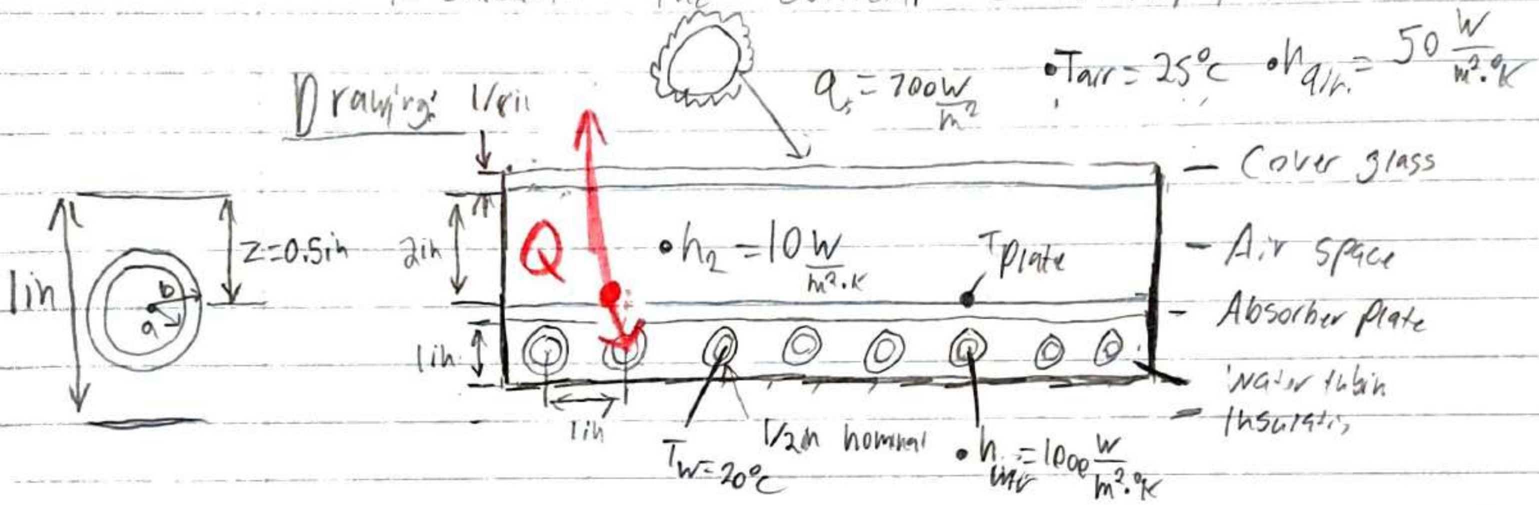
Test 1 Jacob Leonard

Problem 1

Purpose: To calculate the amount of heat collected by one of the tubes. To calculate the air space temperature. To calculate the flow rate of the water in the tubes. To calculate the collector efficiency, η.

Drawing:



Sources:
- Test 1
- Textbook

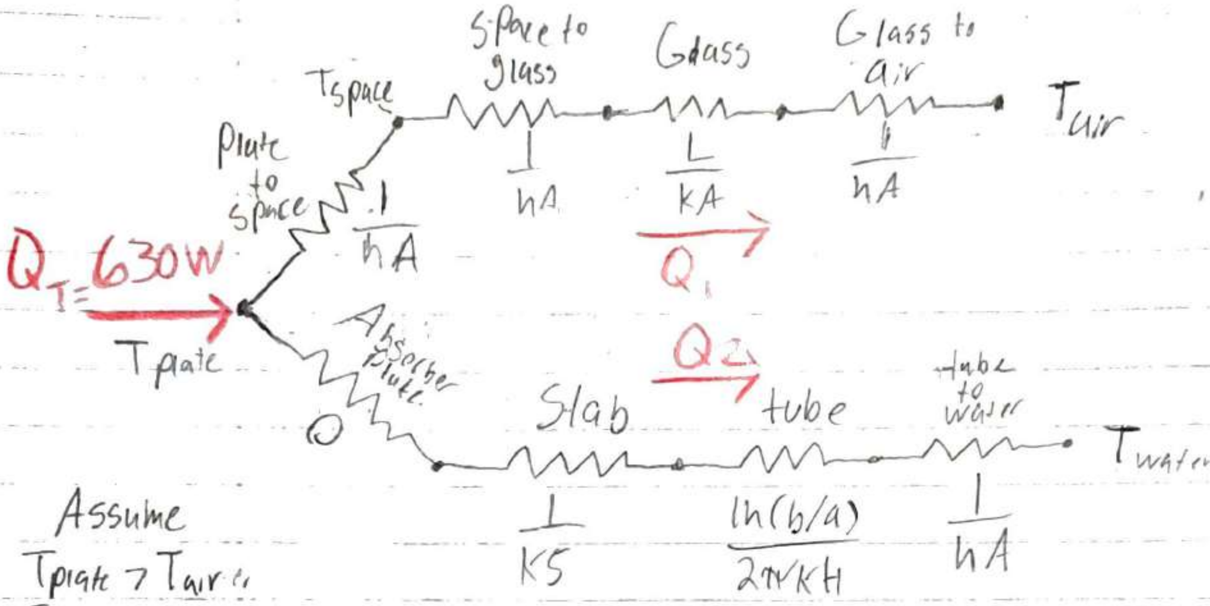
Design Considerations
- Steady State

Data and Variables
- $q_s = 700 W/m^2$
- $\alpha = 90\%$
- $T_{wtr} = 20°C$
- $h_{wtr} = 1000 W/m^2 \cdot °C$
- $T_{air} = 25°C$
- $h_{air} = 50 W/m^2 \cdot °K$
- $h_{space} = 10 W/m^2 \cdot °C$
- Tube OD $= 0.625in = 15.9 \times 10^{-3} m$
- Tube ID $= 0.528in = 13.4 \times 10^{-3} m$
- $K_{copper} = 386 W/m \cdot °C$
- $K_{silver} = 419 W/m \cdot °C$
- $K_{glass} = 0.78 W/m \cdot °C$
- $L_{glass} = 0.125in = 3.175 \times 10^{-3} m$
- $z = 0.5in = 12.7 \times 10^{-3} m$

Procedure:

The first step is to look at the drawing and understand where the heat is going. The second step is to draw the thermal resistance circuit. From the thermal resistane circuit and drawing, the equivalent resistance can be determined for each path in the system. Next, using the formula $Q = \Delta T / R$, the temperature of the absorber plate can be determined. The temperature of the plate is needed to calculate the amount of heat collected by one of the tubes. With the total amount of heat absorbed by the plate and the heat collected by one of the tubes, the amount of heat lost to the atmosphere can be determined. This is need to calculate the temperature of the air space. Next, using the inlet and outlet temperatures of the water, heat collected by a tube, and the specific heat of the water, the mass flow rate of the water can be calculated. Finally, with the amount of solar energy that is incident on the collector and heat absorbed by one of the tubes, the collector efficiency, η, can be computed.

Calculations: $Q_{absorbed} = q_s \alpha A = 700 \frac{W}{m^2} \cdot 0.90 \cdot 1m^2 = 630W$



