Test 4
MET 440 Heat Transfer
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<u>Problem 1</u>

Problem Statement:

A flat plate solar collector is used to heat water. It is constructed with a glass cover plate, an insulating air layer, an absorber plate, and copper tubes imbedded in a silver base through which water passes.

Purpose:

- What is the amount of heat collected by the water in one of the tubes?
- What is the flow rate of the water?
- What is the collector efficiency (η)? In this case, η is defined by the ratio of useful heat collected as compared to the rate of solar energy incident on the collector.

Sources:

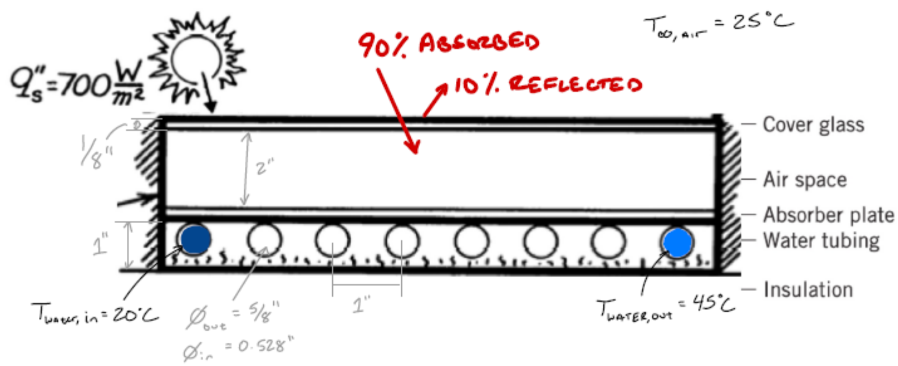
Bayazitoglu, Yildiz, and Necati Ozisik. *A Textbook for Heat Transfer Fundamentals*. Danbury, CT, Begell House Inc, 2012

Holzmann, Tobias. "Temperature-Dependent Thermophysical Properties for Pure Water." *Holzmann CFD*, 06 Dec. 2020, https://holzmann-cfd.com/community/blog-and-tools/cae-blog/thermophysical-properties-water. Accessed 11 Nov. 2023

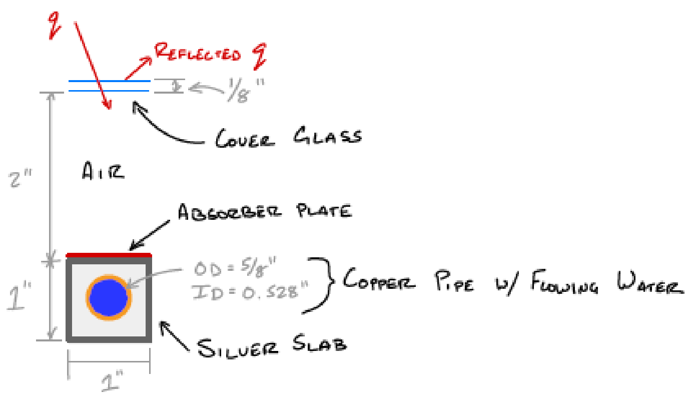
Zografos, A. I., W. A. Martin, and J. E. Sunderland. 1987. Equations of properties as a function of temperature for seven fluids. *Comput. Methods Appl. Mech. Eng.* 61: 177-187.

Drawings:

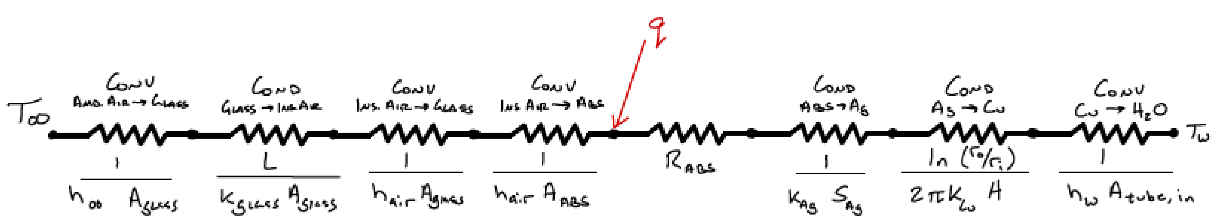
System:



Single Tube Under Inspection:



Heat Resistance Circuit:



Design Considerations:

- The ambient air is stagnant and transfers heat through passive convection
- The insulating air layer is stagnant and transfers heat through passive convection
- The system is steady state
- The insulation underneath the silver block is complete and suffers no heat loss
- There is no new heat generation
- Contact resistance can be neglected

Data and Variables:

- Dimensions of the system are shown in the diagrams
- Heat From Solar Flux:

$$q_{solar} = 700 \, {}^{W}\!/_{m^2} \qquad q_{abs} = 0.9 q_{solar} = 630 \, {}^{W}\!/_{m^2}$$

- Given Temperatures:

$$T_{amb.\,air} = 25^\circ C \qquad T_{water,\,in} = 20^\circ C \qquad T_{water,\,out} = 45^\circ C$$

- Physical properties from the appendices of the source material:

$$k_{silver} = 419 \, {}^{W}\!/_{m \cdot K} \qquad k_{copper} = 386 \, {}^{W}\!/_{m \cdot K} \qquad k_{glass} = 0.78 \, {}^{W}\!/_{m \cdot K}$$

Procedure:

To solve this problem, many properties of the flowing water and the stagnant air must be either looked up or derived using the available information. Because there are no given values for the convective heat transfer coefficients for the ambient air, insulating air gap, or flowing water, these values must be assumed, and an iterative process will be necessary to determine what those values should be. Using the assumed values, the temperature of all the surfaces can be calculated, and the associated temperature differences can be utilized to narrow down possible answers.

- Assume values for the convective heat transfer coefficients for:
  - Ambient air to glass
  - Insulating air to glass
  - Insulating air to absorber plate
  - Water to copper tube
- Use calculated resistances to determine the surface temperatures at all convective boundaries along with the mass flow rate of water
- With calulcated temperature values and flow rate, calculate Nusselt number values for each convective area
- Calculate new convective heat transfer coefficients
- Check for percentage difference between calculated heat transfer coefficients and assumed heat transfer coefficients
- Iterate using the newly calculated heat transfer coefficients to achieve as small a discrepancy as possible between the assumed values and the calculated values
- Finally, the efficiency of the system will be calculated using the final iteration for the heat flow in the system

Calculations:

The calculations below represent one iteration. Further iterations will be displayed on imbedded charts with calculated values rather than fully realized equations.

- All thermophysical properties of the water and air involved in the system have been calculated using polynomial equations provided by Holzmann (2) and Zografos et al. (3)
    - *NOTE: A small change was made to the equation for the thermal conductivity of air provided by Zografos et al. (3) after it was found that a typo existed. The value -23.757762 x 10^{-6} was altered to become 0.23757762 x 10^{-6}. This value was found to better match provided figures in the textbook appendices.*
- Thermophysical properties of the ambient air and the insulated air gap will be defined by:

$$\mu_{aa} = 4.112985 \cdot 10^{-6} + 5.052295 \cdot 10^{-8} T_{aa,\ avg} - 1.43462 \cdot 10^{-11} T_{aa,\ avg}{}^{2} + 2.591403 \cdot 10^{-15} T_{aa,\ avg}{}^{3}$$

$$k_{aa} = -7.488 \cdot 10^{-3} + 1.708186 \cdot 10^{-4} T_{aa,\ avg} - 23.757762 \cdot 10^{-6} T_{aa,avg}{}^{2} + 220.11791 \cdot 10^{-12} T_{aa,avg}{}^{3} - 945.995536 \cdot 10^{-16} T_{aa,avg}{}^{4} + 1579.657437 \cdot 10^{-20} T_{aa,avg}{}^{5}$$

$$c_p = 1061.332 - 0.432819 T_{aa,avg} + 1.02344 \cdot 10^{-3} T_{aa,avg}{}^{2} - 6.47474 \cdot 10^{-7} T_{aa,avg}{}^{3} + 1.3864 \cdot 10^{-10} T_{aa,avg}{}^{4}$$

