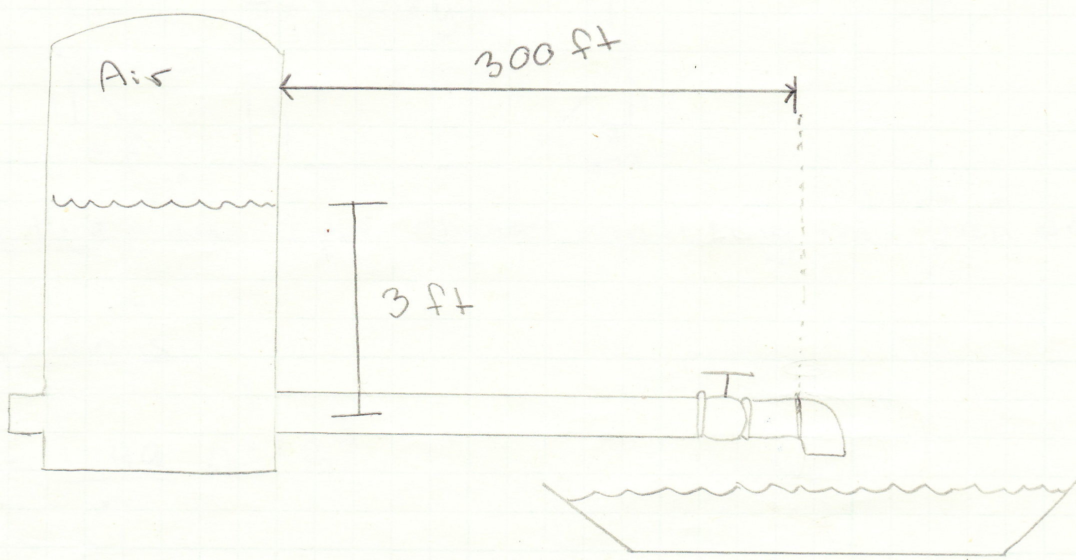
MET 330 | Test 2 | Nina Craig

Purpose

Calculate system parameters in 7 parts:

a. Open channel depth
b. Horizontal + Vertical forces in pipe + elbow
c. Largest log the channel can carry
d. Pressure drop flow nozzle
e. Water hammer, cavitation
f. Drag force on largest log from c.
g. Force on blind flange + force location

Drawing



Sources: Applied Fluid Mechanics, 7th ed. Mott & Untener

Design Considerations

- Incompressible fluid
- Isothermal
- steady state

Data / Variables

- Water @ 60°F
- Q = 75 gpm
- Trapezoidal open channel
- Tank is pressurized
- 1 1/2 schedule 40 steel pipe (300 ft)
- ε (steel) = 1.5×10^{-4} ft
- dia. = 0.1342 ft
- Flow area = 0.01414 ft^2
- E_{steel} = 200 GPa
- $\rho_{Hickory} = 830 \frac{kg}{m^3}$

a) Given data for part a.

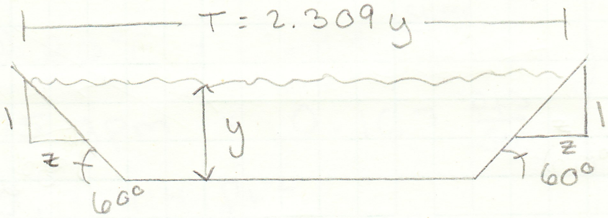
unfinished concrete: $n = 0.017$ (Table 14.1)

Channel slope $= 0.1\%$, $S = 0.001$

lateral walls $= 60°$

$$75 \text{ gpm} \times \frac{0.13368 \text{ ft}^3}{1 \text{ gal}} \times \frac{1 \text{ min}}{60 \text{ sec}} \rightarrow 0.167 \frac{\text{ft}^3}{\text{sec}}$$

Drawing pt a. Calc. depth "y"



