot{w}_{net}}{w_{net}} = \frac{115\,kW}{236.38} \Rightarrow \underline{\dot{m} = .487\ kg/s}$$

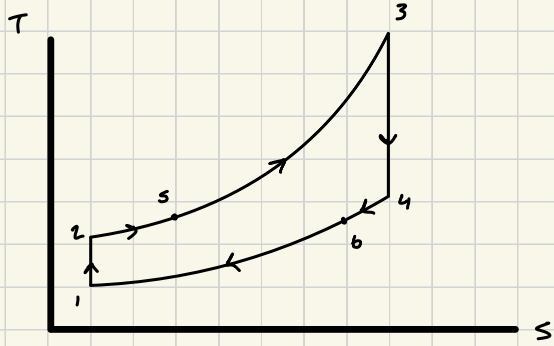
heat addition $\quad Q_{in} = \dot{m} c_p (T_4 - T_3) = .487 \cdot 1.005(1073 - 545.8) \Rightarrow \boxed{Q_{in} = 257.77\ kW}$

heat rejection $\quad Q_{OUT} = \dot{m} c_p (T_6 - T_1) = .487 \cdot 1.005(595 - 303) \Rightarrow \boxed{Q_{OUT} = 142.92\ kW}$

9–107 A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 and 1150 K. Assuming an isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine and an effectiveness of 65 percent for the regenerator, determine (*a*) the air temperature at the turbine exit, (*b*) the net work output, and (*c*) the thermal efficiency. *Answers:* (*a*) 783 K, (*b*) 108 kJ/kg, (*c*) 22.5 percent

$$\frac{P_2}{P_1} = 7$$





$$1 \xrightarrow{\text{ISENT}} 2 \xrightarrow{\text{P CONST}} 5 \xrightarrow{\text{P CONST}} 3 \xrightarrow{\text{ISENT}} 4 \xrightarrow{\text{P CONST}} 6 \longrightarrow$$

1	2	5	3	4	6
$T_1 = 310K$	$T_2 = 540.5K$	$P_5 = P_2$	$T_3 = 1150K$	$P_4 = P_1$	$P_6 = P_1$
	$P_2 = 7P_1$	$T_5 = 702.16$	$P_3 = P_2$	$T_4 = 659.5K$	$T_6 = 663.03K$
	$\uparrow T_2 = T_1(7)^{.4/1.4}$	$\uparrow \epsilon = \frac{T_5 - T_{2a}}{T_{4a} - T_{2a}}$		$\uparrow \frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{.4/1.4}$	$\uparrow T_6 = T_{4a} - (T_5 - T_{2a})$

a) $T_{TURB,EX} = T_{4a} = \boxed{747.8K}$

b) $w_{net} = w_{OUT} - w_{in} = c_p(T_3 - T_{4a}) - c_p(T_3 - T_{4a}) - c_p(T_{2a} - T_1) \Rightarrow \boxed{108\ kJ/kg}$

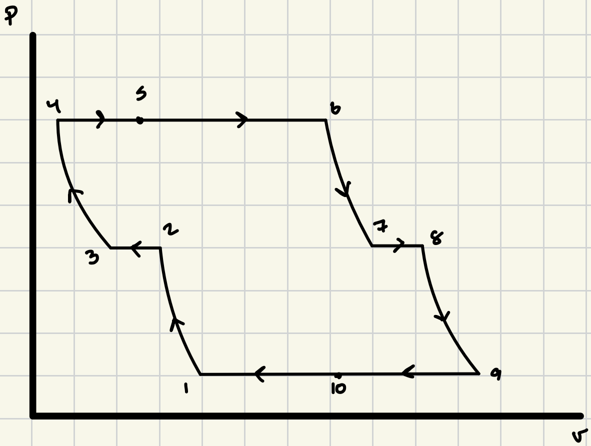
c) $\eta_{TH} = \frac{w_{net}}{q_{in}}$ where $q_{in} = c_p(T_3 - T_5) \Rightarrow \boxed{22.5\%}$