- Further calculations of the physical properties of air necessary for this problem relied on the equations above before they could be derived. The calculations for density and the Prandtl number for the ambient air are below:

$$T_{aa,avg} = 298K$$

$$\mu_{aa} = 1.7963 \cdot 10^{-5}\ Pa \cdot s \qquad k_{aa} = 0.02743\ {}^{W}\!/_{mK} \qquad c_{p,aa} = 1007.2\ {}^{J}\!/_{kgK}$$

$$\rho_{aa} = \frac{P_{aa}}{RT_{aa,avg}} = \frac{(101300\ Pa)}{\left(287\ {}^{J}\!/_{kgK}\right)(298K)} = 1.184\ {}^{kg}\!/_{m^3}$$

$$Pr_{aa} = \frac{\mu c_p}{k} = \frac{\left(1.7963 \cdot 10^{-5} Pa \cdot s\right)\left(1007.2 {}^{J}\!/_{kgK}\right)}{\left(0.02743 {}^{W}\!/_{mK}\right)} = 0.6596$$

$$v_{aa} = \frac{\mu}{\rho} = \frac{\left(1.7963 \cdot 10^{-5} Pa \cdot s\right)}{\left(1.184 {}^{kg}\!/_{m^3}\right)} = 1.571 \cdot 10^{-5} {m^2}\!/_{s}$$

- A similar methodology will be used to derive the thermophysical properties of water using polynomial equations provided by Holzmann (2)

$$\mu_w = -2.80572 \cdot 10^{-9} T_{w,avg}{}^{3} + 2.90283 \cdot 10^{-6} T_{w,avg}{}^{2} + -0.00100532 T_{w,avg} + .116947$$

$$k_w = -9.29827 \cdot 10^{-6} T_{w,avg}{}^{2} + 0.0071857T - 0.710696$$

$$c_{P,\ W} = -0.000127063 T_{w,avg}{}^{3} + 0.13736 T_{w,avg}{}^{2} - 48.6714 T_{w,avg} + 9850.69$$

$$\rho_w = -0.00365471 T_{w,avg}{}^{2} + 1.93017 T_{w,avg} + 746.025$$

- Because the inlet and outlet temperature of water was provided in the problem statement, the properties of the water could be fully expressed:

$$T_{w,avg}=\frac{T_{w,out}+T_{w,in}}{2}=\frac{(318K)+(293K)}{2}=305.5K$$

$$\mu_w=0.0007458\ Pa\cdot s \qquad k_w=0.6167\ {}^{W}/_{mK} \qquad c_{p,\ w}=4178.55\ {}^{J}/_{kgK} \qquad \rho_w=994.597\ {}^{kg}/_{m^3}$$

$$Pr_w=\frac{\mu c_p}{k}=\frac{(0.0007458Pa\cdot s)\left(4178.55{}^{J}/_{kgK}\right)}{\left(0.6167{}^{W}/_{mK}\right)}=5.053$$

With the known properties established, the next step is to make assumptions about the unknown convective heat coefficients related to the ambient air, insulated air gap, and the flowing water. The heat being added to the system must be quantified using the known thermal resistances and the assumed thermal resistances associated with the convective boundaries.

- Known thermal resistances:
  - Glass Plate:

$$R_{glass}=\frac{L_{glass}}{k_{glass}A_{glass}}=\frac{(0.003175m)}{\left(0.78{}^{W}/_{m^2K}\right)\left(0.0254m^2\right)}=0.1603{}^{K}/_{W}$$

  - Silver Plate:

$$R_{Ag}=\frac{1}{k_{Ag}S_{Ag}} \quad where \quad S_{Ag}=\frac{2\pi L}{\ln\left(1.08\frac{W}{D}\right)}=\frac{2\pi(1m)}{\ln\left(1.08\frac{(0.0254m)}{(0.015875m)}\right)}=11.4874m$$

$$=\frac{1}{\left(419{}^{W}/_{m^2K}\right)(11.4874m)}=0.0002078{}^{K}/_{W}$$

  - Copper Tube:

$$R_{Cu}=\frac{\ln\left(\frac{r_o}{r_i}\right)}{2\pi k_{Cu}H}=\frac{\ln\left(\frac{0.015875m}{0.013411m}\right)}{2\pi\left(386{}^{W}/_{m^2K}\right)(1m)}=0.00006955{}^{K}/_{W}$$