Procedure – Use open channel eqn using the given data

Calculations

$$Q = \frac{1.49}{n} A S^{\frac{1}{2}} R^{\frac{2}{3}}$$

move all known values to one side

$$AR^{\frac{2}{3}} = \frac{nQ}{1.49 S^{\frac{1}{2}}} \rightarrow AR^{\frac{2}{3}} = \frac{(0.017)(0.167 \frac{ft^3}{s})}{1.49(0.001)^{\frac{1}{2}}}$$

Given values

$$AR^{\frac{2}{3}} = 0.0603$$

$A = 1.73y^2$; $R = y/2$ (Table 14.3)

$$(1.73y^2)\left(\frac{y}{2}\right)^{\frac{2}{3}} = 0.0603$$

Simplify left hand side (done with online calculator Symbolab.com)

$$1.0898 y^{\frac{8}{3}} = 0.0603$$

Solve for y

$$\frac{1.0898\, y^{8/3}}{1.0898} = \frac{0.0603}{1.0898}$$

$\hookrightarrow y^{8/3} = 0.0553$

$\hookrightarrow \boxed{y = 0.34 \text{ ft}}$

b) Given data for part b

ρ_{H_2O} @ 60°F = 1.94 $\frac{\text{slugs}}{\text{ft}^3}$

$Q = 75$ gpm $= 0.167 \frac{\text{ft}^3}{\text{sec}}$

$A = 0.01414 \text{ ft}^2$

K for globe valve = 340f

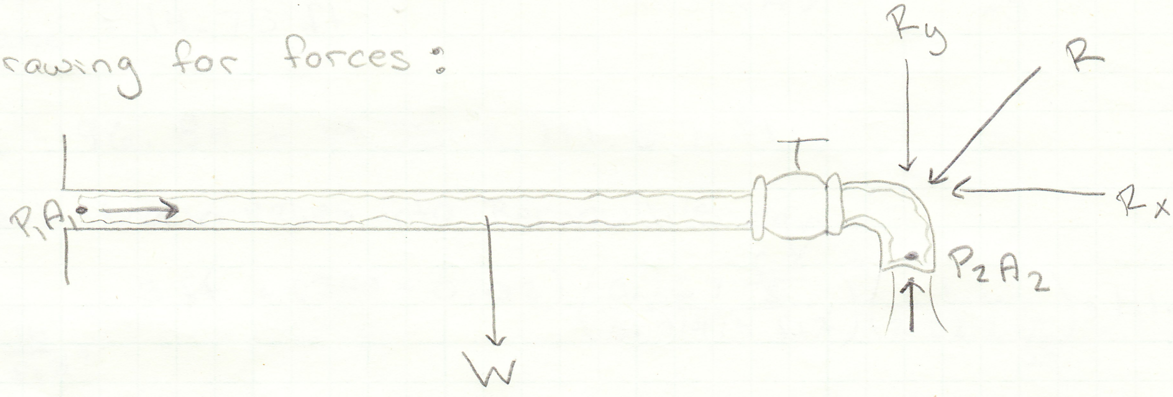
$\gamma_{H_2O} = 62.4 \frac{\text{lb}}{\text{ft}^3}$

dia. of pipe = 0.1342 ft

Friction factor "f" for 1½" pipe = 0.020
(from Table 10.5)

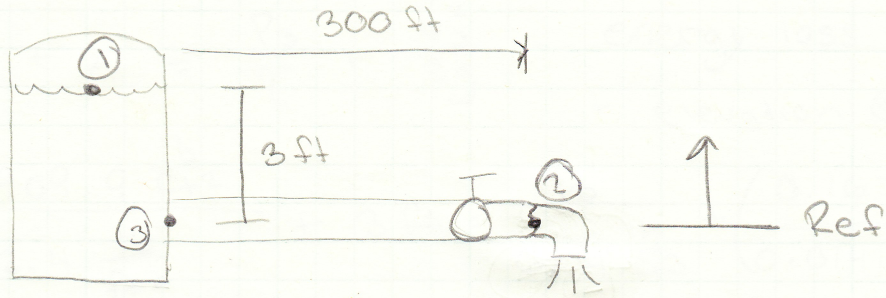
Drawing pt b.

