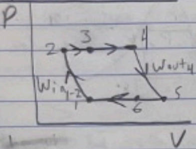
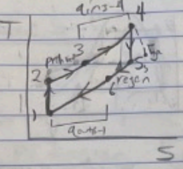
Test 1

Problem 1 Nathan Cherny5

Step 1: plots & diagram

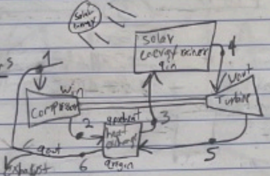




Source: thermodynamics an engineering approach & ayala's excellent teaching

Purpose:
Find η^{th} & ϵ of the system & regenerator.
Find $\dot{m}$.
Find η^{th} of heat exchanger w/ 100% effectiveness

Diagram:



given:
- Braytoncycle
- 1 compression, 1 expansion
- heat regen. Counter flow
- $\eta^{th}_{turbine} = 0.8$
- Cold air standard → @ room temp. → ideal gas laws apply

Step 2: States & Assumptions

assume:
Const C_p/C_v
$C_p = 1.005 \frac{kJ}{kg \cdot K}$
$R = 0.287 \frac{kJ}{kg \cdot K}$
$K = 1.4$

States

P const exhaust → (1) $\xrightarrow[W_{in}]{\text{Isent. comp.}}$ (2) $\xrightarrow[\text{preheating}]{P \text{ const}}$ (3) $\xrightarrow[q_{in}]{P\text{const}}$ (4) $\xrightarrow[W_{out}]{\text{isent. exp.},\ \eta^{th}_T = 0.8}$ (5) $\xrightarrow[\text{regen}]{P\text{const}}$ (6) →

1	2	3	4	5	6
✓ $P_1 = 1$ bar	✓ $P_2 = 5$ bar	✓ $T_3 = 760°K$	✓ $T_4 = 1140°K$	✓ $P_5 = 1$ bar	$P_6 = 1$ bar
$T_1 = 310°K$	$T_2 = 490.98°K$	$P_3 = 5$ bar	$P_4 = 5$ bar	$T_{5s} = 719.779°K$	$T_6 = 534.8°K$
				$T_{5a} = 803.8232°K$	

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1 bar = 100 Kpa

part 1 → get states

calculations:

isentropic process, constant cp/cv $\left\} \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \right.$ → $T_2 = (5)^{\frac{0.4}{1.4}} \cdot 310°K = 490.98°K$

$P_3 = P_2 \quad P_4 = P_2 \quad P_5 = P_1 \quad P6 = P_1$

T_{5s} → ideal
T_{5a} → actual

isentropic process, const cp/cv $\left\} \frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{\frac{k-1}{k}} \right. = \left(\frac{1}{5}\right)^{\frac{0.4}{1.4}} \cdot 1140°K = T_{5s} = 719.779°K$ ← impossible! b/c if this Temp. is lower than $T_3 = 760°K$ then $\boxed{T_3 \text{ heats } T_5}$ must be the ideal Not actual

$\eta^{th}_T = 0.8 = \eta^{th} = \frac{h_4 - h_{5a}}{h_4 - h_{5s}}$ → ~~take out cp~~ → $\frac{T_4 - T_{5a}}{T_4 - T_{5s}}$

$= 0.8(T_4 - T_{5s}) - T_4 = T_{5a}$

$= 0.8(1140°K - 719.779°K) - 1140°K = T_{5a} = -803.8232°K$

✓ $T_{5a} = 803.8232°K$

We can find T_6 by taking the assumption from Q_3 heat exchange w/ 100% effectiveness.