  Note: the absorber plate resistance, R_{abs} *is equal to 0 as the plate is described as both very thin and very conductive, and is therefore negligible*

- Convective coefficient values will now be assumed for four different points in the system: where the ambient air meets the glass surface, where the insulated air meets the glass surface, where the insulated air meets the absorber surface, and where the water meets the copper tube. These values will be used to calculate the thermal resistances at those boundaries.

$$h_{aa}=50{}^{W}/_{m^2K} \qquad h_{ia,glass}=10{}^{W}/_{m^2K} \qquad h_{ia,abs}=10{}^{W}/_{m^2K} \qquad h_w=1000{}^{W}/_{m^2K}$$

  - Ambient air to glass:

$$R_{aa}=\frac{1}{h_{aa}\cdot A}=\frac{1}{\left(50{}^{W}/_{m^2K}\right)\left(0.0254m^2\right)}=0.7874{}^{K}/_{W}$$

- Insulated air to glass:

$$R_{ia,glass} = \frac{1}{\left(10\,{}^{W}\!/_{m^2K}\right)\left(0.0254m^2\right)} = 3.937\,{}^{K}\!/_{W}$$

- Insulated air to absorber:

$$R_{ia,\ abs} = \frac{1}{\left(10\,{}^{W}\!/_{m^2K}\right)\left(0.0254m^2\right)} = 3.937\,{}^{K}\!/_{W}$$

- Water to copper tube:

$$R_w = \frac{1}{h_w \cdot A_{tube,in}} = \frac{1}{\left(1000\,{}^{W}\!/_{m^2K}\right)\left(\pi \cdot 0.013411m \cdot 1m\right)} = 0.02373\,{}^{K}\!/_{W}$$

- With all the resistances in the system calculated and a given value for Q in the problem statement, the heat flow at the absorber plate is known and the temperature of that plate can be calculated using the standard equation for Q:

$$Q = \frac{\Delta T}{\Sigma R}$$

Where ΔT is defined by the difference in temperature between the absorber plate and the ambient air in one direction and the difference between the absorber plate at the inlet water temperature in the other. The total Q for this system can then be represented by:

$$Q_T = Q_{top} + Q_{bottom}$$

Where Q_{top} is the heat flow above the absorber plate and Q_{bottom} is the heat flow below the absorber plate.

$$Q_{top} = \frac{T_{abs} - T_{aa}}{R_{aa} + R_{glass} + R_{ia,glass} + R_{ia,\ abs}} = \frac{T_{abs} - (298K)}{\left(0.7874\,{}^{K}\!/_{W}\right) + \left(0.1603\,{}^{K}\!/_{W}\right)\left(3.937\,{}^{K}\!/_{W}\right) + \left(3.937\,{}^{K}\!/_{W}\right)}$$
$$= \frac{T_{abs} - 298K}{8.8217\,{}^{K}\!/_{W}}$$

$$Q_{bottom} = \frac{T_{abs} - T_{w,in}}{R_{abs} + R_{Ag} + R_{Cu} + R_w} = \frac{T_{abs} - (293K)}{\left(0\,{}^{K}\!/_{W}\right) + \left(0.0002078\,{}^{K}\!/_{W}\right) + \left(0.00006954\,{}^{K}\!/_{W}\right) + \left(0.02373\,{}^{K}\!/_{W}\right)}$$
$$= \frac{T_{abs} - 293K}{0.02401\,{}^{K}\!/_{W}}$$