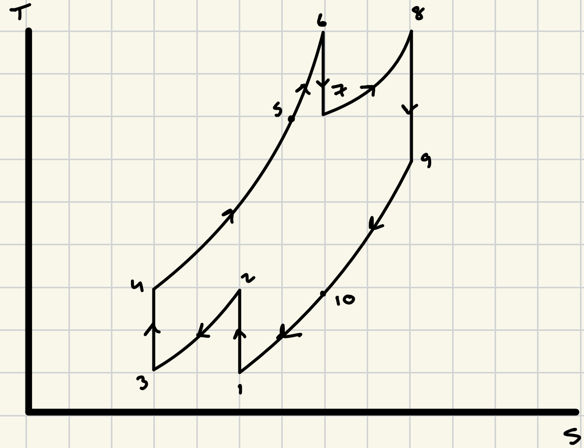
9–119 Consider a regenerative gas-turbine power plant with two stages of compression and two stages of expansion. The overall pressure ratio of the cycle is 9. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Accounting for the variation of specific heats with temperature, determine the minimum mass flow rate of air needed to develop a net power output of 110 MW.
Answer: 250 kg/s

$r_p = \sqrt{9} = \underline{3}$

$\frac{P_2}{P_1} = \frac{P_4}{P_3} = 3 \quad \rightarrow \quad \frac{P_6}{P_7} = \frac{P_8}{P_9}$

c_p, c_v CONST??





$① \xrightarrow{\text{ISENT}} ② \xrightarrow{\text{P CONST}} ③ \xrightarrow{\text{ISENT}} ④ \xrightarrow{\text{P CONST}} ⑤ \xrightarrow{\text{P CONST}} ⑥ \xrightarrow{\text{ISENT}} ⑦ \xrightarrow{\text{P CONST}} ⑧ \xrightarrow{\text{ISENT}} ⑨ \xrightarrow{\text{P CONST}} ⑩$

$T_1 = 300K$ | $T_2 = 410.6K$ | $T_3 = 300K$ | $T_4 = 410.6K$ | $T_5 = 1200K$ | $T_6 = 876.7K$ | $T_7 = 1200K$ | $T_8 = 876.7K$ | $T_9 = 1200K$ | $T_{10} = 410.6K$

$\uparrow \frac{T_2}{T_1} = 3^{.4/1.4}$ $\quad \uparrow \frac{T_4}{T_3} = 3^{.4/1.4}$ $\quad \uparrow \frac{T_5}{T_6} = 3^{.4/1.4}$

* only solved for temp. b/c that was all I had enough info for & that is all that is required for problem statement

$w_c = c_p(T_2 - T_1) + c_p(T_4 - T_3) = 1.005(410.6 - 300) + 1.005(410.6 - 300) \Rightarrow \underline{w_c = 222.3 \text{ kJ/kg}}$

$w_T = c_p(T_5 - T_6) + c_p(T_7 - T_8) = 1.005(1200 - 876.7) + 1.005(1200 - 876.7) \Rightarrow \underline{w_T = 649.8 \text{ kJ/kg}}$

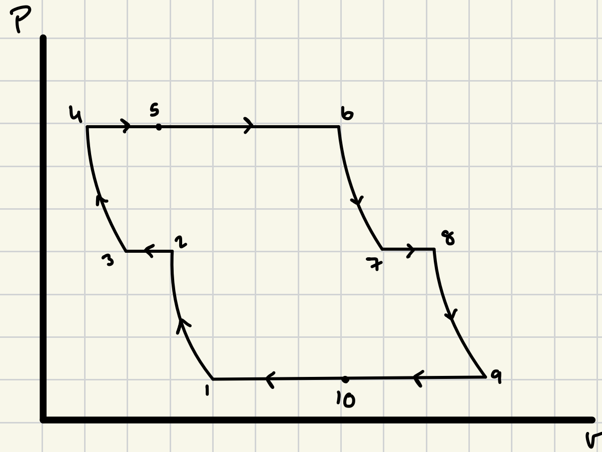
$w_{NET} = w_T - w_c = 649.8 - 222.3 = \underline{427.5} \text{ kJ/kg} \rightarrow \dot{P} = \dot{m} \cdot w_{NET} \Rightarrow$