- drawing for forces:



Procedure – Apply Bernoulli's equation to find pressure at water in the tank's surface, then to find pressure at pipe's entrace. Last, determine forces using force equations.

Calculations



Begin with Bernoulli's eqn:

$$\cancel{h_A} + \frac{P_1}{\gamma} + \cancel{\frac{V_1^2}{2g}} + z_1 = \cancel{\frac{P_2}{\gamma}} + \frac{V_2^2}{2g} + \cancel{z_2} + h_L \qquad z_2 = \text{ref} \therefore 0$$

tank $\therefore V_1 \cong \emptyset$ $\qquad$ (2) $P = \emptyset$

Calculate energy losses from pipe + globe valve

$$h_{L\,pipe} = f\frac{L}{D}\frac{V^2}{2g} \longrightarrow \underbrace{(0.02)}_{f}\underbrace{\left(\frac{300}{0.1342}\right)}_{L/D}\underbrace{\left(\frac{0.167}{0.01414}\right)^2}_{(Q/A)^2}\underbrace{\frac{1}{2\times 32.2}}_{1/2g}$$

$h_{L\,pipe} = 96.84$ ft

$$h_{L\,valve} = K\frac{V^2}{2g} \longrightarrow \underbrace{(340\cdot 0.02)}_{K}\underbrace{\left(\frac{0.167}{0.01414}\right)^2}_{(Q/A)^2}\underbrace{\frac{1}{2\times 32.2}}_{1/2g}$$

$h_{L\,valve} = 14.73$ ft

$$h_L = 96.84 + 14.73 = 111.57 \text{ ft}$$

Insert known values into Bernoulli's eqn

$$\frac{P_1}{62.4\ \frac{lb}{ft^{\cancel{3}\,2}}} + 3\ \cancel{ft} = (340\times 0.02)\left(\frac{0.167\ \frac{ft^{\cancel{3}}}{s}}{0.01414\ \cancel{ft^2}}\right)^2\left(\frac{1}{2\cdot 32.2\ \frac{ft}{\cancel{s^2}}}\right) + 111.57\ \cancel{ft}$$

$$\frac{P_1}{62.4} + 3 = 113.736$$

$$\hookrightarrow \frac{6909.9\ lb/ft^2}{144} = 47.99\ \text{psi} = P_1$$

Calculate pressure at 3 with Bernoulli's

$$\frac{P_1}{\gamma} + z_1 = \frac{P_3}{\gamma} + \frac{V_3^2}{2g}$$

energy loss h_L negligible + elevation @ 3 = ref ∴ Ø

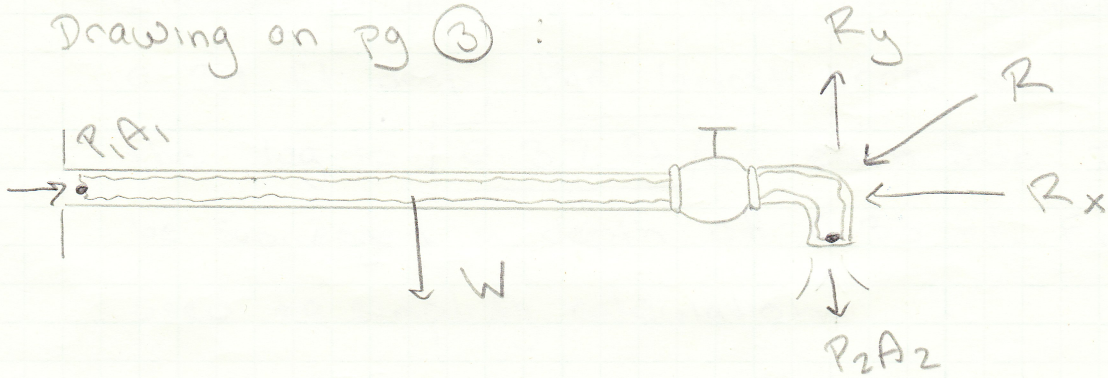
$$\frac{6909.9 \frac{lb}{ft^2}}{62.4 \frac{lb}{ft^3}} + 3\,ft = \frac{P_3}{62.4 \frac{lb}{ft^3}} + \left(\frac{0.167 \frac{ft^3}{s}}{0.01414\,ft^2}\right)^2 \times \frac{1}{2 \cdot 32.2 \frac{ft}{s^2}}$$