- Calculating T_{abs} was made simpler in Excel by finding it in terms of R_{bottom} and R_{top} and relating those values to Q_{total}:

$$Q_T = Q_{top} + Q_{bot} = \frac{T_{abs} - T_{aa}}{R_{top}} + \frac{T_{abs} - T_{w,in}}{R_{bot}}$$
$$\therefore$$
$$T_{abs} = \frac{Q_T R_{bot} R_{top} + T_{aa} R_{bot} + T_{w,in} R_{top}}{R_{bot} + R_{top}} \quad where \quad Q_T = q_{abs} \cdot A = \left(630\,{}^{W}\!/_{m^2}\right)\left(0.0254m^2\right) = 16.002W$$
$$= \frac{(16.002W)\left(0.02401\,{}^{K}\!/_{W}\right)\left(8.8217\,{}^{K}\!/_{W}\right) + (298K)\left(0.02401\,{}^{K}\!/_{W}\right) + (293K)\left(8.8217\,{}^{K}\!/_{W}\right)}{\left(0.02401\,{}^{K}\!/_{W}\right) + \left(8.8217\,{}^{K}\!/_{W}\right)}$$
$$= 293.397K$$

- With an established temperature for the absorber plate, the values of Q_{top} and Q_{bottom} can be found and used to calculate the other temperature values that are necessary for the iteration process

$$Q_{top}=\frac{T_{abs}-T_{aa}}{R_{top}}=\frac{(293.397K)-(298K)}{\left(8.8217 K/W\right)}=-0.5218W$$

$$Q_{bottom}=\frac{T_{abs}-T_{w,in}}{T_{bottom}}=\frac{(293.397K)-(293K)}{\left(0.02401 K/W\right)}=16.535W$$

NOTE: The value of Q_{top} *is negative because the temperature of the ambient air is higher than the temperature of the absorber plate, meaning some of that heat is transferring into the system along with the solar heat.* Q_{top} *will be treated as positive value for future calculations as it still represents energy being added to the system.*

- Surface temperatures can now be established:
  - Temperature of the glass:

$$T_{glass}=Q_{top}R_{aa}+T_{aa}=(0.5218W)\left(8.8217 K/W\right)+(298K)=298.41K$$

  - Temperature of the insulated air layer:

$$T_{ia}=Q_{top}R_{ia,abs}+T_{abs}=(0.5218W)\left(3.937 K/W\right)+(293.397K)=295.45K$$

  - Temperature of the copper tube:

$$T_{Cu}=Q_{bottom}R_{w}+T_{w,in}=(16.535W)\left(0.02373 K/W\right)+(293K)=293.39K$$

Having established temperatures for all the convective boundaries in the system, it is now possible to calculate thermophysical properties for all the fluids located along those boundaries. Using these properties, the value of the convective coefficients for each interface can be found using the associated Nusselt numbers. These calculated values will be compared with the assumed coefficient values and the iterative process will be used to achieve a final answer where the assumed values closely match the calculated values.

- The ambient air and the water have already had some of these physical properties worked out above using the known temperature values. To find the convective coefficient, the Nusselt number must be calculated.
  - The ambient air is stagnant and will be treated as free convection. The Nusselt number for free convection relies on a relationship between the Grashof number and the Prandtl number:

$$Nu_{aa}=c(GrPr)^{n}$$

    Where *c* and *n* are based on experimentally determined values given in the textbook under table 9.2.

$$Gr = \frac{g\beta\left(T_{wall} - T_{\infty}\right)L^3}{v^3} \quad where \quad \beta = \frac{1}{T_f} \quad where \quad T_f = \frac{T_{wall} + T_{\infty}}{2}$$

$$T_{f,aa} = \frac{(298.411K) + (298K)}{2} = 298.21K \quad \therefore \quad \beta = \frac{1}{298.21K} = 0.00335K^{-1}$$

$$Gr_{aa} = \frac{\left(9.81 m/_{s^2}\right)\left(0.00335K^{-1}\right)(298.411K - 298K)(0.0254m)^3}{\left(1.517 m^2/_s\right)^2} = 961.8$$

- Nusselt number based on assumed h_{aa}:

$$Nu_{aa} = 0.54(961.8 \cdot 0.6596)^{0.25} = 2.71$$

- Convective coefficient based on Nusselt number:

$$h_{aa,new} = \frac{k_{aa}}{L} Nu_{aa} = \frac{\left(0.02743 W/_{mK}\right)}{(0.0254)}(2.71) = 2.93 W/_{m^2K}$$

