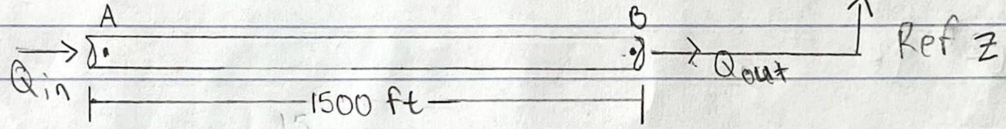
MET 330 Test 3 Micajah Paynter 4/10/22

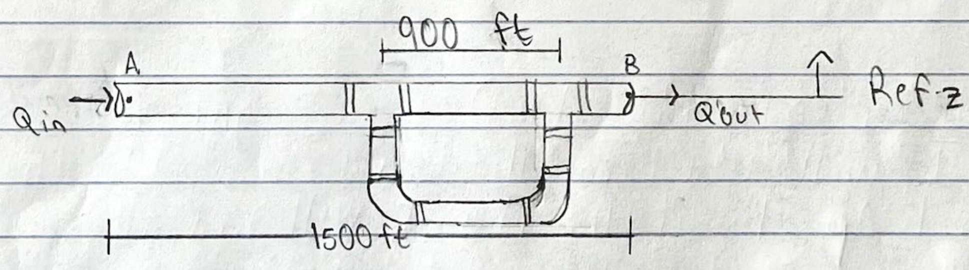
Purpose:

a) 1500 ft of 2 in standard steel tubing has water passing through it at a rate of 65 gpm. Determine the corresponding pressure drop

b) The pipe is modified by adding a loop of 1 1/2 in standard steel tubing that is 900 ft long. What is the expected increase in flow rate through the system for the same pressure as the original pipe?

Drawings + Diagrams:





Sources:

Mott, R., Untener, J. A., "Applied Fluid Mechanics," 7th edition, Pearson Education Inc., (2015)

Design Considerations:

Constant Properties
Incompressible Fluid
Steady State

Room Temperature
Constant Temperature

Data + Variables:

Flow Rate Q_1, Q_2, Q_3
Area A_1, A_2 (2 in, $1\frac{1}{2}$ in Standard steel tubing)
Velocity V_1 V_2 + gravity $g = 32.2\ ft/s^2$
γ_{H_2O} @ 70°F = 62.3 lb/ft^3
ν_{H_2O} @ 70°F = 1.05×10^{-5} ft^2/s
Δp
R_e
f, f_T
D_1, D_2
L_1, L_2 = 1500 ft, 900 ft
h_L
$\varepsilon = 5.0 \times 10^{-6}$ ft

Procedure:

First I will solve for the Δp in the straight tubing. To do so, I will use Bernoulli's and manipulate the equation to get Δp by itself. Then I will find all the unknowns in my equation using the continuity equation, Reynold number equation, + friction factor (f + f_T) equations. All that is left is to plug in and solve.

Next, I will solve for the increase in flow rate using Bernoulli's equation for each branch in the modified system + the $Q_1 = Q_2 + Q_3$ equation. To do this, I will have to assume a Q_1 to start. I will base my Re_1 $(D/\varepsilon)_1$ + f_1 on this Q_1. Then I will have to assume a value for my f_2 and then calculate my new (D/ε), my denomenator + the solve for my Q_2. Using this Q_2 I can find my new f_2 value that I can compare to my assumed. Then I will iterate using my new f_2. This will give me an accurate Q_2. Then Repeat for Q_3.

Calculations:

a) $$\frac{p_A}{\gamma_{H_2O}} + \frac{\cancel{V_A^2}}{\cancel{2g}} + \cancel{z_A} = \frac{p_B}{\gamma_{H_2O}} + \frac{\cancel{V_B^2}}{\cancel{2g}} + \cancel{z_B} + h_{L_{A-B}}$$

$$\Rightarrow \Delta p = (h_{L_{A-B}})\,\gamma_{H_2O}$$

$Q = 65\ gpm$ $\quad 1\ ft^3/s = 449\ gpm$

$\Rightarrow \frac{65\ gpm}{449\ \frac{ft^3}{s}}$ $\quad Q = 0.145\ ft^3/s$

$Q = VA \Rightarrow V = \frac{Q}{A}$ $\quad D = 0.1558\ ft$ $\quad A = 0.01907\ ft^2$

