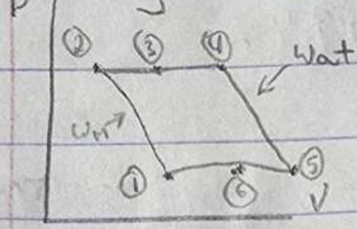
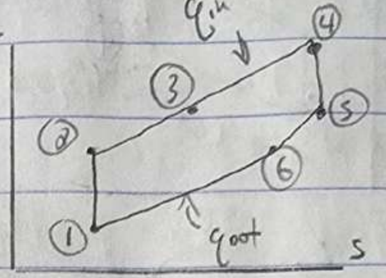
Test 1

Jeffrey Relmud

1a





$c_p = 1.005$ kJ/kg

(1) Isen comp → (2) P const → (3) P const → (4) Isen Turbine → (5) P const → (6) P const

$P_1 = 100$ kPa $P_2 = 1000$ kPa $P_3 = 1000$ kPa $P_4 = 1000$ kPa $P_5 = 100$ kPa $P_6 = 100$ kPa

$T_1 = 303$ K $T_2 = 585$ K $T_3 = 545.8$ K $T_4 = 1073$ K $T_5 = 555.8$ K $T_6 = 595$ K

$$T_2 = T_1 (r_p)^{\frac{(k-1)}{k}} = 303 (10)^{\frac{1.4-1}{1.4}} = 585\text{K}$$

$$T_5 = T_4 \left(\frac{1}{r_p}\right)^{\frac{(k-1)}{k}} = 1073 \left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 555.8\text{K}$$

$$T_3 = T_5 - 10 = 555.8 - 10 = 545.8\text{K} \qquad T_3 - T_2 = T_5 - T_6$$

$$T_6 = T_5 - (T_3 - T_2) = 555.8 - (545.8 - 585.0) = 595\text{K}$$

$$W_{in} = c_p (T_2 - T_1) = 1.005 (585 - 303) = 283.41 \text{ kJ/kg}$$

$$W_{out} = c_p (T_4 - T_5) = 1.005 (1073 - 555.8) = 519.8 \text{ kJ/kg}$$

$$W_{net} = W_{out} - W_{in} = 519.8 - 283.41 = 236.4 \text{ kJ/kg}$$

$$q_{out} = c_p (T_6 - T_1) = 1.005 (595 - 303) = 293.5 \text{ kJ/kg}$$

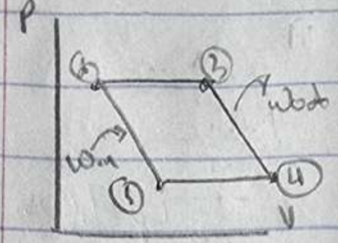
$$q_{in} = c_p (T_4 - T_3) = 1.005 (1073 - 545.8) = 529.8 \text{ kJ/kg}$$

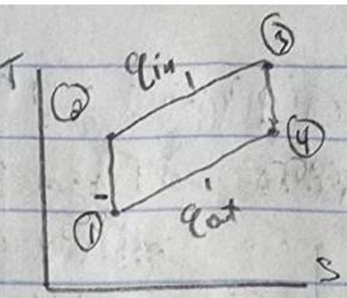
$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.4}{529.8} = 0.446 = 44.6\%$$

$$\varepsilon = \frac{T_3 - T_2}{T_5 - T_2} = \frac{545.8 - 585}{555.8 - 585} = \frac{-39.2}{-29.2} = 1.34$$

The regenerator is cooling the compressed air instead of heating it.

1b





① → Isen comp → ② → Pconst → ③ → Isen Turb → ④ → Pconst

$P_1 = 100kPa$ $P_2 = 1000kPa$ $P_3 = 1000kPa$ $P_4 = 100kPa$

$T_1 = 303K$ $T_2 = 585K$ $T_3 = 1073K$ $T_4 = 555.8K$

$W_{in} = c_p(T_2 - T_1) = 1.005(585 - 303) = 283.4\ kJ/kg$

$W_{out} = c_p(T_3 - T_4) = 1.005(1073 - 555.8) = 519.8\ kJ/kg$

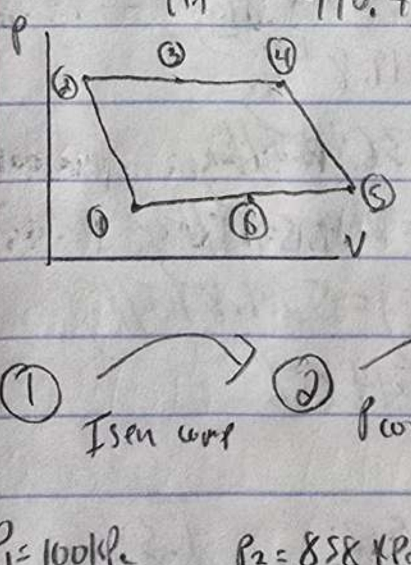
$q_{in} = c_p(T_3 - T_2) = 1.005(1073 - 585) = 490.4\ kJ/kg$