$$113.736\,ft = \frac{P_3}{62.4} + 2.166$$

$$P_3 = 6961.94 \frac{lb}{ft^2} \longrightarrow 48.35\,psi$$

Use force eqns to find vertical + horizontal forces.

Drawing on pg 3 :



$\underline{x}$ $P_1A_1 - R_x = \rho Q V$

$$R_x = P_1A_1 + \rho \frac{Q^2}{A}$$

$$R_x = 6909.9\,lb/\cancel{ft^2} \times 0.01414\,\cancel{ft^2} + \frac{\left(0.167 \frac{ft^{\cancel{3}}}{s}\right)^2}{0.01414\,\cancel{ft^2}} \times 1.94 \frac{slugs}{\cancel{ft^3}}$$

$$R_x = 97.71\,lb + 3.83\,lb$$

$\uparrow$ $\frac{slug \times ft}{s^2}$

$$\boxed{R_x = 101.54\,lb}$$

$\underline{y}$ $R_y - P_2\cancel{A_2} - W = \rho \frac{Q^2}{A}$

$P_2 = \emptyset$

$$R_y = \left[1.94 \times \frac{0.167^2}{0.01414}\right] lb + W$$

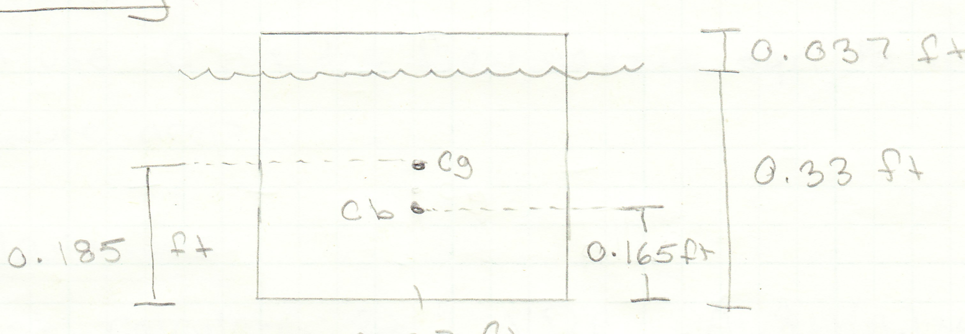
$$\boxed{R_y = 3.83 \text{ lb} + \text{Weight}}$$

c) Largest hickory log

According to problem statement, log must "barely float." Assumed 90% of the cross section is submerged and 10% is above water.

Knowing from part a) that the channel is 0.34 ft deep, the largest cross section for the log is $\boxed{0.37 \text{ ft}}$ on each side. It will be submerged a depth of 0.33 ft. Excel was used to simplify calculation.

Drawing



$$V_d = \underset{h_{below}}{(0.333} - \underset{h_{above}}{0.037)}\underset{Area}{(0.1369)} = 0.0405$$

$MB = I / V_d$

$$I = \frac{bh^3}{12} = \frac{0.37(0.37^3)}{12} = 0.00156 \text{ ft}^4$$

$$MB = \frac{I}{V_d} = \frac{0.00156\ ft^4}{0.0405\ ft^3} = 0.0385\ ft$$

Center of gravity lies at 0.185 ft from the bottom of the x-section ∴ log is not stable