$V = \frac{0.145\ \frac{ft^3}{s}}{0.01907\ ft^2}$ $\quad R_e = \frac{VD}{\nu} = \frac{7.6\ \frac{ft}{s} \cdot 0.1558\ ft}{1.05\times10^{-5}\ \frac{ft^2}{s}}$

$V = 7.6\ \frac{ft}{s}$ $\quad R_e = 112769$

$$f = \frac{0.25}{\left[\log\left(\frac{1}{3.7(D/\varepsilon)} + \frac{5.74}{Re^{0.9}}\right)\right]^2} \qquad f = \frac{0.25}{\left[\log\left(\frac{1}{3.7\left(\frac{0.1558\ ft}{5.0\times10^{-6}\ ft}\right)} + \frac{5.74}{(112769)^{0.9}}\right)\right]^2}$$

$f = 0.0176$

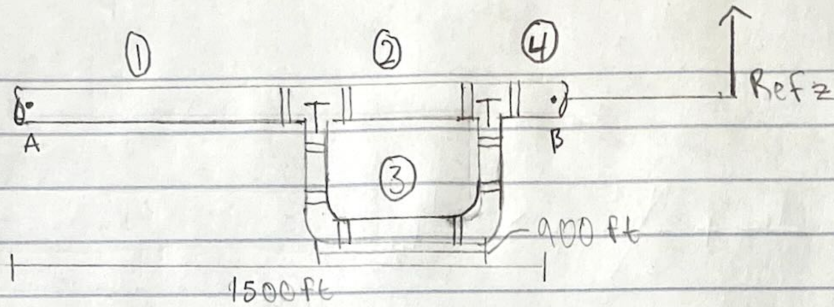
$$h_{L_{A-B}} = f\frac{L}{D}\frac{V^2}{2g} = 0.0176\left(\frac{1500\ ft}{0.1558\ ft}\right)\left(\frac{(7.6\ \frac{ft}{s})^2}{2\cdot 32.2\ ft/s^2}\right)$$

$h_{L_{A-B}} = 151.98\ ft$

$\Delta p = (151.98\ ft)\ 62.3\ lb/ft^3$

$\Delta p = 9468.4\ lb/ft^2$

b)



$1 \to 2 \to 4$

$$\frac{\Delta p}{\gamma_{H_2O}} = f_1 \frac{L_1}{D}\frac{V_1^2}{2g} + 2K_{Tee,1}\frac{V_2^2}{2g}$$

$1 \to 3 \to 4$

$$\frac{\Delta p}{\gamma_{H_2O}} = f_1 \frac{L_1}{D}\frac{V_1^2}{2g} + 2K_{Tee,2}\frac{V_1^2}{2g} + 2K_{Elbow}\frac{V_3^2}{2g}$$

$$Q_1 = Q_2 + Q_3$$

$$\frac{\Delta p}{\gamma_{H_2O}} = f_1 \frac{L_1}{D_1}\frac{8Q_1^2}{g\pi^2 D_1^4} + 2K_{Tee,1}\frac{8Q_2^2}{g\pi^2 D_2^4} + f_2\frac{L_2}{D_1}\frac{8Q_2^2}{g\pi^2 D_2^4}$$

$$\frac{\Delta p}{\gamma_{H_2O}} = f_1 \frac{L_1}{D_1}\frac{8Q_1^2}{g\pi^2 D_1^4} + 2K_{Tee\,2}\frac{8Q_3^2}{g\pi^2 D_3^4} + 2K_{Elbow}\frac{8Q_3^2}{g\pi^2 D_3^4} + K_{Red}\frac{8Q_3^2}{g\pi^2 D_3^4} + K_{Exp}\frac{8Q_3^2}{g\pi^2 D_3^4}$$
$$+ f_3\frac{L_3}{D_3}\frac{8Q_3^2}{g\pi^2 D_3^4}$$

Procedure:

1) Assume $Q_1 \to Re_1, (D/\varepsilon)_1 \to f_1$
2) Solve for Q_2 using (2) Iteration
   - 2.1) Assume f_2
   - 2.2) Solve for Q_2
   - 2.3) Compare $Re_2, (D/\varepsilon)_2$
   - 2.4) compute f_2 (New)
   - 2.5) compare old and new