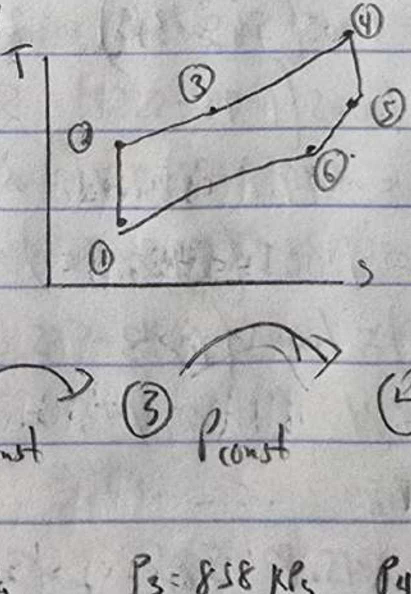
$q_{out} = c_p(T_4 - T_1) = 1.005(555.8 - 303) = 254.1\ kJ/kg$

$W_{net} = W_{out} - W_{in} = 519.8 - 283.4 = 236.4\ kJ/kg$

$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.4}{490.4} = 0.482 = 48.2\%$

1c





① → Isen comp → ② → Pconst → ③ → Pconst → ④ → Isen Turbine → ⑤ → Pconst → ⑥ → Pconst

$P_1 = 100kPa$ $P_2 = 858kPa$ $P_3 = 858kPa$ $P_4 = 858kPa$ $P_5 = 100kPa$ $P_6 = 100kPa$

$T_1 = 303K$ $T_2 = 560K$ $T_3 = 570.6$ $T_4 = 1073K$ $T_5 = 580.6K$ $T_6 = 570$

$T_2 = T_1(r_p)^{\frac{k-1}{k}} = 303(8.58)^{\frac{1.4-1}{1.4}} = 560K$

$T_5 = T_4\left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = 1073\left(\frac{1}{8.58}\right)^{\frac{1.4-1}{1.4}} = 580.6K$

$T_3 = T_5 - 10 = 580.6 - 10 = 570.6$

$T_6 = T_5 - (T_3 - T_2) = 580.6 - (570.6 - 560) = 570$ k

$W_{in} = c_p(T_2 - T_1) = 1.005(560 - 303) = 258.3$ kJ/kg

$W_{out} = c_p(T_4 - T_5) = 1.005(1073 - 580.6) = 494.9$ kJ/kg

$W_{net} = W_{out} - W_{in} = 494.9 - 258.3 = 236.6$ kJ/kg

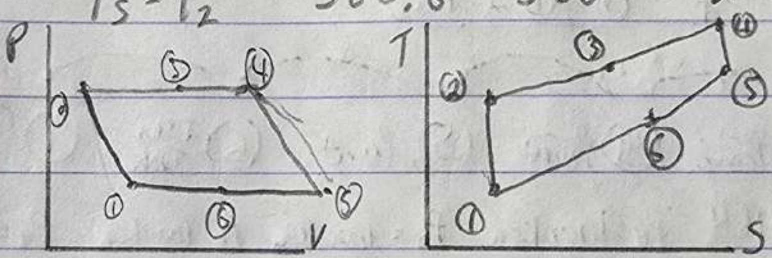
$q_{out} = c_p(T_6 - T_1) = 1.005(570 - 303) = 268.3$ kJ/kg

$q_{in} = c_p(T_4 - T_3) = 1.005(1073 - 570.6) = 504.9$ kJ/kg

$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{236.6}{508.9} = 0.469 = 46.9\%$

$\varepsilon = \frac{T_3 - T_2}{T_5 - T_2} = \frac{570.6 - 560}{580.6 - 560} = \frac{10.6}{20.6} = 0.515 = 51.5\%$

1d



① Isen comp → ② P-const → ③ P const → ④ Isen Turb → ⑤ P const → ⑥ P const

$P_1 = 100$ kPa $P_2 = 1000$ kPa $P_3 = 1000$ kPa $P_4 = 1000$ kPa $P_5 = 100$ kPa $P_6 = 100$ kPa

$T_1 = 303$ k $T_2 = 585$ k $T_3 = 595.5$ k $T_4 = 1169$ $T_5 = 605.5$ $T_6 = 595$ K

