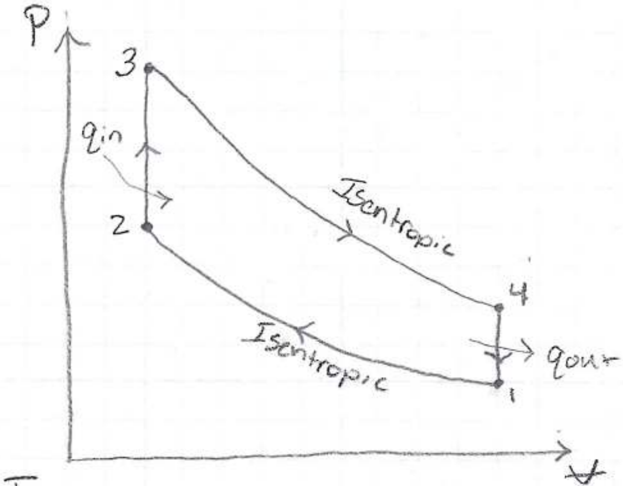
Rhiley Silvis | HW 1.3 | MET 350

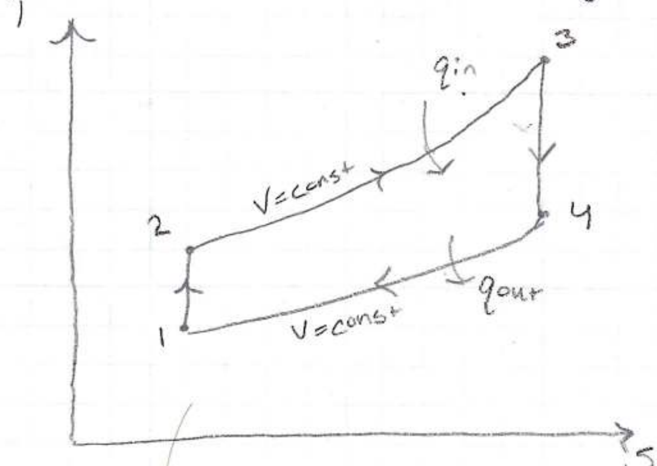
Problem 9-33

An Ideal Otto cycle has a compression ratio of 8. At the begining of the compression process, air is at 95 kPa and 27°c, and 750 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Taking into account the variation of specific heats with temperature.



given:

$r = 8$
$P_1 = 95$ kPa
$T_1 = 300$ k
$q_{in} = 750$ kJ/kg
(Variable Specific Heats)



a) the pressure and temperature at the end of the heat addition process.

1→2) Isentropic, Ideal gas, Var cp, cv

$$\frac{V_2}{V_1} = \frac{V_{r2}}{V_{r1}}$$

From A-17 @ 300k : $u_1 = 214.07 \frac{kJ}{kg}$

$V_{r1} = 621.2$

$$V_{r2} = \left(\frac{V_2}{V_1}\right) V_{r1} = \left(\frac{1}{r}\right) V_{r1}$$

$V_{r2} = \left(\frac{1}{8}\right) 621.2 = 77.65$

A-17 @ $V_{r2} = 77.65$

V_{r2}	T
78.61	670
77.65	T_2
75.50	680

$$\frac{77.65 - 78.61}{75.50 - 78.61} = \frac{T_2 - 670}{680 - 670}$$

$T_2 = 673.09\,K$

V_{r2}	u
78.61	488.81
77.65	u_2
75.50	496.62

$$\frac{77.65 - 78.61}{75.50 - 78.61} = \frac{u_2 - 488.81}{496.62 - 488.81}$$

$u_2 = 491.22\ kJ/kg$

$$\frac{P_1 V_1}{R T_1} = \frac{P_2 V_2}{R T_2} \qquad P_1\left(\frac{V_1}{T_1}\right) = P_2\left(\frac{V_2}{T_2}\right)$$

$$P_1\left(\frac{V_1}{T_1}\right)\left(\frac{T_2}{V_2}\right) = P_2 \qquad P_2 = P_1\left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)$$

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{V_{r1}}{V_{r2}}\right)$$

$$P_2 = P_1\left(\frac{V_{r1}}{V_{r2}}\right)\left(\frac{T_2}{T_1}\right) = 95\left(\frac{621.2}{77.65}\right)\left(\frac{673.09}{300}\right)$$