Assume
$T_{plate} > T_{air}$
$T_{plate} > T_{water}$

$$Q_T = Q_1 + Q_2 \qquad S = \frac{2\pi L}{\ln\left(\frac{8Z}{\pi D}\right)}$$

$$Q = \frac{\Delta T}{R}$$

$$Q_1 = \frac{T_{plate} - T_{air}}{\frac{1}{h_{space} \cdot A} + \frac{1}{h_{space} A} + \frac{L_{glass}}{K_{Glass} \cdot A} + \frac{1}{h_{air} A}}$$

$$Q_2 = \frac{T_{plate} - T_{water}}{\frac{1}{K_{silver}\left[\frac{2\pi H}{\ln\left(\frac{8Z}{\pi D}\right)}\right]} + \frac{\ln(b/a)}{2\pi K_{copper} H} + \frac{1}{h_{water}(2\pi a \cdot H)}}$$

$$Q_T = \frac{T_{plate} - T_{air}}{\frac{1}{h_{space} A} + \frac{1}{h_{space} A} + \frac{L_{glass}}{K_{Glass} A} + \frac{1}{h_{air} A}} + \frac{T_{plate} - T_{water}}{\frac{1}{K_{silver}\left[\frac{2\pi H}{\ln\left(\frac{8Z}{\pi D}\right)}\right]} + \frac{\ln(b/a)}{2\pi K_{copper} H} + \frac{1}{h_{water}(2\pi a H)}}$$

- Need to find temperature of the absorber plate first

$$630\,W = \frac{T_{plate} - 25°C}{\frac{1}{10\frac{W}{m^2 \cdot °C} \cdot 1m^2} + \frac{1}{10\frac{W}{m^2 \cdot °C} \cdot 1m^2} + \frac{3.175 \times 10^{-3} m}{0.78\frac{W}{m \cdot °C} \cdot 1m^2} + \frac{1}{50\frac{W}{m^2 \cdot °C} \cdot 1m^2}} +$$

$$+ \frac{T_{plate} - 20°C}{\frac{1}{419\frac{W}{m \cdot °C}\boxed{\frac{2\pi \cdot 1m}{\ln\left(\frac{8 \cdot 0.5\cancel{in}}{\pi \cdot 0.625\cancel{in}}\right)}}} + \frac{\ln\left(\frac{0.3125in}{0.264in}\right)}{2 \cdot \pi \cdot 386\frac{W}{m \cdot °C} \cdot 1m} + \frac{1}{1000\frac{W}{m^2 \cdot °C}(2 \cdot \pi \cdot 6.71 \times 10^{-3} m \cdot 1m)}}$$

$$630\,W = \frac{T_{plate} - 25°C}{0.224\frac{°C}{W}} + \frac{T_{plate} - 20°C}{0.0241\frac{°C}{W}}$$

$$630\,W = \frac{T_{plate}}{0.224\frac{°C}{W}} - 111.57\,W + \frac{T_{plate}}{0.0241\frac{°C}{W}} + 830.76\,W$$

$$1572.33\,W = T_{plate}\left(\frac{1}{0.224\frac{°C}{W}} + \frac{1}{0.0241\frac{°C}{W}}\right)$$

$$1572.33\,W = T_{plate}\left(46.0\frac{W}{°C}\right)$$

$$T_{plate} = 34.18°C$$

a) $$Q_2 = \frac{T_{plate} - T_{water}}{R_{eq_2}} = \frac{34.18°C - 20°C}{0.0241\frac{°C}{W}}$$

$$Q_2 = 589.03\,W$$

b)

$$Q_T - Q_2 = Q_1$$

$$630W - 589.03W = Q_1$$

$$Q_1 = 40.97\,W$$

$$Q_1 = \frac{T_{plate} - T_{space}}{\frac{1}{h_{space}A}}$$

$$\frac{Q_1}{h_{space}A} = T_{plate} - T_{space}$$

$$T_{space} = T_{plate} = \frac{Q}{h_{space}A}$$

$$T_{space} = 34.18°C - \frac{40.97W}{10\,\frac{W}{m^2 \cdot °C} \cdot 1\,m^2}$$

$$T_{space} = 30.08\,°C$$

c) $T_{water,in} = 20°C$ $T_{water,out} = 45°C$ $C_p = 4179\,\frac{J}{Kg \cdot °K}$

$$Q = \dot{m} \cdot c_p \cdot \Delta T$$

$$\dot{m} = \frac{Q}{c_p \Delta T} = \frac{58903\,\frac{J}{s}}{4179\,\frac{J}{Kg \cdot °C}(45°C - 20°C)}$$

mass flow rate $\dot{m} = 0.00564\,\frac{Kg}{s}$

d) $\eta = \frac{\text{Useful heat collected}}{\text{Solar energy incident on collector}}$