- Comparing the calculated *h* value to the assumed *h* value:

$$\% \, diff = \frac{\left|h_{aa} - h_{aa,new}\right|}{\frac{h_{aa} + h_{aa,new}}{2}} \cdot 100 = \frac{\left|50 W/_{m^2K} - 2.93 W/_{m^2K}\right|}{\frac{50 W/_{m^2K} + 2.93 W/_{m^2K}}{2}} \cdot 100$$

$$= 177\%$$

- The value $h_{aa,new}$ will be used in place of h_{aa} and the iterative process will be repeated until the %diff becomes very small
- The Nusselt number for water has a different procedure which is based on the Reynolds number (*Re*) of the flowing water.
- Calculating *Re*:

$$Re = \frac{\mu_m D}{v} \quad where \quad \mu_m = \frac{\dot{m}}{\rho A} \quad and \quad v = \frac{\mu}{\rho}$$

$$\therefore$$

$$Re = \frac{\frac{\dot{m}}{\rho\left(\pi\left(D/2\right)^2\right)} D}{\frac{\mu}{\rho}} = \frac{4\dot{m}}{\pi D \mu}$$

- The mass flow rate of the water can be calculated using the assumed Q_{bottom}:

$$Q_{bottom} = \dot{m} c_p \left(T_{out} - T_{in}\right) \quad \therefore \quad \dot{m}_w = \frac{Q_{bottom}}{c_{p,w}\left(T_{w,out} - T_{w,in}\right)}$$

$$\dot{m}_w = \frac{(16.524W)}{\left(4178.55 J/_{kgK}\right)(318K - 293K)} = 0.000158 kg/_s$$

- The Reynolds number is:

$$Re = \frac{4 \cdot \left(0.000158 kg/_s\right)}{\pi(0.013411m)(0.0007458 Pa \cdot s)} = 20.11$$

- When *Re*<2300 the flow of water is considered to be laminar, which results in a much simpler solution to Nusselts number. For a pipe with a constant wall heat and laminar flow:

$$Nu = 3.66$$

- The convective coefficient for laminar flow is:

$$h = Nu\frac{k}{D} = (3.66)\frac{\left(0.6167 W/mK\right)}{(0.013411m)} = 168.3 W/m^2K$$

- %diff:

$$\%\,diff = \frac{\left|1000 W/m^2K - 168.3 W/m^2K\right|}{\frac{1000 W/m^2K + 168.3 W/m^2K}{2}} \cdot 100 = 142\%$$

- For both *h* values of the insulated air gap layer, a more involved process is necessary. The temperatures upon which the thermophysical properties of that layer are based were calculated with the assumed *h* values that started this process. Using those derived temperatures, the polynomial equations used for the ambient air was applied to the insulated air gap on both the top and the bottom resulting in the following properties:
  - Insulated air, glass side:

$$T_{f,\,ia\,glass} = 296.93K \quad \mu_{ia\,glass} = 1.79178 \cdot 10^{-5} Pa\cdot s \quad k_{ia\,glass} = 0.02735 W/mK$$

$$c_{p,ia\,glass} = 1007.18 J/kgK \quad \rho_{ia\,glass} = 1.1887 kg/m^3 \quad \nu_{ia\,glass} = 1.1507 \cdot 10^{-5} m^2/s \quad Pr_{ia\,glass} = 0.6598$$

  - Insulated air, absorber side:

$$T_{f,\,ia\,abs} = 294.424K \quad \mu_{ia\,abs} = 1.7811 \cdot 10^{-5} Pa\cdot s \quad k_{ia\,abs} = 0.02715 W/mK$$

$$c_{p,ia\,abs} = 1007.13 J/kgK \quad \rho_{ia\,abs} = 1.1988 kg/m^3 \quad \nu_{ia\,abs} = 1.4856 \cdot 10^{-5} m^2/s \quad Pr_{ia\,glass} = 0.6606$$

  - The same procedure used to find a new *h* value for ambient air will be followed to find new *h* values for both sides of the insulated air gap. First, a Grashof number will be calculated, then a Nusselt number, and finally, an *h* value that can be compared to the assumed *h* value at the beginning of the process:

$$Gr_{ia\,glass} = 7052.55 \qquad Nu_{ia\,glass} = 4.600 \qquad h_{ia\,glass\,new} = 4.8025 W/m^2K$$
$$\%\,diff_{ia\,glass} = 70.23\%$$

$$Gr_{ia\,abs} = 5081.89 \qquad Nu_{ia\,abs} = 4.110 \qquad h_{ia\,abs\,new} = 4.3941 W/m^2K$$
$$\%\,diff_{ia\,abs} = 77.89\%$$

