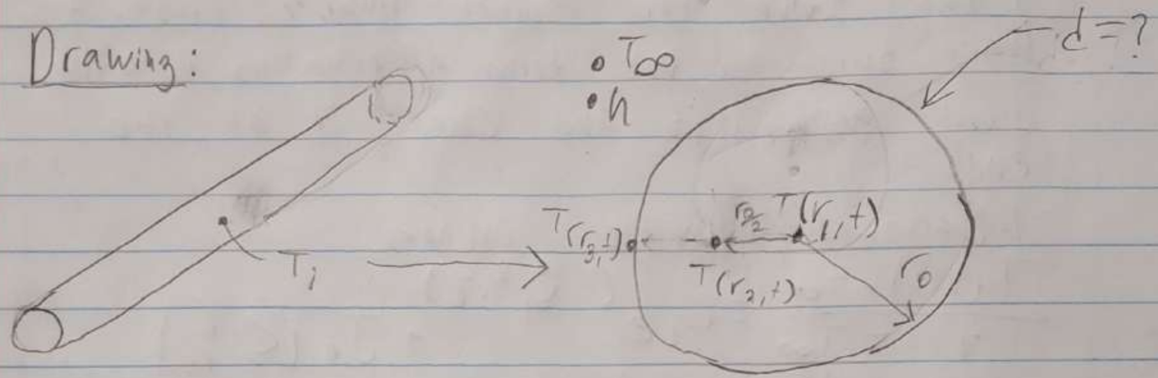
Test 3

Problem 1)

Purpose: To determine the diameter of the plastic rod whose center temperature is 30°C after 1338 seconds. To determine the temperature at the surface and at half the radius.

Drawing:



Sources:

- Test 3
- Bayazitoglu, Y., Ozisik, N., "A Textbook for Heat Transfer Fundamentals" Begell House Inc. (2012)

Design Considerations

- Transient Conduction
- One-dimensional heat transfer.
- No heat generation

Data and Variables

- $T_i = 70°C$
- $T_\infty = 25°C$
- $h = 20.42\ W/m^2 \cdot K$
- $t = 1338\ s$
- $T(r_1, t) = 30°C$
- $\rho = 1190\ kg/m^3$
- $C_p = 1465\ J/kg \cdot K$
- $k = 0.19\ W/m \cdot K$

Procedure:

- To solve for the diameter of the rod, I will use the equation for 1 Dimensional transiant conduction within an infinite cylinder.
  - Since diameter of the rod is unknown, the radius is unknown. This means that I can't solve for Fourier number first to make sure a one term approximation is valid. I will assume it is and check it at the end.
  - Equation for infinite cylinder

$$\frac{T(r,t)-T_\infty}{T_i-T_\infty}=C_1 e^{(-\zeta_1^2 \cdot \tau)} \cdot J_0\left(\zeta \frac{r}{r_0}\right)$$

- I will need an iterative process to find r_0 since it is needed to calculate Fourier number and Biot number

$$\tau=\frac{\alpha t}{r_0^2} \qquad Bi=\frac{h r_0}{k}$$

- Biot number is then used to read C_1 and ζ_1 from table 5.1

Steps for iteration

1. Solve left hand side of the equation
2. guess value of R_0 and calculate τ and Bi
3. Read C_1 and ζ_1 from table 5.1 using Bi
4. With these values, I can calculate the right side of the equation
5. Compare RHS to LHS and calculate percent error, If they are the same, I will need to return to step 2 and repeat the process until percent difference is less than 1%.

- After I obtain r_0, I can calculate τ and check if a one term approximation is valid.
- If it is, the diameter of the rod can easily be found now using the radius r_0.

Using r_0, I can calculate the temperature at the surface and at half of the radius using the equation for an infinite cylinder.

Solution

a) $$\frac{T(r,t)-T_\infty}{T_i-T_\infty}=C_1 e^{(-\zeta_1^2 \tau)} \cdot J_0\left(\zeta_1 \cdot \frac{r}{r_0}\right)$$

$$\tau=\frac{\alpha t}{r_0^2} \qquad Bi=\frac{h r_0}{K} \qquad \alpha=\frac{K}{\rho c_p}$$

- C_1 and ζ_1 are read from table 5.1 using Biot Number.

