

Key Fluid Mechanics

12.4. In the branched system shown in Fig P12.4 1350 gal/min of benzene ($\rho = 0.87$ at 140°F) is flowing in the 8 in pipe. Calculate the volume flow rate in the 6 in and the 2 in pipes. All pipes are standard schedule 40 steel pipes.

given: $Q = 1350 \text{ L/min} = 11.7 \text{ L/s} = 0.0117 \text{ m}^3/\text{s}$, DN 100 40 ($D = 0.043 \text{ m}$)

DN 50 schedule 40 ($D = 0.0603 \text{ m}$)

Solution: step 1

upper branch 6 in pipe

Diameter $D = 6 \text{ in} = 0.1524 \text{ m}$

minor loss

elbow $K = 1.0$

swing type check valve $K = 2$

1/2 bend $K = 0.45$

total minor loss coefficient

$$K_{\text{upper}} = 1.0 + 2 + 0.9 = 14.7$$

lower branch (2 in pipe, for 8 in)

$D = 2 \text{ in} = 0.0508 \text{ m}$

$L = 50 \text{ ft} = 15.24 \text{ m}$

$K = 0.4 \times 2$

total minor loss

$K_{\text{lower}} = 2 \times 0.4 = 1.8$

step 2

8 in pipe (6 in pipe)

$$Q_1 = 0.01 \text{ m}^3/\text{s}$$

$$\text{cross section: } A = \frac{\pi (D/2)^2}{4} = 0.00152 \text{ m}^2$$

$$\text{velocity } v = \frac{Q_1}{A} = 0.549 \text{ m/s}$$

$$h_{\text{minor upper}} = K_{\text{upper}} \frac{v^2}{2g} = 14.7 \frac{0.549^2}{2 \times 9.81} = 0.216 \text{ m}$$

7.46 m/s (2 inch pipe, 500 ft)

$$Q_2 = 0.01 \text{ m}^3/\text{s}$$

$$\text{Cross sectional Area } A = \frac{\pi (D/2)^2}{4} = 0.0002 \text{ m}^2$$

$$\text{Velocity } v = \frac{Q_2}{A} = 1.95 \text{ m/s}$$

$$\text{Reynolds number: } Re = \frac{vD}{\nu} = \frac{1.95 \times 0.0508}{1.31 \times 10^{-6}} = 7.4 \times 10^5$$

$$\text{Friction factor } f = 0.02$$

Head loss due to minor loss

$$h_{\text{minor loss}} = K_{\text{loss}} \frac{v^2}{2g} = 1.8 \frac{1.95^2}{2 \times 9.81} = 0.23 \text{ m}$$

total head loss

$$h_{\text{total loss}} = h_{\text{friction}} + h_{\text{minor loss}} = 1.2 + 0.23 = 1.43 \text{ m}$$

Step 3: Adjust flow rate

Head loss difference

$$\Delta h = h_{\text{total loss}} - h_{\text{minor loss}} = 1.43 - 0.23 = 1.2 \text{ m}$$

12.10 In the branched pipe system shown in fig 12.3 850 L/min of water at 10°C in DN 100 schedule 40 pipe at A. The flow splits into two DN schedule 40 pipes as shown and then rejoins at B. Calculate (a) the flow rate in each of the branches and (b) the pressure difference $p_A - p_B$ taking the effect of the minor losses into account. The total length of pipe in the lower branch is 60 m.

Given: $Q = 850 \text{ L/min} = 0.01417 \text{ m}^3/\text{s}$
 10°C density $\rho = 999.7 \text{ kg/m}^3$, $\nu = 1.31 \times 10^{-6} \text{ m}^2/\text{s}$

Solution = Initial Flow Assumption

$$Q_1 + Q_2 = Q$$

$$Q_1 = 0.007 \text{ m}^3/\text{s} \text{ and } Q_2 = 0.00717 \text{ m}^3/\text{s}$$