NOTE: The equations used above for the properties and new h *values for the insulated air gap were identical to those used for ambient air and were therefore left out of the explanation. See equations for* Gr, Nu, h, *and* %diff *under the ambient air explanation for a full breakdown of how to achieve these values.*

Tables with fully iterated values:

Iterated *h* values and the corresponding resistance calculated, started with the assumed *h* values

Ambient Air		**Insulated Air, Glass**		**Insulated Air, Absorber**		**Water**	
h (W/m^2K)	**Resistance** (K/W)	**h** (W/m^2K)	**Resistance** (K/W)	**h** (W/m^2K)	**Resistance** (K/W)	**h** (W/m^2K)	**Resistance** (K/W)
50	0.7874	10	3.937	10	3.937	1000	0.02373
2.9284165	13.444	4.8024726	8.1978603	4.3940560	8.9598310	168.30788	0.14101
3.821505	10.30222	4.8644578	8.0933994	3.460567	11.376745	168.30788	0.14101
3.599880	10.93647	4.658853	8.4505778	3.6981142	10.645966	168.30788	0.14101
3.646176	10.79761	4.7130029	8.3534850	3.629533	10.847125	168.30788	0.14101
3.635590	10.82905	4.6997788	8.3769898	3.6475709	10.79348	168.30788	0.14101
3.638194	10.82130	4.703100	8.3710735	3.6430224	10.806960	168.30788	0.14101

Iterated surface temperatures and *Q* values started with assumed *h* values

Surface Temperatures				**Associated *Q* Values**	
Absorber (K)	**Glass** (K)	**Insulated Air Gap** (K)	**Copper Pipe** (K)	**Q Top** (W)	**Q Bottom** (W)
293.40	298.41	295.45	293.39	0.5218	16.5238
295.27	299.19	296.07	295.27	0.0886	16.0907
295.27	298.94	296.31	295.27	0.0911	16.0931
295.27	298.99	296.24	295.27	0.0903	16.0923
295.27	298.98	296.25	295.27	0.0904	16.0924
295.27	298.98	296.25	295.27	0.0904	16.0924
295.27	298.98	296.25	295.27	0.0904	16.0924

Ambient Air Iterated Properties

Ambient Air			
Gr	**Nu**	**h_{new}** (W/m^{2}K)	**%diff**
963.63	2.7113	2.9284	177.87
2794.58	3.5381	3.8215	26.46
2200.55	3.3329	3.5999	5.97
2315.95	3.3758	3.6462	1.28
2289.17	3.3660	3.6356	0.29
2295.74	3.3684	3.6382	0.07
2294.11	3.3678	3.6375	0.02

Water Iterated Properties

Water			
Re	**Nu**	**h_{new}** (W/m^{2}K)	**%diff**
20.14	3.66	168.31	142.38
19.61	3.66	168.31	0.00
19.61	3.66	168.31	0.00
19.61	3.66	168.31	0.00
19.61	3.66	168.31	0.00
19.61	3.66	168.31	0.00
19.61	3.66	168.31	0.00

Insulated Air Gap, Glass Side

Thermophysical Properties Iterations						
$T_{f,}$ (K)	**μ** (Pa*s)	**k** (W/mK)	**C_p** (J/kgK)	**P** (kg/m^3)	***v*** (m^2/s)	**Pr**
296.93	1.7918E-05	0.02735	1007.18	1.1887	1.5073E-05	0.6598
297.63	1.7948E-05	0.02741	1007.19	1.1859	1.5134E-05	0.6596
297.62	1.7947E-05	0.02740	1007.19	1.1859	1.5134E-05	0.6596
297.61	1.7947E-05	0.02740	1007.19	1.1860	1.5132E-05	0.6596
297.62	1.7947E-05	0.02740	1007.19	1.1860	1.5133E-05	0.6596
297.61	1.7947E-05	0.02740	1007.19	1.1860	1.5133E-05	0.6596
297.61	1.7947E-05	0.02740	1007.19	1.1860	1.5133E-05	0.6596