Part d) Flow nozzle

Known Data

pipe dia. = 0.1342 ft

$\beta = 0.5$ ∴ nozzle dia = 0.0671 ft

$\gamma_{H2O} = 62.4$, $\rho_{H2O} = 1.94$, $\eta_{H2O} = 2.35 \times 10^{-5}$

$A_{pipe} = 0.01414\ ft^2$

$A_{nozzle} = 0.0035\ ft^2$

$Q = 0.167\ \frac{ft^3}{s}$

$v = \frac{Q}{A} = 11.81\ \frac{ft}{s}$

Procedure – we will use the eqn for flow nozzles to solve for pressure drop. First, we will need to find Re, and then use it to find C. Finally, we will use C in the flow nozzle eqn to find pressure drop.

Calculations

First, calculate Re: $\frac{VD\rho}{\eta} \rightarrow \frac{(11.81)(0.1342)(1.94)}{2.35 \times 10^{-5}}$

Re = 130839

Now use Re to find C

$$C = 0.9975 - 6.53\sqrt{\beta/Re}$$

$$= 0.9975 - 6.53\sqrt{0.5/130839}$$

$$C = 0.985$$

Now rearrange flow nozzle eqn to solve for pressure drop $(P_1 - P_2)$

$$V = C\sqrt{\frac{2g(P_1-P_2)/\gamma}{(A_1/A_2)^2-1}}$$

$$\left(\frac{V}{C}\right)^2 = \frac{2g(P_1-P_2)/\gamma}{(A_1/A_2)^2-1}$$

$$\frac{\left(\frac{V}{C}\right)^2 \times \left[(A_1/A_2)^2-1\right]\gamma}{2g} = (P_1 - P_2)$$

Insert known values

$$\frac{\left(\frac{11.81}{0.985}\right)^2 \times \left[\left(\frac{0.01414}{0.0035}\right)^2 - 1\right] \times 62.4}{2 \times 32.2} = (P_1 - P_2)$$

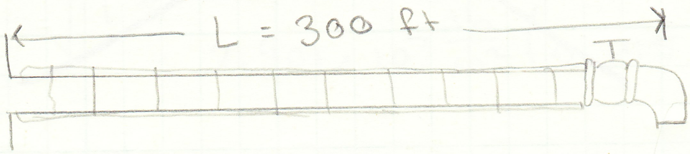
$$\frac{2134.18\ \frac{lb}{ft^2}}{144}$$

$$\boxed{14.82\text{ psi}}$$

Part e) Pressure incriment (water hammer)

<u>Purpose</u> - Determine pressure incriment after sudden valve closure

<u>Drawing</u>



<u>Known / Given Data</u>

$\rho_{H_2O} = 1.94 \frac{slugs}{ft^3} = 0.036 \frac{lb}{in^3}$

$V = 11.81 \frac{ft}{s}$ (calculated previously) $= \frac{0.167}{0.01414}$ $\frac{Q}{A}$

$L = 300$ ft

δ (Pipe thickness) $= 0.145$ in $= 0.0121$ ft (Table F.1)

$E = 200$ GPa $= 2.901 \times 10^7$ psi $= 4.177 \times 10^9 \frac{lb}{ft^2}$

pipe dia. $= 0.1342$ ft $= 1.610$ in

Bulk Modulus (E_o) of water $= 316000$ psi (Table 1.3)

<u>Procedure</u> We will need to first calculate C, the speed of the pressure wave. Then we will be able to calculate pressure incriment using the equation for water hammer.