$T_3 = T_2 + 269.02°K \rightarrow (T_3 - T_2) = 269.02°K$ → heat preheated

therefore w/ 100% effectiveness, that heat taken from T_5 creates $T_6 = T_5 - 269.02°K$

$T_6 = 803.8232°K - 269.02°K = 534.8032°K$

Part 2 Solutions

(a) $\eta_{th} = \frac{W_{net}}{q_{in}}$ $\quad W_{net} = W_{out} - W_{in} \rightarrow W_{out\,4-5} - W_{in\,1-2}$

Wout process 1st law

(4-5) $\cancel{q_{in}^0} - \cancel{q_{out}} + \cancel{w_{in}^0} - W_{out} = \Delta h$

$-W_{out} = h_5 - h_4 \rightarrow -W_{out} = c_p(T_5 - T_4)$

$-W_{out} = 1.005\frac{kJ}{kg}(803.8032K - 1140°K)$

$\cancel{-}W_{out} = \cancel{-}337.857\frac{kJ}{kg}$

Win process 1st law

(1-2) $\cancel{q_{in}^0} - \cancel{q_{out}} + w_{in} - \cancel{W_{out}^0} = \Delta h$

$W_{in} = h_2 - h_1 \rightarrow w_{in} = c_p(T_2 - T_1)$

$W_{in} = 1.005\frac{kJ}{kg \cdot K}(490.98°K - 310°K)$

$W_{in} = 181.885\frac{kJ}{kg}$

$W_{net} = W_{out} - W_{in}$

$W_{net} = 337.857\frac{kJ}{kg} - 181.885\frac{kJ}{kg}$

$W_{net} = 155.972\frac{kJ}{kg}$

qin process 1st law

(3-4) $q_{in} - \cancel{q_{out}^0} + \cancel{w_{in}^0} - \cancel{W_{out}^0} = \Delta h$

$q_{in} = c_p(T_4 - T_3)$

$q_{in} = 1.005\frac{kJ}{kg \cdot K}(1140K - 760K) = 381.9\frac{kJ}{kg}$

$\eta_{th} = \frac{155.972\frac{kJ}{kg}}{381.9\frac{kJ}{kg}}$

$\boxed{\eta_{th} = 0.41}$

effectiveness of regenerator $= \epsilon = \frac{T_3 - T_2}{T_{5a} - T_2} = \frac{760°K - 490.98°K}{803.8032K - 490.98°K} = \boxed{0.86 = \epsilon}$

Under cold air assumptions

(b) air $\dot{m} = \dot{W}_{net}/w_{net}$ if $\dot{W}_{net} = 500KW$

$\dot{m} = \frac{500\,kW}{155.972\frac{kJ}{kg}} = \boxed{3.21\,kg/s = \dot{m}}$

(c) η_{th} under cold air assumptions $= \eta_{th} = 1 - \left(\frac{T_1}{T_4}\right)\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 1 - \left(\frac{310°K}{1140°K}\right)(5)^{0.4/1.4}$

$\boxed{\eta_{th} = 0.569}$

technical writing: in each cycle the thermal efficiency is 0.41 → 41% and the regenerator under cold air is as effective as 86% so with a net work of 156 $\frac{kJ}{kg}$ we can expect 41% efficiency w/ 86% of the recycled energy being capable of use. if the effectiveness of the cycle is 100% we can expect the overall thermal efficiency to improve to 57%.

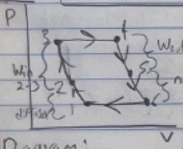
Test 1
Problem 2

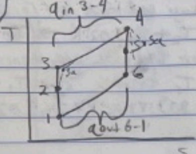
Ethan Cherneys

Purpose: Find Pressure after turbine, Find Velocity at nozzle exit, Find thrust if $d = 1.6M$

Step 1: Plots & Diagram

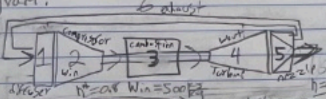
Source: class notes, thermodynamics an engineering approach textbook, Yunus Cengel





assumptions:
- $\eta_c^{th} = 0.8$ $\eta_T^{th} = 0.9$
- Variable cp/cv
- air is an ideal gas
- V_2 at the end of the diffuser $= 0$
- Turbine produces $500 \frac{kJ}{kg}$
- all W_{out} goes into W_{in}; $W_{in} = W_{out}$
- $V_5 = 0\,m/s$ ⇒ beginning of nozzle

Diagram:



Step 2: States:

$\eta_c^{th} = 0.8$ $W_{in} = 500 \frac{kJ}{kg}$ $\eta = 0.9$ $W_{out} = 500 \frac{kJ}{kg}$

① diffuser, isent. → ② compressor, isent → ③ Combustion, P const → ④ expansion, isent. → ⑤ nozzle, isent. → ⑥ exhaust, P-const

$R = 0.287 \frac{kJ}{kg \cdot K}$

①	②	③	④	⑤	⑥
$V_1 = 900\,km/h$	$V_2 = 0$	$P_3 = 4196.5\,kPa$	$T_4 = 1223\,°K$	$T_5 = 824.84\,°K$	$P_6 = 40\,kPa$
$T_1 = 238\,°K$	$T_2 = 269.16\,°K$	$T_3 = 865.38\,°K$	$P_4 = 4196.4\,kPa$	$P_5 = 3563.8\,kPa$	$T_6 = 238\,°K$
$P_1 = 40\,kPa$	$P_2 = 61.43\,kPa$	$h_{3a} = 894.27 \frac{kJ}{kg}$	$h_4 = 1304.853 \frac{kJ}{kg}$	$h_{5a} = 854.853 \frac{kJ}{kg}$	$V_6 = 1110.7\,m/s$
$h_1 = 238.02 \frac{kJ}{kg}$	$h_2 = 269.27 \frac{kJ}{kg}$	$h_{3s} = 769.27 \frac{kJ}{kg}$		$h_{5s} = 804.853$	

$\boxed{\text{Calculations}}$ $\boxed{V_5 = 0\,m/s}$

(1-2) 1st law

Source: Chap 9 thermodynamics an engineering approach

$$h_1 + \frac{V_1^2}{2} = h_2 + \cancel{\frac{V_2^2}{2}} \rightarrow 238.02 \frac{kJ}{kg} + \frac{250^2}{2 \cdot 1000} = 269.27 \pm h_2 \rightarrow \text{interpolate} \rightarrow T_2, P_{r2}$$

h_1 @ 238 K ⇒ interpolated app $= 238.02 \frac{kJ}{kg}$

conversion factor W/ this 1st law

$V_1\,km/h \rightarrow m/s = 250\,m/s$

$T_2 = 269.16\,°K$

$P_{r2} = 0.949$

isent var. cp/cv

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \quad P_{r1} = 0.61794 \quad P_{r2} = 0.949 \rightarrow \frac{0.949}{0.61794} \cdot 40\,kPa = P_2 = 61.43\,kPa$$

(4-5)

1st law to find ideal h_{5s} — ideal

$$W_{out} = 500\,kJ/kg = \Delta h_{4-5} \rightarrow 500 \frac{kJ}{kg} = h_4 - h_5$$

h_4 @ 1223 K $= 1304.853 \frac{kJ}{kg}$ ideal! $h_{5s} = 1304.853 \frac{kJ}{kg} - 500 \frac{kJ}{kg} = 804.853 \frac{kJ}{kg}$

actual — isent. efficiency of turbine

$$\eta = \frac{h_4 - h_{5a}}{h_4 - h_{5s}} = 0.9 = \frac{1304.853 \frac{kJ}{kg} - h_{5a}}{1304.853 \frac{kJ}{kg} - 804.853} = 854.853 \frac{kJ}{kg} = h_{5a}$$

$T_5 = \text{interpolate } h_{5a} = 824.84\,°K = T_5$

(2-3) ideal

compression isentropic

$$W_{in} = 500 \frac{kJ}{kg} = \Delta h_{2-3} \rightarrow 500 \frac{kJ}{kg} = h_{3s} - h_2 = 500 \frac{kJ}{kg} + 269.27 \frac{kJ}{kg} = 769.27 \frac{kJ}{kg} = h_{3s}$$

actual thermal efficiency

$$\eta^{th} = 0.8 = \frac{h_{3s} - h_2}{h_{3a} - h_2} = \frac{769.27 \frac{kJ}{kg} - 269.27 \frac{kJ}{kg}}{0.8} + 269.27 \frac{kJ}{kg} = h_{3a} = 894.27 \frac{kJ}{kg} = h_{3a}$$

