

HW 3.3

**By**

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MET 440 - Heat Transfer

Dr. Ayala

## Ch6 Problems

### Question 2-35

- 6-35** A fireclay brick in the dimensions of semi infinite strip,  $0 < x < \infty$ , as shown in Figure P6-35 within  $0 < y < 8$  cm is initially at a uniform temperature  $T_i = 250^\circ\text{C}$ . Suddenly it is subjected to an ambient convective cooling at  $T_\infty = 25^\circ\text{C}$ , with a heat transfer coefficient  $h = 100 \text{ W}/(\text{m}^2 \cdot ^\circ\text{C})$ . Calculate the temperature  $T_0$  of a point  $P$  located along the midplane at a distance  $x = L = 5$  cm from the surface, after  $t = 2$  h it starts cooling. The fireclay brick has  $k = 1 \text{ W}/(\text{m} \cdot ^\circ\text{C})$  and  $\alpha = 5.4 \times 10^{-7} \text{ m}^2/\text{s}$   
*Answer:*  $28.8^\circ\text{C}$

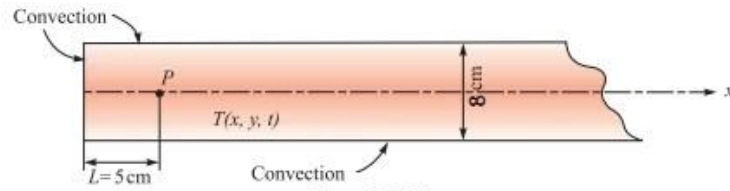
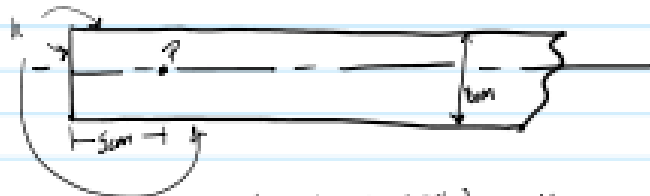


Figure P6-35

# Solution

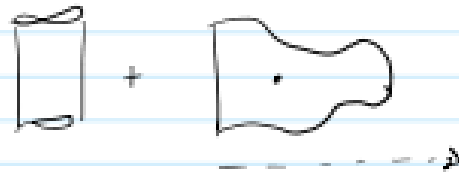
6-35

fireclay Brick semi-infinite strip  $0 \leq x < \infty$   
 $0 \leq y \leq 8 \text{ cm}$ ,  $T_i = 250^\circ\text{C}$ ,  $T_\infty = 25^\circ\text{C}$ ,  
 $h = 100 \text{ W/m}^2\cdot^\circ\text{C}$  calculate  $T_o$  of P @  
 $x = L = 5 \text{ cm}$  after  $t = 2 \text{ hrs}$   
 Brick:  $k = 1 \text{ W/m}\cdot^\circ\text{C}$  &  $\alpha = 5.4 \times 10^{-7} \text{ m}^2/\text{s}$



$$\text{Biot \#} : \frac{hb}{k} = \frac{(100 \text{ W/m}^2\cdot^\circ\text{C} \cdot 0.04 \text{ m})}{1 \text{ W/m}\cdot^\circ\text{C}} = 4.0$$

$$\tau : \frac{\alpha t}{b^2} = \frac{(5.4 \times 10^{-7} \text{ m}^2/\text{s})(7200 \text{ s})}{0.04 \text{ m}^2} = 2.43$$



$$\frac{T(x,t) - T_\infty}{T_i - T_\infty} = \theta(x,t) \times C(r,t)$$

$$\theta(x,t) = \text{erf}\left(\frac{\xi}{\sqrt{4\alpha t}}\right)$$

$$\xi = \frac{x}{\sqrt{4\alpha t}} = \frac{0.05 \text{ cm}}{\sqrt{4(5.4 \times 10^{-7} \text{ m}^2/\text{s})(7200 \text{ s})}} = 0.401$$