Convection Coefficient Iterations			
Gr	**Nu**	**h_{new}** (W/m^{2}K)	**%diff**
7052.55	4.4600	4.8025	70.23
7366.88	4.5085	4.8645	1.28
6198.51	4.3180	4.6589	4.32
6492.68	4.3684	4.7130	1.16
6419.84	4.3561	4.6998	0.28
6438.08	4.3592	4.7031	0.07
6433.53	4.3584	4.7023	0.02

Insulated Air Gap, Absorber Side

Thermophysical Properties Iterations						
$T_{f,}$ (K)	**μ** (Pa*s)	**k** (W/mK)	**C_p** (J/kgK)	**P** (kg/m^3)	***v*** (m^2/s)	**Pr**
294.42	1.7810E-05	0.02715	1007.13	1.1988	1.4857E-05	0.6606
295.67	1.7864E-05	0.02725	1007.15	1.1938	1.4964E-05	0.6602
295.79	1.7869E-05	0.02726	1007.16	1.1933	1.4975E-05	0.6602
295.75	1.7867E-05	0.02726	1007.16	1.1934	1.4972E-05	0.6602
295.76	1.7868E-05	0.02726	1007.16	1.1934	1.4972E-05	0.6602
295.76	1.7867E-05	0.02726	1007.16	1.1934	1.4972E-05	0.6602
295.76	1.7867E-05	0.02726	1007.16	1.1934	1.4972E-05	0.6602

Convection Coefficient Iterations			
Gr	**Nu**	**h_{new}** (W/m^{2}K)	**%diff**
5081.89	4.1104	4.3941	77.89
1928.10	3.2255	3.4606	23.77
2511.18	3.4457	3.6981	6.64
2330.99	3.3822	3.6295	1.87
2377.42	3.3989	3.6476	0.50
2365.65	3.3947	3.6430	0.12
2368.59	3.3957	3.6442	0.03

- With the above results, the heat that is transferred into the water in a single pipe is the result of the final iteration for Q_{bottom}:

$$Q_w = Q_{bottom} = \boxed{16.0924W}$$

- The flow rate of the water is the result of the final iteration for m_dot:

$$\dot{m}_w = \boxed{0.00015405\ {}^{kg}/_{s}}$$

- The efficiency of the system can be calculated by comparing the amount of heat energy reaching the water to the amount of heat energy entering the system:

$$\eta = \frac{Q_w}{Q_{total}} = \frac{(16.0924W)}{(16.002W)} = 1.006 = \boxed{100.6\%}$$

Summary:

The initial values chosen proved to be fairly far off based on the percentage difference calculated at the end of the process. Utilizing the physical properties to hone in on the correct values proved effective as the assumed values were well within .1% in just 6 iterations. The efficiency was greater than 100%.

Materials:

- Silver plate
- Glass plate
- Copper tubes
- Water
- Ambient and trapped air

Analysis:

This system had very small temperature differences which meant that the values for the Grashof number were also very small. The result was that the convective coefficient values initially used were very far off, but the changes to the system when they were iterated with very small. A larger temperature differential between the ambient air and the incoming water would have changed how the system absorbed heat and resulted in larger differences during the final analysis. Because the temperatures were so close, the system was nearly in equilibrium, resulting in almost no heat transfer at all. Even though the initial *h* values were all more than 70% higher than the final values, the overall heat transfer changed by less than 0.2 watts, and the mass flow of the water only shifted by 0.000004 kg/s.

The efficiency of the system was also an interesting result as it was more than 100%. This is a result of the temperature differences between the ambient air and the insulated air gap. Some of the heat from the ambient air was also impacting the insulated air gap because it was colder inside the gap than outside. As the water temperature increases due to the heat gathered from Q_{bottom} the efficiency will drop. The bulk temperature of the water was only a tiny bit above the ambient temperature, so this difference had not yet impacted the system above the absorber plate resulting in a lower efficiency.