$\eta = \frac{589.03 W}{700 W}$

$\eta = 0.842$

Summary:

The amount of heat collected by one of the tubes in the solar collector is 589.03 watts. The temperature of the air space is 30.08°C. The mass flow rate of the water is 0.00564 kg/s. The efficiency of the solar collector is 84.2%. The incident radiation is absorbed by the absorber plate, most of the heat goes through the absorber plate, silver slab, copper tubes, and finally to the water where this heat can be used. A smaller portion of the heat is lost through convection to the air space, through the cover glass, and finally transfered to the atmosphere through convection.

Materials:

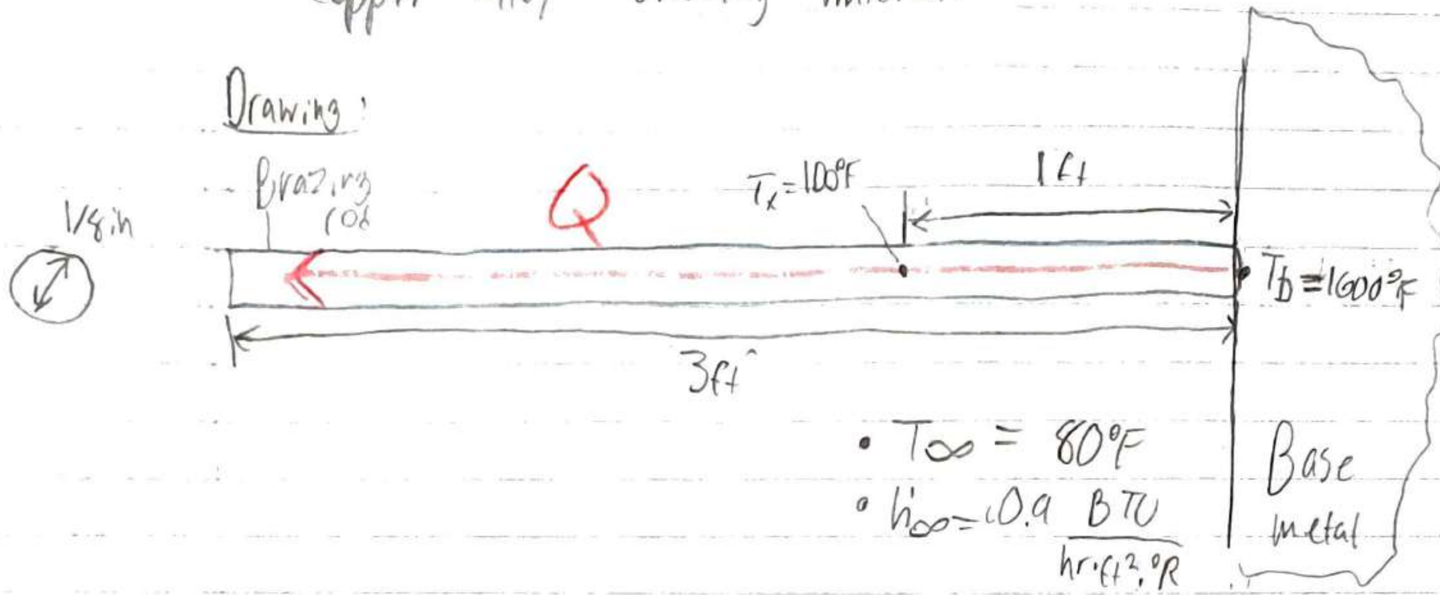
- Glass
- Silver
- Copper
- Water

Analysis: This design is fairly efficient at $\eta = 0.842$. This is due to the low thermal resistance of the copper, silver, and water flowing through the tubes. A small amount is lost to the higher thermal resistance of air and glass. Two ways to improve the efficiency would be to lower the temperature of the water and to increase the thickness of the cover glass. These two changes would allow more useful heat to be collected by the water and less heat to be lost to the atmosphere.