<u>Calculations</u>

$$C = \frac{\sqrt{\frac{E_o}{\rho}}}{\sqrt{1 + \frac{E_o D}{E \delta}}}$$

$$\cancel{C = \frac{\sqrt{\dfrac{316000 \frac{lb}{in^2}}{0.036 \frac{lb}{in^3}}}}{\sqrt{1 + \dfrac{316000 \frac{lb}{in^2}(1.61\ in)}{2.901 \times 10^7 \frac{lb}{in^2}(0.145\ in)}}}}$$

$$* \; E_0 = 316000\ psi \times 144 = 4.5504 \times 10^7 \ \frac{lb}{ft^2}$$

$$C = \frac{\sqrt{\dfrac{4.5504 \times 10^7 \ \frac{lb}{ft^2}}{1.94 \ \frac{slugs}{ft^3}}}}{\sqrt{1 + \dfrac{(4.5504 \times 10^7 \ \frac{lb}{ft^2})(0.1342\ ft)}{(4.177 \times 10^9 \ \frac{lb}{ft^2})(0.0121\ ft)}}}$$

Previous eqn format discarded & redone to keep all length units in ft.

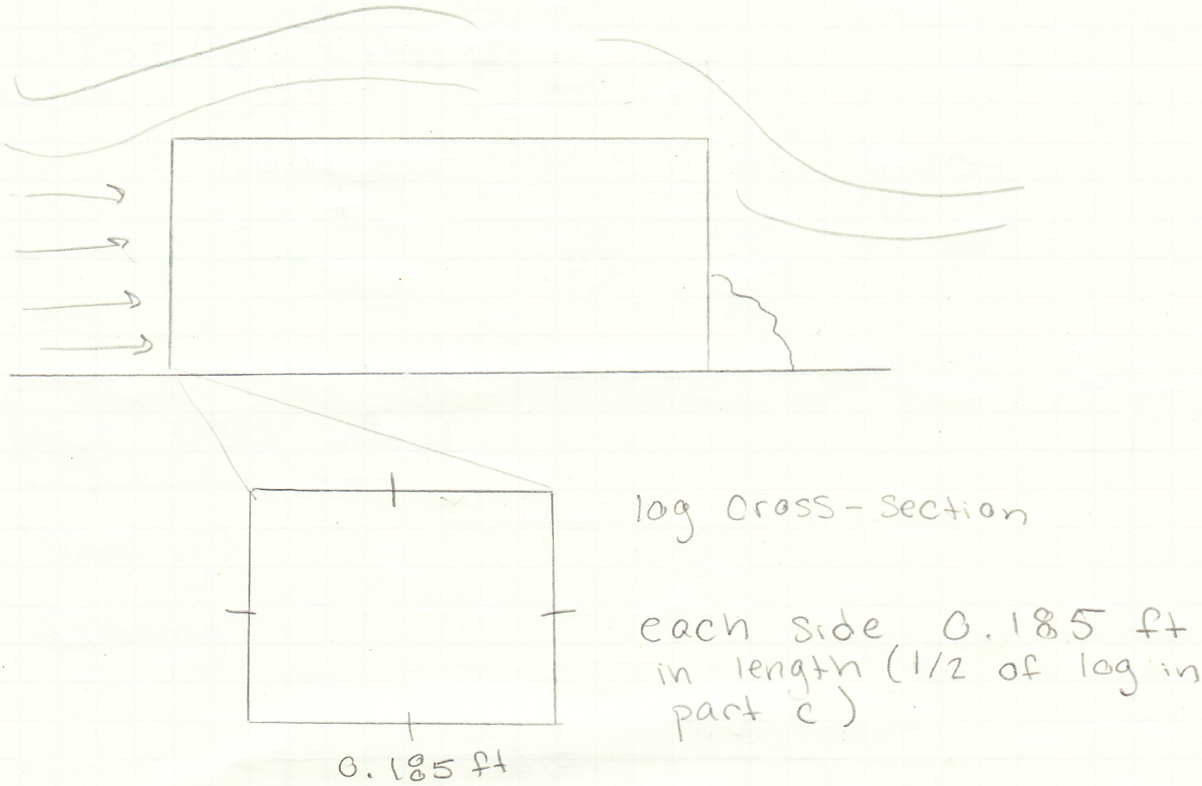
$$C = \frac{4843.11}{1.059} = 4574.62$$

Calculate pressure increment

$$\Delta P = \rho C V \rightarrow (1.94)(4574.62)(11.81)$$

$$= \frac{104811}{144} \longrightarrow \boxed{\Delta P = 727.85\ psi}$$

Part f) drawing



Purpose Determine drag force

Assumptions

Log is stuck with the cross section facing the flow from the open channel. It is also fully submerged. Flow is perpendicular to log face. Channel is not inclined at the bottom.