$T_2 = T_1(r_p)^{\frac{k-1}{k}} = 303(10)^{\frac{1.4-1}{1.4}} = 585$ k

$T_5 = T_4\left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = 1169\left(\frac{1}{10}\right)^{\frac{1.4-1}{1.4}} = 605.5$ k

$T_3 = T_5 - 10 = 605.5 - 10 = 595.5$ K

$T_6 = T_5 - (T_3 - T_2) = 605.5 - (595.5 - 585) = 595$

$W_{in} = c_p(T_2 - T_1) = 1.005(585 - 303) = 283.4$ kJ/kg

$W_{out} = c_p(T_4 - T_5) = 1.005(1169 - 605.5) = 566.3$ kJ/kg

$W_{net} = W_{out} - W_{in} = 566.3 - 283.4 = 282.9$ kJ/kg

$q_{out} = c_p(T_6 - T_1) = 1.005(595 - 303) = 293.5$ kJ/kg

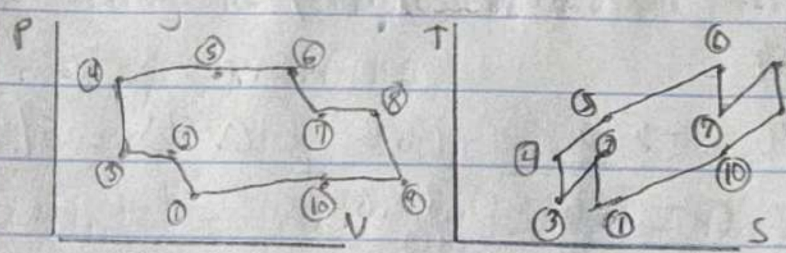
1-d cont

$q_{in} = c_p(T_9 - T_3) = 1.005(1169 - 595.3) = 576.6 \text{ kJ/kg}$

$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{282.9}{576.6} = 0.490 = 49\%$

$\varepsilon = \frac{T_3 - T_2}{T_5 - T_2} = \frac{595.5 - 585}{605.5 - 585} = \frac{10.5}{20.5} = 0.512 = 51.2\%$

1-e



$r_p = \sqrt{10} = 3.162$

① Isen comp → ② P const → ③ Isen comp → ④ P const → ⑤ P const → ⑥ Isen Turb → ⑦ P const

$P_1 = 100\text{kPa}$ $P_2 = 316\text{kPa}$ $P_3 = 316\text{k}$ $P_4 = 1000\text{kPa}$ $P_5 = 1000\text{kPa}$ $P_6 = 1000\text{kPa}$ $P_7 = 316\text{kPa}$

$T_1 = 303\text{k}$ $T_2 = 421\text{k}$ $T_3 = 303\text{k}$ $T_4 = 421\text{k}$ $T_5 = 431\text{k}$ $T_6 = 1073\text{k}$ $T_7 = 772\text{k}$

→ ⑧ Isen Turb → ⑨ P const → ⑩ P const

$P_8 = 316\text{kPa}$ $P_9 = 100\text{kPa}$ $P_{10} = 100\text{kPa}$

$T_8 = 1073\text{k}$ $T_9 = 772\text{k}$ $T_{10} = 762$

$T_2 = T_1 (r_p)^{\frac{k-1}{k}} = 303(3.162)^{\frac{1.4-1}{1.4}} = 421\text{k}$

$P_2 = P_1 \cdot r_p = 100(3.162) = 316\text{kPa}$ $T_5 = T_4 + 10 = 421 + 10 = 431$

$P_4 = P_3 \cdot r_p = 316(3.162) = 1000\text{kPa}$

$T_7 = T_6 \cdot \left(\frac{P_7}{P_6}\right)^{\frac{k-1}{k}} = 1073\left(\frac{316}{1000}\right)^{\frac{1.4-1}{1.4}} = 772\text{k}$ $T_{10} = T_9 - 10 = 772 - 10 = 762$

$T_9 = T_8 \left(\frac{P_9}{P_8}\right)^{\frac{k-1}{k}} = 1073\left(\frac{100}{316}\right)^{\frac{1.4-1}{1.4}} = 772\text{k}$

$W_{in} = c_p(T_2 - T_1) + (T_4 - T_3) = 1.005(421-303) + (421-303) = 237.2 \text{ kJ/kg}$

$W_{out} = c_p(T_6 - T_7) + (T_8 - T_9) = 1.005(1073-772) + (1073-772) = 605 \text{ kJ/kg}$

$W_{net} = W_{out} - W_{in} = 605 - 237.2 = 367.8 \text{ kJ/kg}$

$q_{in} = c_p(T_6 - T_5) + (T_8 - T_7) = 1.005(1073-431) + (1073-772) = 946.2 \text{ kJ/kg}$