- Going to an iterative process in excel to obtain r_0

$$\alpha=\frac{K}{\rho \cdot c_p}=\frac{0.19\ \frac{W}{m \cdot K}}{1190\ \frac{kg}{m^3} \cdot 1465\ \frac{J}{kg \cdot K}}=1.1 \times 10^{-7}\ \frac{m^2}{s} \quad \frac{W \cdot m^2}{J}=\frac{W \cdot m^2}{W \cdot s}=\frac{m^2}{s}$$

$$LHS=\frac{T(r,t)-T_\infty}{T_i-T_\infty}=\frac{30^\circ C-25^\circ C}{70^\circ C-25^\circ C}=0.11$$

Using data from Table 5.1, equations developed for ζ_1 and C_1 for an infinite cylinder for Biot Numbers of 0.5 through 2.

$$\text{Zeta } 1=-0.1941 \cdot Bi^2+0.9209 \cdot Bi+0.5336$$

$$C_1=-0.0364 \cdot Bi^2+0.2402x+1.0034$$

After iterating in excel, $r_0=0.010222\,m$

$$RHS = C_1 \cdot e^{(-\zeta_1^2 \cdot \tau)} \cdot J_0\left(\zeta_1 \frac{r}{r_0}\right)$$

$$\tau = \frac{\alpha t}{r_0^2} = \frac{1.1 \times 10^{-7} \frac{m^2}{s} \cdot 13388}{0.010222 m^2} = 1.396$$

$$Bi = \frac{h \cdot r_0}{k} = \frac{20.42 \frac{W}{m^2 \cdot K} \cdot 0.010222 m}{0.19 \frac{W}{m \cdot K}} = 1.096$$

Using formulas in excel for ζ_1 and C_1

$C_1 = 1.223$ $\quad \zeta_1 = 1.31$

$$RHS = 1.223\, e^{(-(1.31^2 \cdot 1.396))} \cdot J_0\left(1.31 \cdot \frac{0}{1.0222 m}\right)$$

$RHS = 0.111121$

$D = 2 \cdot r_0 = 2 \cdot 0.010222 m$

$$\boxed{D = 0.020444 m = 20.44 mm}$$

$\tau = 1.396 > 0.2$

One term approximation is Valid.

b) Temp at Surface $T(r_3, t)$ $\quad r_3 = r_0 = 0.01022$

$$\frac{T(r_3,t) - T_\infty}{T_i - T_\infty} = C_1 e^{(-\zeta_1^2 \cdot \tau)} \cdot J_0\left(\zeta_1 \frac{r_3}{r_0}\right)$$

$$\frac{T(r_3,t) - 25°C}{70°C - 25°C} = 1.223 \cdot e^{(-(1.31^2 \cdot 1.396)} \cdot J_0\left(1.31 \cdot \frac{0.01022}{0.01022}\right)$$

$$\frac{T(r_{3},t)-25^\circ C}{70^\circ C-25^\circ C} = 0.068263$$

$$T(r_{3},t)=0.068263(70^\circ C-25^\circ C)+25^\circ C$$

$$\boxed{T(r_{3},t)=28.07^\circ C}$$

c) Temp at half of radius $T(r_2,t)$

$$r_2 = r_0/2 = 0.010222m/2 = 0.005111m$$

$$\frac{T(r_2,t)-T_\infty}{T_1-T_\infty} = C_1 e^{(-\zeta_1^2\cdot\tau)}\cdot J_0\left(\zeta_1\frac{r_2}{r_0}\right)$$

$$\frac{T(r_2,t)-25^\circ C}{70^\circ C-25^\circ C} = 1.223e^{(-(1.31)^2\cdot 1.396)}\cdot J_0\left(1.31\cdot\frac{0.0051m}{0.01022m}\right)$$

$$\frac{T(r_2,t)-25^\circ C}{70^\circ C-25^\circ C} = 0.0995$$

$$T(r_2,t)=0.0995(70^\circ C-25^\circ C)+25^\circ C$$

$$\boxed{T(r_2,t)=29.48^\circ C}$$

Summary

- The diameter of the plastic rod with a center temperature of 30°C after 1338 seconds of cooling is 20.44mm.
- The temperature of the surface of the rod after 1338 seconds of cooling is 28.07°C.
- The temperature at half the radius of the rod after 1338 seconds of cooling is 29.48°C.

Materials

- Plastic

Analysis

- Since Fourier number was greater than 0.2, it was valid to use a one term approximation to find the diameter of the rod given the center temperature.
- The temperature solutions at the surface and at half the radius make sense because the rod was being cooled, so the center temperature of 30°C was a maximum. The solutions must be between the center temperature and the fluid temperature of 25°C. The temperature within the rod must decrease radially which is what the solutions show. The temperature at half the radius was 29.48°C which is less than the center temperature. The temperature at the surface of the rod 28.07°C which was lowest temperature within the rod which makes sense because it was in contact with the cooling fluid.