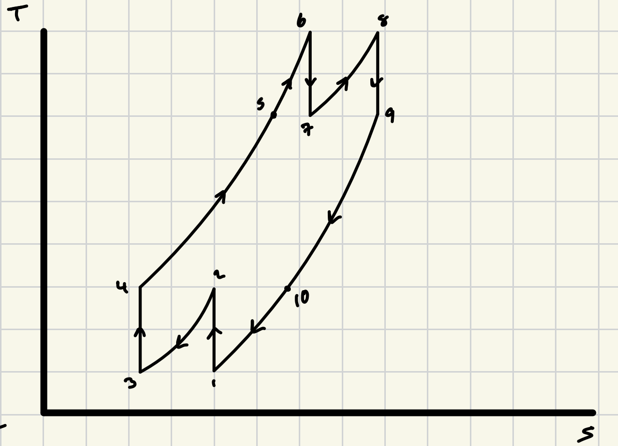
$110E3 = \dot{m} \cdot 427.5$

$\boxed{\dot{m} = 257.3 \text{ kg/s}}$

9–121 Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and turbine is 3. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Determine the back work ratio and the thermal efficiency of the cycle, assuming (*a*) no regenerator is used and (*b*) a regenerator with 75 percent effectiveness is used. Use variable specific heats.

$r_p = 3$





$$1 \xrightarrow{\text{ISENT}} 2 \xrightarrow{\text{P CONST}} 3 \xrightarrow{\text{ISENT}} 4 \xrightarrow{\text{P CONST}} 5 \xrightarrow{\text{P CONST}} 6 \xrightarrow{\text{ISENT}} 7 \xrightarrow{\text{P CONST}} 8 \xrightarrow{\text{ISENT}} 9 \xrightarrow{\text{PCONST}} 10$$

$T_1 = 300K$, $h_1 = 300.19\ kJ/kg$

$T_2 = 410K$, $h_2 = 411.12\ kJ/kg$

$$3 = \frac{P_{r_2}}{1.386}$$

$P_{r_2} = 4.158$

$T_2 \cong 410K$

$T_3 = 300K$, $h_3 = 300.19\ kJ/kg$

$T_4 = 410K$, $T_4 = 411.12\ kJ/kg$

$$3 = \frac{P_{r_4}}{P_{r_3}}$$

$T_5 = 1200K$, $h_5 = 1277.79\ kJ/kg$

$T_6 = 911K$, $h_6 = 946.36\ kJ/kg$

$$P_{r_6} = \frac{P_{r_5}}{r_p}$$

		X	Y
1	900	75.29	932.93
	917.	79.33	?
2	920	82.05	955.38

$T_7 = 1200K$, $h_7 = 1277.79\ kJ/kg$

$T_8 = 911K$, $h_8 = 946.31\ kJ/kg$

$T_9 = 1200K$, $h_9 = 1277.79\ kJ/kg$

$T_{10} = 410K$, $h_{10} = 411.12\ kJ/kg$

a) $w_c = 2(429.34 - 300.19) = 258.3\ kJ/kg$ $\qquad \frac{w_c}{w_T} \Rightarrow .433$ or 43.3%

$w_T = 2(1277.79 - 979.5) = 596.58\ kJ/kg$

$Q_{in} = (1277.79 - 429.34) + (1277.79 - 979.5) = 1146.74\ kJ/kg$

$$\rightarrow \eta_{TH} = \frac{w_T - w_c}{Q_{in}} = 29.5\%$$

b) $Q_{in} = 1146.74 - .75(979.4 - 429.34) = 717.69\ kJ/kg$

$$\eta_{TH} = \frac{w_T - w_c}{Q_{in}} = 47.13\%$$

$B.W.R = 43.3\%$