$P_2 = 1705.16\ kPa$

$q_{in} = (u_3 - u_2) \qquad q_{in} + u_2 = u_3$

$750\ kJ/kg + 491.22\ kJ/kg = u_3 = 1241.22\ kJ/kg$

From A-17 @ 1241.22 kJ/kg

u	T
1223.87	1520
1241.22	T_3
1242.43	1540

$$\frac{1241.22 - 1223.87}{1242.43 - 1223.87} = \frac{T_3 - 1520}{1540 - 1520}$$

$\boxed{T_3 = 1538.70\,K}$

T	Vr
1520	6.854
1538.70	Vr3
1540	6.569

$$\frac{1538.7 - 1520}{1540 - 1520} = \frac{Vr_3 - 6.854}{6.569 - 6.854}$$

$$Vr_3 = 6.588$$

$$P_3 = P_2\left(\frac{T_3}{T_2}\right) = 1705.16\left(\frac{1538.7}{673.09}\right) = \boxed{3898.04 \text{ kPa}}$$

3→4) Isentropic

$$\frac{V_4}{V_3} = \frac{Vr_4}{Vr_3} \qquad \left(\frac{V_4}{V_3}\right) = r$$

$$Vr_3(r) = Vr_4 = 6.588(8) = 52.704$$

From A-17 @ 52.704

Vr	u
55.54	560.01
52.704	u_4
51.64	576.12

$$\frac{52.704 - 55.54}{51.64 - 55.54} = \frac{u_4 - 560.01}{576.12 - 560.01}$$

$$u_4 = 571.72 \ \frac{kJ}{kg}$$

b) Net work output

$$W_{net} = q_{in} - q_{out} = (u_3 - u_2) - (u_4 - u_1)$$

$$(1241.22 - 491.22) - (571.72 - 214.07)$$

$$\boxed{W_{net} = 392.35 \ kJ/kg}$$

c) The thermal efficiency

$$\eta_{Th} = \frac{w_{net}}{q_{in}} = \frac{392.35}{750} = \boxed{52.3\ \%}$$

d) Mean effective pressure for the cycle

$$MEP = \frac{w_{net}}{v_d} = \frac{w_{net}}{(v_1 - v_2)} \qquad (v_1 - v_2) = v_1\left(1 - \frac{1}{r}\right)$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287\ \frac{kJ}{kg \cdot K} \cdot 300K}{95\ kPa} = 0.906\ \frac{m^3}{kg}$$

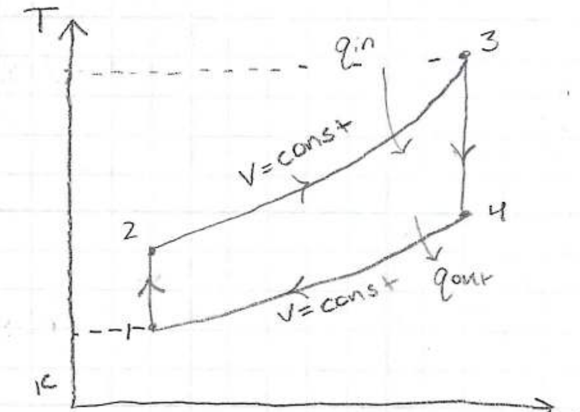
$$\frac{392.35}{0.906\left(1 - \frac{1}{8}\right)} = \boxed{494.92\ kPa}$$

Rhiley Silvis | HW 1.3 | MET 350

Problem 9-36

A six-cylinder, four stroke, spark ignition engine operating on the ideal Otto cycle takes in air at 14 psia and 105°F, and is limited to maximum cycle temperature of 2400°F. Each cylinder has a bore of 3.5 in, and each piston has a stroke of 3.9 in. The minimum enclosed volume is 9.8 percent of the maximum enclosed volume. How much will this engine produe when operated at 2500 rpm?

(constant specific heats @ 300 K)



given:

$P_1 = 14$ psia

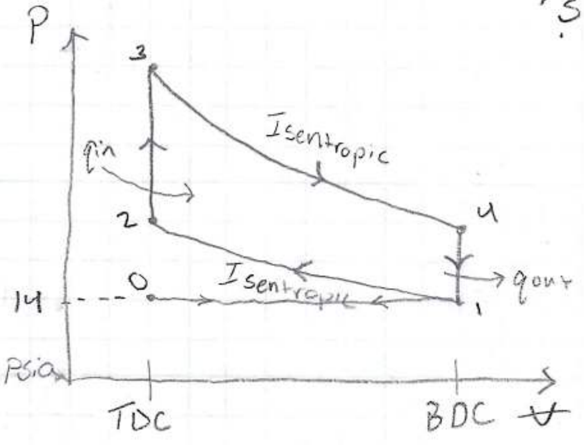
$T_1 = 105\,°F = 564.67\ R$

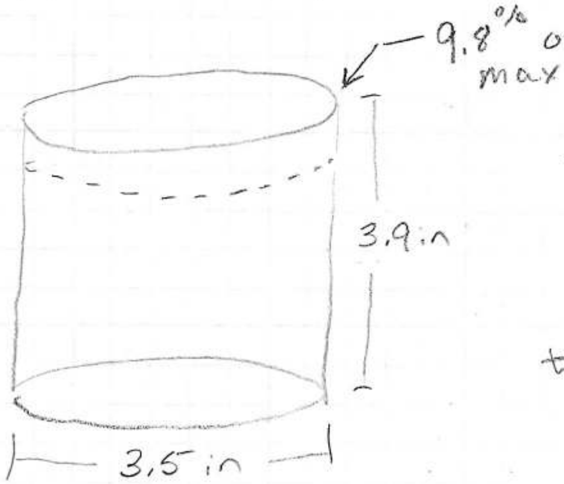
$T_3 = 2400\,°F = 2859.67\ R$

Bore = 3.5 in

Stroke = 3.9 in

rpm = 2500





$V_{max} = \pi r^2 h$

$= \pi (1.95)^2\, 3.9$

$= 46.59\ in^3$

$V_{min} = 46.59 \cdot 0.098$

$= 4.566\ in^3$

Process 1->2) Isentropic

$T_1 = 564.67\ R$

$P_1 = 14$ psia $\qquad k = 1.4$

$\forall_1 = 46.59\ in^3 \qquad \forall_2 = 4.566\ in^3$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{k-1} \qquad T_2 = 564.67\left(\frac{46.59\ in^3}{4.566\ in^3}\right)^{1.4-1}$$

$T_2 = 1429.87\ R$

$$\frac{P_2 \forall_2}{R T_2} = \frac{P_1 \forall_1}{R T_1} \qquad P_2 = \frac{P_1 \forall_1}{T_1} \cdot \frac{T_2}{\forall_2}$$

$$\frac{14\ psia \cdot 46.59 in^3}{564.67\ R} \cdot \frac{1429.87\ R}{4.566\ in^3} = P_2 = 361.73\ psia$$

Process 2 → 3) V = const $\qquad T_3 = 2859.67\ R$

$$\frac{P_2}{P_3} = \frac{T_2}{T_3} \qquad P_3 = \frac{P_2 T_3}{T_2} = \frac{361.73 \cdot 2859.67}{1429.87}$$

$P_3 = 723.44$ psia

Process 3 → 4) Isentropic

$$\frac{T_4}{T_3} = \left(\frac{\forall_3}{\forall_4}\right)^{k-1} \qquad T_4 = 2859.67\left(\frac{4.566}{46.59}\right)^{1.4-1}$$

$T_4 = 1129.31\ R$

Calculating for power)

$$W_{net} = q_{in} - q_{out} = c_v(T_3 - T_2) - c_v(T_4 - T_1)$$

$$0.171(2859.67 - 1429.87) - 0.171(1129.31 - 564.67)$$

$$W_{net} = 244.5 - 96.55 = 147.95 \frac{BTU}{lbm}$$

$$V_1 = \frac{RT_1}{P_1} = \frac{0.3704 \frac{psia \cdot ft^3}{lbm \cdot R} \cdot 564.7 R}{14\ psia} = 14.9 \frac{ft^3}{lbm}$$

$$m = \frac{V_d}{V_1} = \frac{0.0262\ ft^3 \cdot 6\ cylinders}{14.9 \frac{ft^3}{lbm}} = m_{Total} = 0.01059\ lbm$$

$$W_{net}\ per\ cycle = W_{net} \cdot m_{Total}$$

$$147.95 \frac{BTU}{lbm} \cdot 0.01059\ lbm = 1.567\ BTU$$

$$\dot{W}_{net} = N \cdot W_{net}$$

$$2500 \left(\frac{rev}{min}\right) \cdot 1.567 \left(\frac{BTU}{cycle}\right)\left(\frac{1\ min}{60 sec}\right)\left(\frac{1\ cycle}{2\ rev}\right) = 32.65 \frac{BTU}{sec}$$

$$32.65 \frac{BTU}{sec} = 46.12\ hp$$