Head loss

$$D = 0.1023 \text{ m}$$

$$A = \pi D^2/4 = 0.00816 \text{ m}^2$$

$$V_1 = \frac{Q_1}{A} = \frac{0.007}{0.00816} = 2.45 \text{ m/s}$$

Lower Branch:

$$D = 0.1023 \text{ m}$$

$$A = \pi D^2/4 = 0.00816 \text{ m}^2$$

$$V_2 = \frac{Q_2}{A} = \frac{0.00717}{0.00816} = 2.51 \text{ m/s}$$

$$R_2 = \frac{V_2 D}{\nu} = \frac{2.51 \times 0.1023}{1.31 \times 10^{-6}} = 1.98 \times 10^5$$

$$f = \frac{0.02}{V_2} = 0.02 \frac{60}{0.00816} \frac{2.51^2}{2 \times 9.81} = 2.54$$

minor loss Darcy friction

$$\text{total minor loss } k = 10.4 \times 0.1 = 13.6$$

$$h_{\text{minor}} = K \frac{v^2}{2g} = 13.6 \frac{2.51^2}{2 \times 9.81} = 1.29 \text{ m}$$

total head loss in lower branch

$$h_{\text{total}} = h_f + h_{\text{minor}} = 2.54 + 1.29 = 6.83 \text{ m}$$

Energy method
flow adjustment

$$\text{head loss in pipe } h_{\text{total lower}} - h_{\text{upper}} = 6.83 - 0 = 6.83 \text{ m}$$

upper branch $h_{\text{upper}} = 0$

lower branch $k_{\text{lower}} = 13.6$

$$\text{flow adjustment factor } \Delta Q = \frac{Ah}{2.5 \left(\frac{K}{PA} \right)}$$

Flow rates Q_1 and Q_2 in $Q = 0.07 \text{ m}^3/\text{s}$

$$Q_2 = 0.06717 \text{ m}^3/\text{s}$$

Head loss

$$h_{\text{down}} = 6.83 \text{ m}$$

Flow adjustment

$$\Delta Q = \frac{6.83}{2 \left(\frac{13.6}{0.06717} \right)} = 4.45 \times 10^{-6} = 6.69 \times 10^{-6} \text{ m}^3/\text{s}$$

Adjusted flow rate:

$$Q_1 = Q + \Delta Q = 0.067 + 6.69 \times 10^{-6} = 0.0670669 \text{ m}^3/\text{s}$$

$$Q_2 = Q - \Delta Q = 0.067 - 6.69 \times 10^{-6} = 0.0669331 \text{ m}^3/\text{s}$$

Flow rates in each branch

$$\text{upper branch } Q_1 = 0.0670669 \text{ m}^3/\text{s}$$

$$\text{lower branch } Q_2 = 0.0669331 \text{ m}^3/\text{s}$$

$$P_A - P_B = 67.01 \text{ kPa}$$

1A.12: A trapezoidal channel with bottom width of 3.0 ft and side slopes having ratio of 1.0:0.75 carries 0.80 ft³/s of water and is made from steel lined concrete. use $n = 0.015$ ft/s

Given: $b = 3.0$ ft, $z = 0.75$, $Q = 0.80$ ft³/s, $n = 0.015$ ft/s

Solution: Step 1 calculate cross-sectional Area

$$A = y(b + zy) = 0.05(3.0 + 0.75 \times 0.05) = 0.05(3.0 + 0.0375) = 0.05 \times 3.0375 = 0.151875 \text{ ft}^2$$

Wetted perimeter p

$$P = b + 2y\sqrt{1 + z^2} = 3.0 + 2 \times 0.05 \sqrt{1 + 0.75^2} = 3.0 + 0.1 \sqrt{1.5625} = 3.0 + 0.1 \times 1.25 = 3.0 + 0.125 = 3.125 \text{ ft}$$

Hydraulic Radius

$$R = \frac{A}{p} = \frac{0.151875}{3.125} = 0.0486 \text{ ft}$$

calculate slope S :

$$Q = 0.80 \text{ ft}^3/\text{s}$$

$$n = 0.015$$

Manning's equation slope S

$$Q = \left(\frac{1.49 A R^{2/3}}{n} \right) \sqrt{S}$$

$$R^{2/3} = \left(\frac{Q n}{1.49 A} \right)^{3/2} = 0.072$$

$$S = \left(\frac{Q n}{1.49 A R^{2/3}} \right)^2 = 0.05$$

M.36 T.B. is to carry $1.25 \text{ ft}^3/\text{s}$ of water at a velocity of 2.75 ft/s . Design the channel cross section for each of the slopes shown in Table M.3 which lists the most efficient sections for open channels.