Data

$C_D = 1.60$ (Table 17.1)

$\rho_{H_2O} = 1.94$ slugs/ft3

$V_{H_2O} = 11.81$ ft/s (calculated previously)

$A = 0.034 \text{ ft}^2$

Calculations

$$F_D = C_D\left(\tfrac{1}{2}\rho v^2\right)A$$

$$= 1.60\left(\tfrac{1}{2} \times 1.94 \times 11.81^2\right) 0.034$$

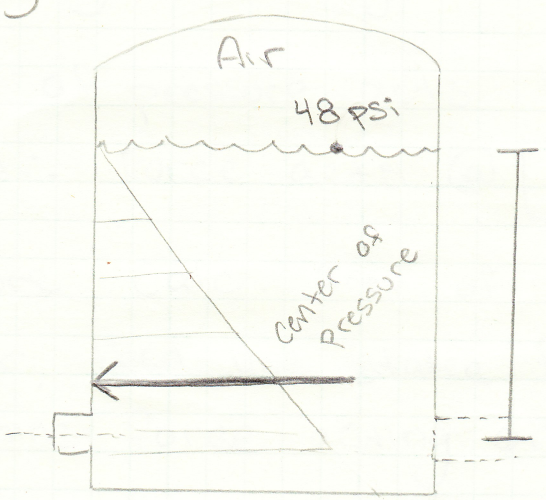
$\frac{slug}{\cancel{ft^3}} \quad \frac{\cancel{ft^2}}{s^2} \quad \cancel{ft^2}$

$\frac{slug \cdot ft}{s^2} = lb$

$$\boxed{F_D = 7.36 \text{ lbf}}$$

Part g)

Drawing



Purpose: Determine force on blind flange

Data

~~Air pressure in tank, calculated in pt b~~

~~→ 48 psi = 6912 lb/ft²~~

* Air pressure not necessary

$\gamma_{H_2O} = 62.4 \frac{lb}{ft^3}$

Flange dia. = 0.1342 ft

Flange Area = 0.01414 ft²

Calculations

Force on flange — static fluid

$F = [\gamma \times h_c \times A]$

$= (62.4 \tfrac{lb}{\cancel{ft^3}})(3 \cancel{ft})(0.01414 \cancel{ft^2})$

$\boxed{F = 2.63\ lb}$

Distance from flange to bottom of tank is unknown. For the sake of this problem, I am assuming the distance is very small. Thus I am estimating total height of fluid to be 3 ft.

Center of pressure acts 1/3 height of fluid from bottom ∴ Force acts @ $\boxed{1\ ft}$ from bottom.

Procedure Calc. Area of flange from its diameter, then use static fluid equation to determine force acting on it.

Materials for all parts

water

schedule 40 steel pipe

unfinish concrete

Hickory wood

Summary

Depth of channel is 0.34 ft. Total forces in x & y direction are 101.54 lb & 3.83 lb (plus weight) respectively. Largest hickory log supported by the channel is 0.37 ft in length on each side of its cross-section; it is not stable. Pressure drop measured with the flow nozzle is 14.82 psi. Pressure incriment due to water hammer is 727.85 psi; Cavitation should not be a major concern because it would require the fluid to heat past its boiling point, as well as low pressure conditions, and this should not occur in the system regularly. Drag force experienced by the smaller log would be 7.36 lbf. Force experienced by the blind flange is 2.63 lb, force acts 1 ft from the bottom.

Analysis

Designing an open channel system requires only a few specific variables for the channel itself, though for to serve the desired purpose, it requires a number of different components. Each of these in turn require analysis of the internal forces caused by moving and static fluid to function as best as possible.