$T_3 = \text{interpolate } h_{3a} = 865.38\,°K = T_3$

States: continued

(2-3)

isent, Var cp/cv: $\frac{P_3}{P_2} = \frac{Pr_3}{Pr_2} \rightarrow \begin{matrix} Pr_3 @ T_3 = 64.675 \\ Pr_2 @ T_2 = 0.949 \end{matrix} \rightarrow \frac{64.675}{0.949} \cdot 61.43\,kPa = P_3 = 4186.5\,kPa$

$P_3 = P_4$

(4-5)

isent, Var cp/cv: $\frac{P_5}{P_4} = \frac{Pr_5}{Pr_4}$ $\quad Pr_5 @ T_5 = 55.055$ (interpolate) $\rightarrow \frac{55.055}{64.675} \cdot 4186.5\,kPa = \boxed{P_5 = 3563.8\,kPa}$

(5-6)

isent, Var cp/cv: $\frac{P_6}{P_5} = \frac{Pr_6}{Pr_5} \rightarrow \frac{40\,kPa}{3563.8\,kPa} \cdot 55.055 = 0.6179 \rightarrow$ interpolate for $T_6 = 238°K$

$h_1 = h_6 \rightarrow T_1 = T_6$

1st law → jet turbine w/ actual h_5: $h_6 + \frac{V_6^2}{2} = h_{5a} + \cancel{\frac{V_5^2}{2}}^{0} = V_6 = \sqrt{2 \cdot (h_{5a} - h_6)}$

$= \sqrt{2 \cdot (854.853 - 238.02\,\frac{kJ}{kg}) \cdot 1000} \rightarrow$ conversion factor

$\boxed{V_6 = 1110.7\,m/s}$ Much faster than the initial velocity

Thrust → $F = \dot{m}(V_6 - V_1)$

$\dot{m} = \rho_1 \cdot V_1 \cdot A_1 \longrightarrow d = 1.6\,m = \frac{\pi d^2}{4} = A_1 = \frac{\pi \cdot 1.6^2}{4} = 1.0053\,m^2$

$\longrightarrow \rho = \frac{1}{v} = Pv = RT_1$

$R = 0.287\,kJ/kg \cdot K$ $\quad \frac{238°K \cdot 0.287\,kJ/kg \cdot K}{40\,kPa} = v_1 = 1.70765\,m^3/kg$

$\rho = \frac{1}{1.70765\,m^3/kg} = 0.5856\,kg/m^3$

$\dot{m} = 0.5856\,\frac{kg}{m^3} \cdot 250\,m/s \cdot 1.0053\,m^2 = 147.177\,kg/s$

$F = \dot{m}(V_6 - V_1) \rightarrow F = 147.177\,kg/s\,(1110.7\,m/s - 250\,m/s) = \boxed{126675.53\,N = F}$

Technical writing: As the air enters the diffuser at 250 m/s it exits it at 0 m/s for the isentropic compression. This increases the Temp & enthalpy. Knowing the compressor has an efficiency of 0.8, and W_{in} we can find the actual enthalpy to find the Temp before combustion. Then we can interpolate to find the pressures of States 3 and 4. After the combustion in the isentropic expansion between States 4 & 5 shows a drop in enthalpy by the W_{out} given and the efficiency giving an ideal & real enthalpy, similar in process to State 3. Through interpolation w/ T_5 we can find Pr_5 to get pressure at State 5 b/c it is an isentropic & variable specific heat problem. →

technical writing continued

given that we can use the same equation for the next process which is also isentropic w/ variable specific heats, this results in a Temp for state 6 in which it is equal to state 1. Which is unlikely. but is calculated due to the compressor & turbine great efficiencies. Following these calculations the final velocity calculated through the 1st law ($h_1 + \frac{V_1^2}{2} = h_n + \frac{V_n^2}{2}$) the velocity at the exit is more than 4 times the inlet velocity, and w/ an inlet diameter of 1.6m, the thrust is 126,675 kN of force.