MET330 | HW 1.3

6.79 Oil with a specific gravity of 0.90 is flowing downward through the venturi meter (Fig 6.33). If the manometer deflection h is 28 in, calculate volume flow rate of oil.

Given: $D_1 = 4\text{in}$

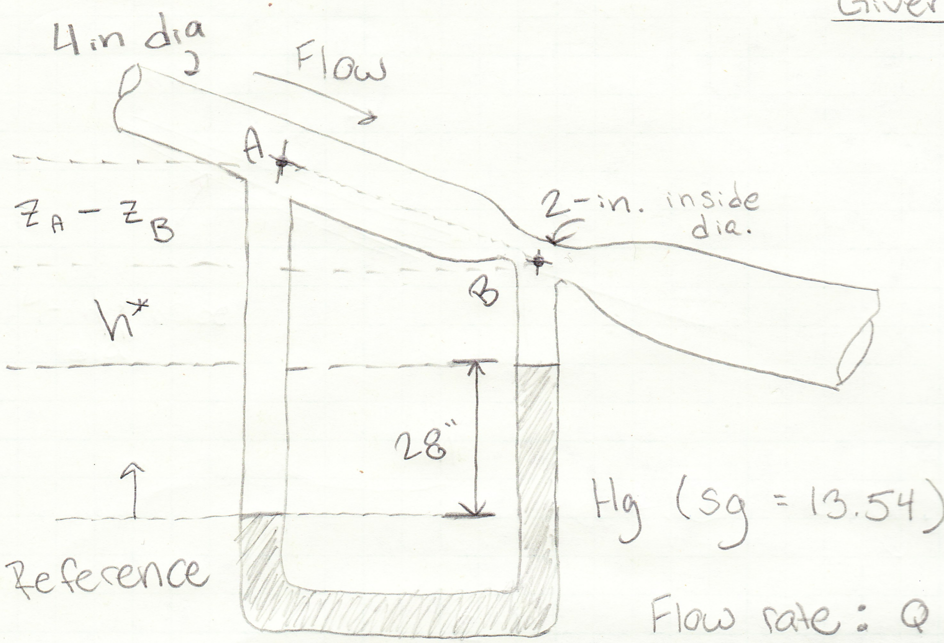
$D_2 = 2\text{in}$

$h = 28\text{ in}$

$sg_{oil} = 0.90$

$sg_{Hg} = 13.54$

$\gamma_{H_2O} = 9.81\ kN/m^3$



Flow rate: $Q = V \times A$

$\Delta P = \gamma h$

$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + z_1 = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + z_2$$

→ Plug $Q = v \times A$ into Bernoulli's eqn

$v = \frac{Q}{A}$

→ $\frac{V_A^2}{2g} \Rightarrow \frac{Q^2}{2g}\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)$

→ $$\frac{Q^2}{2g}\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right) = \frac{P_B - P_A}{\gamma} + (z_A - z_B)$$

Solve for Q: i) Divide RHS by $\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right)$

ii) Multiply RHS by $2g$

iii) Sqrt of RHS

$$Q = \sqrt{\frac{2g\left[\frac{P_B - P_A}{\gamma} + (z_A - z_B)\right]}{\frac{1}{A_A^2} - \frac{1}{A_B^2}}} \quad (1)$$

Point A to Point B Δp:

$$P_A + \gamma_{oil}((z_A - z_B) + h^* + h) - [\gamma_{Hg} h] - [\gamma_{oil} h^*] = P_B$$

Solve for Pressure Difference.

$$P_B - P_A = \gamma_{oil}((z_A - z_B) + h^* + h) - \gamma_{Hg} h - \gamma_{oil} h^*$$

$$\frac{P_B - P_A}{\gamma_{Hg}} = (z_A - z_B) + \left(1 - \frac{\gamma_{Hg}}{\gamma_{oil}}\right) h \quad (2)$$