$$\theta(x,t) = \text{erf}(0.401) = 0.429$$

$$C(r,t) = \theta_o^* J_o\left(\beta_o \frac{r}{r_o}\right)$$

$$\theta_o^* = C_1 e^{-\beta_o^2 \tau} J_o\left(\beta_o \frac{r_o}{r_o}\right)$$

$$\beta_o = \frac{\alpha t}{r_o^2} \quad J_o = 1?$$

$$\theta_o^* = 2.72 \times 10^{-2}$$

$$C(r,t) = 5.19 \times 10^{-2}$$

$$B_o = \frac{hb}{k} = 4.0$$

$$J_o = 1.9081$$

$$C_1 = 1.4698$$

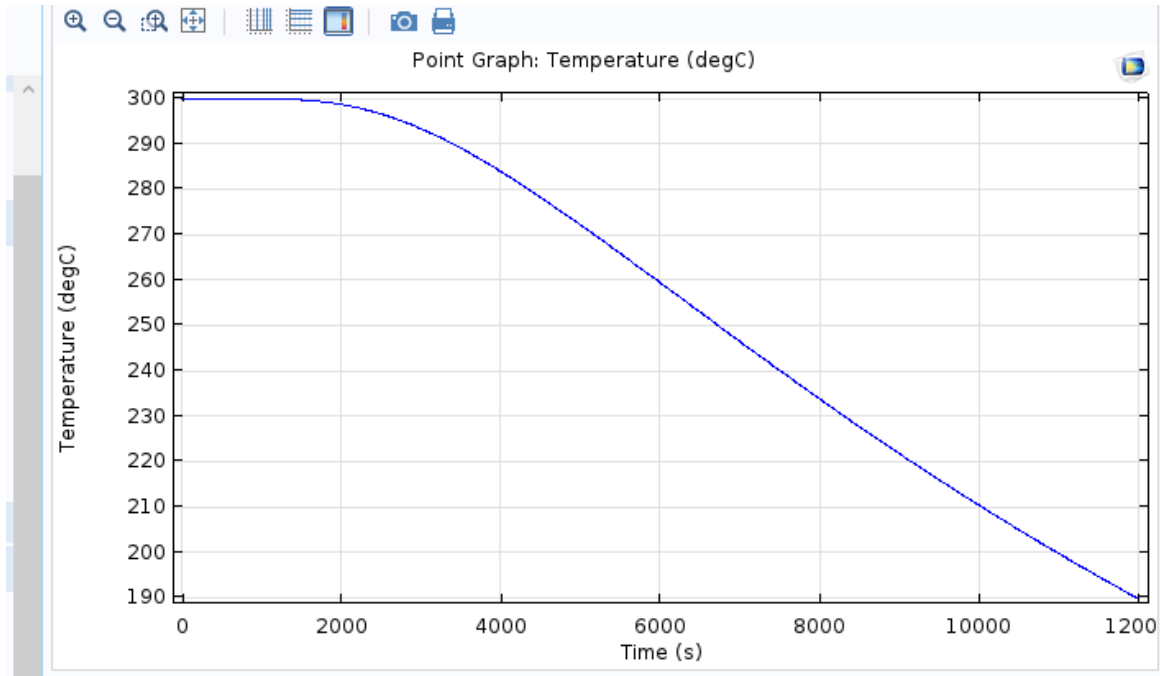
$$\begin{aligned}
 & \text{Handwritten: } h_0 = 2.72 \times 10^{-2} \\
 & \text{Handwritten: } C(r,t) = 5.19 \times 10^{-2} \\
 & \frac{T(r,x,t) - T_\infty}{T_i - T_\infty} = 0.429 \times 5.19 \times 10^{-2} \\
 & T(r,x,t) = \left[ (0.429 \times 5.19 \times 10^{-2}) \times (250 - 25) \right] + 25 \\
 & T(r,x,t) = 30.01^\circ\text{C}
 \end{aligned}$$

y=	8 cm	0.08 m					
x=	5 cm	0.05 m			Biot #	4.0000	
r=	4 cm	0.04 m			$\tau$	2.43E+00	
Ti=	250 degC				$\zeta$	1.9081	
Tinf=	25 degC				C1	1.4698	
h=	100 w/m^2C				J0	1	
To=					$\theta^*$	2.72E-02	
delta t	2 hr	7200 sec			C(r,t)	5.19E-02	
k_brick	1 w/mC						
alpha	5.40E-07 m^2/s				$\xi$	4.01E-01	
biot for ta	4				$\theta(x,t)=$	0.42929364	
					(T0-Tinf)/(Ti-Tinf)	2.23E-02	
					T(r,x,t)=	30.01 degC	

### Question 6-27

**6-27** A large brick wall ( $\alpha = 5 \times 10^{-7} \text{ m}^2/\text{s}$ ) that is 20 cm thick is initially at a uniform temperature of  $300^\circ\text{C}$ . Suddenly its surfaces are lowered to  $50^\circ\text{C}$  and maintained at that temperature. Using an explicit finite-difference scheme and a mesh size  $\Delta x = 5 \text{ cm}$ , determine the time required for the center temperature to reach  $200^\circ\text{C}$  after the start of cooling.

### Solution



**$t=10875\text{s}=3.02 \text{ hours}$**

## Activity

The concept of multidimensional effects covered over the past few days has been much more relevant to the project. The goal of this project is to lower the temperature of the cans in five minutes. If the can and its contents can be modeled as one cylinder, we can use the concept of multidimensional effects to determine the temperature profile in each can. The heat conduction within the can will be multidimensional since heat will be removed from the cans on the lateral sides and the tops and bottoms. There will be convection on all surfaces of the cans. The effects of an infinite cylinder multiplied by the effects of a plane wall can be used to determine the temperature anywhere within the can. This will allow us to determine any temperature within the can at any time. Using this concept we can check the temperature profile within the can at 5 minutes to determine if it has reached the desired temperature.

We would definitely be able to use COMSOL for the project. Comsol would allow us to plot the temperature profile at 5 minutes after specifying the geometry, material, initial temperature, and boundary conditions of each can. The boundary conditions would be convection on the top, bottom, and lateral sides of the cans. The great thing about COMSOL is we can use it to test different convective heat transfer coefficients and fluid temperature quite easily by just changing values until we achieve the desired temperature inside. Being able to develop temperature profiles quickly for different values of  $h$  and  $T_{\infty}$  would save us a lot of time and work versus solving it by hand where we would have to recalculate for each change in position,  $h$ , and  $T_{\infty}$ .

Domain for each can