Given Flowrate $Q = 1.25 \text{ ft}^3/\text{s}$

Velocity $V = 2.75 \text{ ft/s}$

Cross section Area

$$A = \frac{Q}{V} = \frac{1.25 \text{ ft}^3/\text{s}}{2.75 \text{ ft/s}} = 0.4545 \text{ ft}^2$$

channel slope

a. Rectangular channel

Rectangular channel section in hydraulic square $b = 2y$

$$\text{Area } A = b \cdot y = 2y \cdot y = 2y^2$$

$$\text{Set area } = 0.4545 \text{ ft}^2$$

$$2y^2 = 0.4545$$

$$y^2 = 0.22725$$

$$y = \sqrt{0.22725}$$

$$y = 0.4767 \text{ ft}$$

$$b = 2y = 2 \times 0.4767 = 0.9534 \text{ ft}$$

rectangular channel

$$\text{Area } A = \frac{b^3}{2} = y^2 = 0.4545 \text{ ft}^2 = \frac{b^3}{2} = \sqrt{0.4545} = 0.674 \text{ ft}$$

triangular channel

$$\text{Depth } y = 0.674 \text{ ft}$$

$$t = 2y = 2 \times 0.674 = 1.348 \text{ ft}$$

$$\text{trapezoidal channel } = 76 = 1.55y \text{ and } t = 2.309y, z = 0.571, A = 1.73y^2$$

$$\text{area } A = 1.73y^2$$

$$1.73y^2 = 0.4545$$

$$y^2 = \sqrt{0.2627} = 0.513 \text{ ft}$$

$$b = 1.55y = 1.5 \times 0.513 = 0.593 \text{ ft}$$

$$t = 2.309y = 2.309 \times 0.513 = 1.184 \text{ ft}$$

15.4: A sharp-edged orifice is placed in a 10 in diameter pipe carrying ammonia. If the volume flow rate is 25 gal/min, calculate the deflection of a water manometer
 a) if the orifice diameter is 1.0 in and (b) if the orifice diameter is 7.0 in. The specific gravity of ammonia has a specific gravity of 0.63 and a dynamic viscosity of 2.5×10^{-6} lb/ft·s.

Given $D = 10$ in, $Q = 25$ gal/min, $S_g = 0.63$, $\mu = 2.5 \times 10^{-6}$ lb/ft·s

convert flow rate:

$$1 \text{ gallon} = 0.13368 \text{ ft}^3$$

$$Q = 25 \text{ gal/min} \times 0.13368 \text{ ft}^3/\text{gal} = 3.342 \text{ ft}^3/\text{min} = \frac{3.342 \text{ ft}^3}{60} = 0.0557 \text{ ft}^3/\text{s}$$

Flow Velocity

$$A = \frac{\pi D^2}{4} = \frac{\pi (10/12)^2}{4} \text{ ft}^2 = \frac{\pi (5/6)^2}{4} \text{ ft}^2 = 0.545 \text{ ft}^2$$

Flow Velocity

$$V = \frac{Q}{A} = \frac{0.0557}{0.545} \text{ ft/s} = 0.102 \text{ ft/s}$$

orifice diameter 1 in

$$B = d = 1.0 \text{ in} = \frac{1}{12} \text{ ft}$$

$$\beta = \frac{d}{D} = \frac{1/12}{10/12} = \frac{1}{10} = 0.1$$

pressure drop

$$\Delta p = \frac{\rho V^2}{2} \left(\frac{1}{\beta^2} - 1 \right) = \frac{51.792 \text{ lb/ft}^3 (0.102)^2}{2} \left(\frac{1}{0.1^2} - 1 \right) = 0.1693 \times 99 = 16.76 \text{ lb/ft}^2$$

manometer deflection

$$h = \frac{\Delta p}{\rho_{\text{man}} g} = \frac{16.76 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3 \times 32.2 \text{ ft/s}^2} = 0.083 \text{ ft}$$