$\frac{Sg_{Hg}}{Sg_{oil}}$

Plug (2) into (1)

$$Q = \sqrt{\frac{2g\left(1 - \frac{Sg_{Hg}}{Sg_{oil}}\right) h}{\frac{1}{A_A^2} - \frac{1}{A_B^2}}}$$

A_A: 4 in = 0.33 ft

$\frac{\pi D^2}{4} = 0.087 \text{ ft}^2$

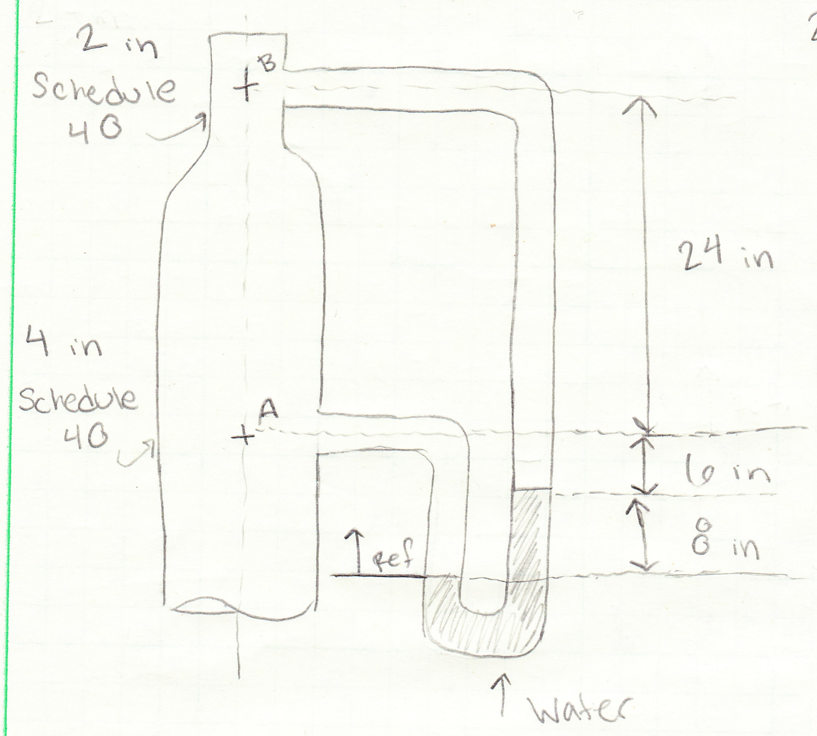
A_B: 2 in = 0.16667 ft

$\frac{\pi D^2}{4} = 0.0218 \text{ ft}^2$

$$Q = \sqrt{\frac{2 \times 32.2 \frac{ft}{s^2}\left(1 - \frac{13.54}{0.9}\right) \times \frac{28 \text{ in}}{12 \text{ in}}}{\frac{1}{(0.087 \text{ ft}^2)^2} - \frac{1}{(0.0218 \text{ ft}^2)^2}}}$$

$$Q = 1.07 \frac{ft^3}{s}$$

6.82 Oil with specific weight of 55.0 lb/ft3 flows from A to B through the system shown (fig 6.35). Calc the volume flow rate of the oil.



2 in: flow area = $0.02333\ ft^2$

4 in: flow area = $0.08840\ ft^2$

$\gamma_{oil} = 55\ lb/ft^3$

$\gamma_{H_2O} = 62.4\ lb/ft^3$

From Appendix

$Q = vA$

$\Delta p = \gamma h$

$$\frac{p_B}{\gamma_{oil}} + \frac{v_B}{2g} + z_B = \frac{p_A}{\gamma_{oil}} + \frac{v_A}{2g} + z_A$$

Plug $Q = vA$ into Bernoulli's eqn.