$q_{out} = c_p(T_{10} - T_1) + (T_2 - T_3) = 1.005(762-303) + (421-303) = 579.2 \text{ kJ/kg}$

$$\eta_{th} = \frac{W_{net}}{q_{in}} = \frac{367.8}{946.2} = 0.388 = 38.8\%$$

$$\varepsilon = \frac{T_5 - T_4}{T_9 - T_4} = \frac{431-421}{772-421} = \frac{10}{351} = 0.0285 = 2.85\%$$

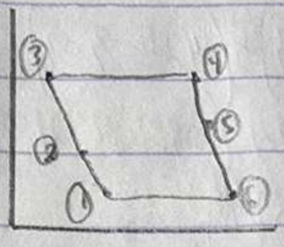
The highest thermal efficiency is option d at 49%. Its complexity to design would not be that bad because it is still a single stage compressor and turbine, but due to its high temperature the material cost may be higher, requiring higher grade material. Additionally, this set up has the highest effectiveness of 51.2% ε

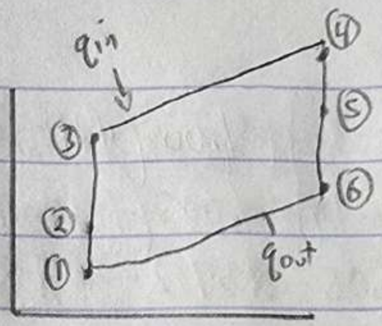
$\varepsilon = 0.512$

$$NTU = \frac{\varepsilon}{1-\varepsilon} = \frac{0.512}{1-0.512} = 1.05$$

$c_{min}/c_{max} \approx 0.85$

2





(1) Isen comp (2) Isen comp (3) P_{const} (4) Isen Turb (5) Isen Turb (6) P_{const}

$P_1 = 40\,kPa$ $P_2 = 61.4\,kPa$ $P_3 = 4060.8\,kPa$ $P_4 = 4060.8\,kPa$ $P_5 = 822.6\,kPa$ $P_6 = 40\,kPa$

$T_1 = 238\,K$ $T_2 = 269\,K$ $T_3 = 766.5\,K$ $T_4 = 1223\,K$ $T_5 = 725.5\,K$ $T_6 = 326.7\,K$

$V_1 = 250\,m/s$

$$T_2 = T_1 + \frac{V_1^2}{2c_p} = 238 + \frac{250^2}{2(1005\,J)} = 269\,K$$

$$P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = 40\left(\frac{269}{238}\right)^{\frac{1.4}{1.4-1}} = 61.4\,kPa$$

$$T_3 = T_2 + \frac{W_{in}}{c_p} = 269 + \frac{500}{1.005} = 766.5\,K$$

$$\eta_c = \frac{T_3 - T_2}{T_3' - T_2} = 0.8 = \frac{766.5 - 269}{T_3' - 269} = T_3' = 891\,K$$

$$P_3 = P_2\left(\frac{T_3'}{T_2}\right)^{\frac{k}{k-1}} = 61.4\left(\frac{891}{269}\right)^{\frac{1.4}{1.4-1}} = 4060.8\,kPa$$

$$T_5 = T_4 - \frac{W_{out}}{c_p} = 1223 - \left(\frac{500}{1.005}\right) = 725.5\,K$$

$$\eta_T = \frac{T_4 - T_5'}{T_4 - T_5} = 0.9 = \frac{1223 - T_5'}{1223 - 725.5} = T_5' = 775\,K$$

a) $$P_5 = P_4\left(\frac{T_5'}{T_4}\right)^{\frac{k}{k-1}} = 4060.8\left(\frac{775}{1223}\right)^{\frac{1.4}{1.4-1}} = 822.6\,kPa$$

$$T_6 = T_5'\left(\frac{P_6}{P_5}\right)^{\frac{k}{k-1}} \; 775\left(\frac{40}{822.6}\right)^{\frac{1.4-1}{1.4}} = 326.7\,K$$

b) $V_6 = \sqrt{2 c_p (T_5 - T_6)} = \sqrt{2(1005)(725.5 - 326.7)} = 895.3 \text{ m/s}$

c) $A = \pi \frac{D^2}{4} = \pi \frac{(1.6)^2}{4} = 2.01 \text{ m}^2$ $\quad \dot{m} = \rho_1 V_1 A = 0.585 \cdot 250 \cdot 2.01 =$

$\dot{m} = 293.5 \text{ Kg/s}$

$F = \dot{m}(V_{exit} - V_{in}) = 293.5(895.3 - 250) = 189396 \text{ N}$ or 189.4 KN