Orifice Diameter 7.0 in

$$d = 7.0 \text{ in} = \frac{7}{12} \text{ ft}$$

$$\beta = \frac{d}{D} = \frac{7 \text{ in}}{10 \text{ in}} = \frac{7}{10} = 0.7$$

pressure drop ΔP

$$\Delta P = \frac{\rho V^2}{2} \left(\left(\frac{1}{\beta^2} - 1 \right) \right) = \frac{51.7 \text{ lb/ft}^3 \times 0.102^2}{2} \left(\left(\frac{1}{0.7^2} - 1 \right) \right) = 0.775 \text{ lb/ft}^2$$

manometer deflection

$$h = \frac{\Delta P}{\rho_{\text{water}} g} = \frac{0.775 \text{ lb/ft}^2}{62.4 \text{ lb/ft}^3 \times 32.2 \text{ ft/s}^2} = 0.00145 \text{ ft} = 0.00168 \text{ inch}$$

15. A flow nozzle is to be installed in a 5 in Typ 1 copper tube carrying the sec oil at 77°F. A mercury manometer is to be used to measure the pressure difference across the nozzle when the expected range of the flow rate is from 700 to 1000 gal/min. The manometer scale ranges from 0 to 5.6 of mercury. Determine an appropriate diameter of the nozzle.

Given Data

5 in dia, fluid at 77°F, flow rate = 700 to 1000 gal/min
manometer reading 0 to 5.6 inches of mercury

Solution:

$$1 \text{ gal} = 0.13368 \text{ ft}^3$$

$$\text{Flow rate (range) } 700 \text{ to } 1000 \text{ gal/min}$$

$$Q_{min} = 700 \text{ gal/min} \times 0.13368 \text{ ft}^3/\text{gal} \times \frac{1 \text{ min}}{60 \text{ s}} = 1.558 \text{ ft}^3/\text{s}$$

$$Q_{max} = 1000 \text{ gal/min} \times \frac{0.13368 \text{ ft}^3/\text{gal}}{1 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ s}} = 2.228 \text{ ft}^3/\text{s}$$

2. In sec oil

sg at 77°F is 0.93

$$\text{Density } \rho = \text{sg} \times \rho_{water} = 0.93 \times 62.416 \text{ lb/ft}^3 = 58.032 \text{ lb/ft}^3$$

manometer fluid

1 in dia

$$\Delta p = h \rho g \times \rho_{fluid} \times g$$

$$\rho_{Hg} = 845.7 \text{ lb/ft}^3$$

$$g = 32.2 \text{ ft/s}^2$$

$$\Delta p_{max} = 8 \text{ in} \times \frac{1 \text{ ft}}{12 \text{ in}} \times 845.7 \text{ lb/ft}^3 \times 32.2 \text{ ft/s}^2 = 1922 \text{ lb/ft}^2 = 12.8 \text{ psi}$$

pressure difference between nozzle diameter

$$\Delta p = \frac{\rho V^2}{2} \Rightarrow V = \frac{Q}{A_{nozzle}} \Rightarrow A_{nozzle} = \frac{Q^2}{2 \Delta p} \Rightarrow \frac{\pi d^2}{4} = \frac{Q^2}{2 \Delta p} \Rightarrow d = \sqrt{\frac{4 Q^2}{\pi \Delta p}}$$

$$\Delta p_{max} = 58.032 \cdot 2.228^2 = 287.7 \text{ ft}^2/\text{s}^2$$

$$d = \sqrt{\frac{4 \cdot 2.228^2}{\pi \cdot 287.7}} = 0.0456 \text{ ft} \Rightarrow B^2 = 0.00208 \Rightarrow B = 0.0457 \text{ ft} = 0.548 \text{ in}$$

$$h = 0.93 \times 0.548 \text{ ft} = 0.51 \text{ ft} = 6.12 \text{ in}$$