$$\frac{Q^2}{2g}\left(\frac{1}{A_A^2} - \frac{1}{A_B^2}\right) = \frac{p_A - p_B}{\gamma_{oil}} + \underbrace{(z_B - z_A)}_{2\ ft}$$

$$Q = \sqrt{\frac{2g\left[\frac{p_B - p_A}{\gamma} + (z_A - z_B)\right]}{\frac{1}{A_B^2} - \frac{1}{A_A^2}}}$$

$$Q = \sqrt{\frac{2g\left[\frac{P_A - P_B}{\gamma} + (z_B - z_A)\right]}{\frac{1}{A_B^2} - \frac{1}{A_A^2}}}$$

$$P_B + \gamma_{oil}(\underset{30\text{ in}}{2.5\text{ ft}}) + \gamma_{H_2O}(\underset{8\text{ in}}{0.667\text{ ft}}) - \gamma_{oil}(\underset{14\text{ in}}{1.1667\text{ ft}}) = P_A$$

$$\to P_B + 55\frac{\text{lb}}{\text{ft}^3}(2.5\text{ ft}) + 62.4\frac{\text{lb}}{\text{ft}^3}(0.6667\text{ ft}) - 55\frac{\text{lb}}{\text{ft}^3}(1.1667\text{ ft}) = P_A$$

$$\to P_B + 114.935 = P_A$$

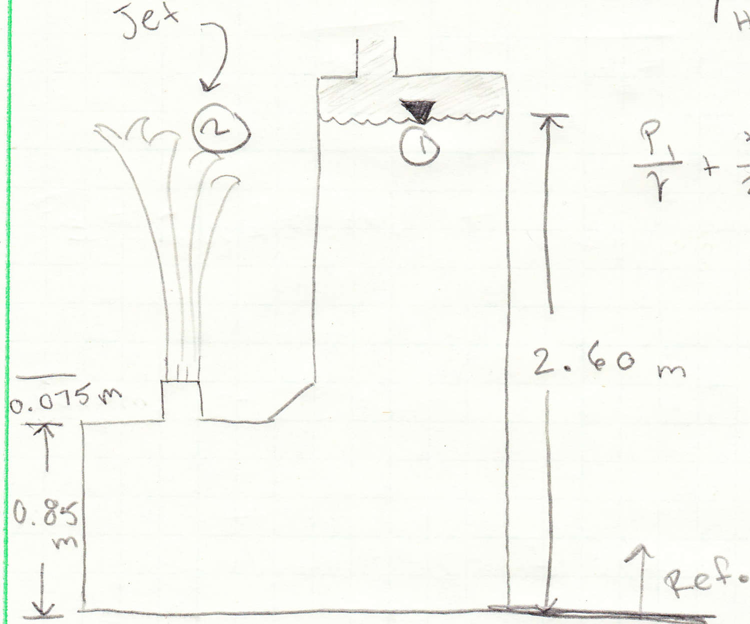
$$P_A - P_B = 114.935\ \frac{\text{lb}}{\text{ft}^2} \quad \text{or} \quad 0.798\text{ psi}$$

$$Q = \sqrt{\frac{2 \times 32.2\frac{\text{ft}}{\text{s}^2}\left[\frac{114.935\frac{\text{lb}}{\text{ft}^2}}{55\frac{\text{lb}}{\text{ft}^3}} + 2\text{ ft}\right]}{\frac{1}{(0.02333\text{ ft}^2)^2} - \frac{1}{(0.08840\text{ ft}^2)^2}}}$$

$$Q = \sqrt{0.15409}$$

$$= \boxed{0.393\ \frac{\text{ft}^3}{\text{s}}}$$

6.91 To what height will the jet of fluid rise for the conditions shown?



$\gamma_{H_2O} = 9.81 \ kN/m^3$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2$$

solve for height of pt 2

$$z_2 = \frac{P_1 - P_2}{\gamma} + \frac{\cancel{V_1^2} - \cancel{V_2^2}}{2g} + z_1$$

large tank = velocity is zero

velocity @ 2 is zero

$$z_2 = \frac{\cancel{P_1} - \cancel{P_2}}{\gamma} + z_1$$

Both pressures are atmospheric

$$z_2 = z_1$$

$$\boxed{z_2 = 2.6 \ m}$$

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