3) Solve for Q_3 using ③ Iteration
4) Compute Q_1 using ①
5) Compare new and old Q_1's

$D_1 = 0.1558\ ft$ $D_2 = 0.1558\ ft$ $D_3 = 0.1142\ ft$ $D_4 = 0.1558 ft$
$L_1 = 600 ft$ $L_2 = 900\ ft$ $L_3 = 900\ ft$
$\nu_{H_2O} = 1.05 \times 10^{-5}\ ft^2/s$ $\gamma_{H_2O}\ 62.3\ lb/ft^3$ $\varepsilon = 5.0 \times 10^{-6}\ ft$
$\Delta P = 9468.4\ lb/ft^2$
$K_{Tee_1} = 20 f_T$ $K_{Tee_2} = 60 f_T$ $K_{Elbow} = 30 f_T$
$K_{Red} = 0.04$ @ $40°$ + $\frac{0.1558 ft}{0.1142 ft}$ $K_{Exp} = 0.4$ @ $40°$ + $\frac{0.1558 ft}{0.1142 ft}$

$$Q_1 = Q_2 + Q_3 \quad ①$$

$$\frac{\Delta P}{\gamma_{H_2O}} = \frac{8 Q_1^2}{g \pi^2 D_1^4}\left(f_1 \frac{L_1}{D_1}\right) + \frac{8 Q_2^2}{g \pi^2 D_2^4}\left(2 K_{Tee_1} + f_2 \frac{L_2}{D_2}\right) \quad ②$$

$$\frac{\Delta P}{\gamma_{H_2O}} = \frac{8 Q_1^2}{g \pi^2 D_1^4}\left(f_1 \frac{L_1}{D_1}\right) + \frac{8 Q_3^2}{g \pi^2 D_3^4}\left(2 K_{Tee_2} + 2 K_{Elbow} + K_{Red} + K_{Exp} + f_3 \frac{L_3}{D_3}\right) \quad ③$$

$$Q_2 = \sqrt{\frac{\frac{\Delta P}{\gamma_{H_2O}} - \frac{8 Q_1^2}{g \pi^2 D_1^4}\left(f_1 \frac{L_1}{D_1}\right)}{\left(2 K_{Tee_1} + f_2 \frac{L_2}{D_2}\right)\frac{8}{g \pi^2 D_2^4}}}$$

$\Rightarrow$ ~~numerator = know~~

$$Q_3 = \sqrt{\frac{\frac{\Delta P}{\gamma_{H_2O}} - \frac{8 Q_1^2}{g \pi^2 D_1^4}\left(f \frac{L_1}{D_1}\right)}{\left(2 K_{Tee_2} + 2 K_{Elbow} + K_{Red} + K_{Exp} + f_3 \frac{L_3}{D_3}\right)\frac{8}{g \pi^2 D_3^4}}}$$

After Iteration

$Q_1 = 0.1751\ \frac{ft^3}{s}$ ✓
$Q_2 = 0.1221\ \frac{ft^3}{s}$ $Q_1 = Q_2 + Q_3 \Rightarrow 0.1221 + 0.0530 = 0.1751\ \frac{ft^3}{s}$ ✓
$Q_3 = 0.0530\ \frac{ft^3}{s}$

$Q_1 = 0.1751\ (449)$
$Q_1 = 78.62$ gpm

$Q_{original} = 65$ gpm
$\boxed{Q_1 = 78.62 \text{ gpm}}$

Summary:

a) *Given the pipe dimensions & flow rate for the straight piep, the change in pressure was found using Bernoulli's

$\boxed{\Delta p = 9468.4\ lb/ft^2}$

b) Using Bernoulli's again & iterating in excel the new flow rate was found to be $\boxed{78.62 \text{ gpm}}$ or $\boxed{\text{an increase of } 13.62 \text{ gpm}}$

Materials used:

Water
steel tubing

Analysis

a) There will always be a change inpressure through a pipe with moving Fluid.

b) The had to be an increase in flow rate because the over-all area in the system was being increased, therefore, the total volume in the system was increased. Thus, the system could support the additional 13.62 gpm.