Problem 2)

Purpose: To calculate the conductivity of the copper alloy brazing material.

Drawing:



Sources:
- Test 1

Design Considerations:
- Steady state
- Assume that rod is infinitely long

Data and Variables:
- $L = 3\,ft$
- $x = 1\,ft$
- $T_b = 1600°F$
- $T_x = 100°F$
- $T_\infty = 80°F$
- $h_\infty = 0.9\,BTU/(hr \cdot ft^2 \cdot °R)$
- $D = 0.125\,in = 0.0104\,ft$

Procedure:

The brazing rod in this example can be seen as an extended surface. Assuming that the rod is infinitely long and the temperature of the tip is the same as the ambient, the following equation can be used to calculate the the thermal conductivity of the rod:

$$\frac{\theta}{\theta_b} = e^{-mX}$$

Where:

- $\theta = T_x - T\infty$
- $\theta_b = T_b - T\infty$
- $m = \sqrt{hP/KA_c}$

Calculations

$$\frac{T_x - T\infty}{T_b - T\infty} = e^{-m \cdot X}$$

$$\frac{100°F - 80°F}{1600°F - 80°F} = e^{-m \cdot 1ft}$$

$$0.013 = e^{-m \cdot 1ft}$$

$$\ln(0.013) = \ln(e^{-m \cdot 1ft})$$

$$\ln(0.013) = -m \cdot 1ft$$

$$m = -\frac{\ln(0.013)}{1ft}$$

$$m = \frac{4.33}{1ft}$$

$$m = \sqrt{\frac{hP}{KA_c}} \qquad m = \frac{4.33}{1ft}$$

$$\left(\sqrt{\frac{hP}{KA_c}}\right)^2 = \left(\frac{4.33}{1ft}\right)^2$$

$$\frac{hP}{KA_c} = \frac{18.76}{1ft^2}$$

$$hP \cdot 1ft^2 = 18.76 \cdot KA_c$$

$$K = \frac{hP \cdot 1ft^2}{18.76 \cdot A_c}$$

$$K = \frac{\left(0.9\,\frac{BTU}{hr \cdot ft^2 \cdot {}^\circ R}\right)(\pi D)(1ft^2)}{(18.76)\left(\frac{\pi}{4} \cdot D^2\right)}$$

$$K = \frac{\left(0.9\,\frac{BTU}{hr \cdot \cancel{ft^2} \cdot {}^\circ R}\right)(\pi \cdot 0.0104\cancel{ft})(1\cancel{ft^2})}{(18.76)\left(\frac{\pi}{4} \cdot 0.0104\cancel{ft}^2\right)} \qquad \frac{BTU}{hr \cdot ft \cdot {}^\circ R}$$

$$K = 18.43\,\frac{BTU}{hr \cdot ft \cdot {}^\circ R}$$

Summary:

The thermal Conductivity of the brazing material is 18.43 BTU/(hr.f.°R). The heat applied to the base material is transfered to the base of the rod and the heat travells through the rod where the heat is dissipated to the air along the length of the rod.

Materials:

- Copper alloy brazing material

Analysis: In this problem a large portion of the heat is transfered to the ambient air along the rod from the base to a distance of 1 foot from the base. This is due to the large delta between the base temperature of the rod and ambient air temperature. The Conductivity of the rod is relatively low because most of the heat has been tranferred to the ambient air through convection. If the Conductivity of the rod was higher, the temperature of the rod at a distance of 1ft would also be higher.