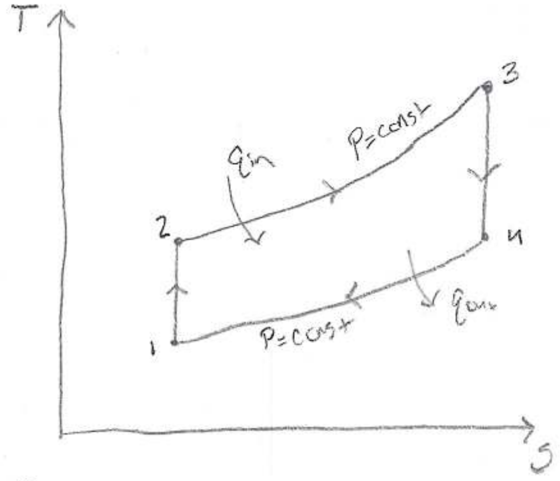
Rhiley Silvis | HW 1.4 | MET 350

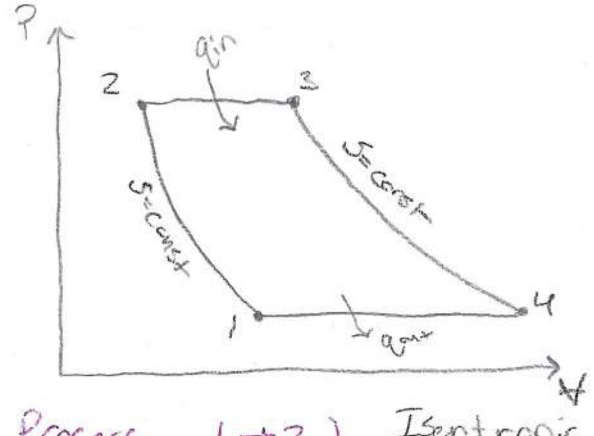
Problem 9-88

An aircraft engine operates on a simple ideal Brayton cycle with a pressure ratio of 10. Heat is added to the cycle at a rate of 500 kW and the air passes through the engine at a rate of 1 kg/s; and the air at the begining of the compression is at 70 kpa and 0°C. Determine the power produced by this engine and its thermal efficiency. (Use constant Specific heats at room temperature)



Given:

$r_p = 10$
$\dot{Q}_{in} = 500$ kW
$\dot{m}_{air} = 1$ kg/s
$P_1 = 70$ kpa
$T_1 = 0^{\circ}C = 273$ K



Process 1→2) Isentropic

$$r_p = \left(\frac{P_2}{P_1}\right) = 10 \qquad 10 = \left(\frac{P_2}{70\text{ kpa}}\right) \qquad P_2 = 700 \text{ kpa}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad T_2 = 273\left(10\right)^{\frac{1.4-1}{1.4}} = 527.1 \text{ K}$$

Process 2 → 3) P = const

$P_2 = P_3$ $\qquad P_3 = 700$ kpa

$Q_{in} = 500$ kW

$Q_{in} = C_p \cdot (T_3 - T_2) \cdot \dot{m}$

$$500\ \mathrm{k\cancel{W}}\left(\frac{1\ \mathrm{kJ/s}}{1\ \cancel{\mathrm{kW}}}\right) = 1.005\ \frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\left(T_3 - 527.1\mathrm{K}\right) \cdot 1\ \frac{\mathrm{kg}}{\mathrm{s}}$$

$$\frac{500\ \mathrm{kJ}/\cancel{\mathrm{s}}}{1\ \mathrm{kg}/\cancel{\mathrm{s}}} = 1.005\ \frac{\mathrm{kJ}}{\mathrm{kg\cdot K}}\left(T_3 - 527.1\mathrm{K}\right)$$

$$\frac{500\ \frac{\cancel{\mathrm{kJ}}}{\cancel{\mathrm{kg}}}}{1.005\ \frac{\cancel{\mathrm{kJ}}}{\cancel{\mathrm{kg}}\cdot \mathrm{K}}} = \left(T_3 - 527.1\mathrm{K}\right)$$

$497.61\ \mathrm{K} + 527.1\mathrm{K} = T_3 = 1024.61\ \mathrm{K}$

Process 3 → 4) Isentropic expansion

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} \qquad T_4 = \frac{T_3}{(r_p)^{\frac{k-1}{k}}}$$

$$T_4 = \frac{1024.61}{(10)^{\frac{1.4-1}{1.4}}} = 358.61\ \mathrm{K}$$

$P_4 = P_1$ $\qquad P_4 = 70$ kpa

Calculating Questions)

$Q_{in} = 500$ kJ

$$Q_{out} = C_p\left(T_4 - T_1\right) \cdot \dot{m}$$
$$= 1.005\ \frac{\mathrm{kJ}}{\cancel{\mathrm{kg\cdot K}}}\left(358.61\ \cancel{\mathrm{K}} - 273\ \cancel{\mathrm{K}}\right) \cdot 1\ \frac{\cancel{\mathrm{kg}}}{\mathrm{s}}$$
$Q_{out} = 86.04$ kJ

$W_{net} = Q_{in} - Q_{out} = 500\ \mathrm{kJ} - 86.04\ \mathrm{kJ} = \boxed{413.96\ \mathrm{kW}}$

$$\eta = \frac{W_{net}}{Q_{in}} \cdot 100\% \qquad \frac{413.96}{500} = \boxed{82.8\ \%}$$

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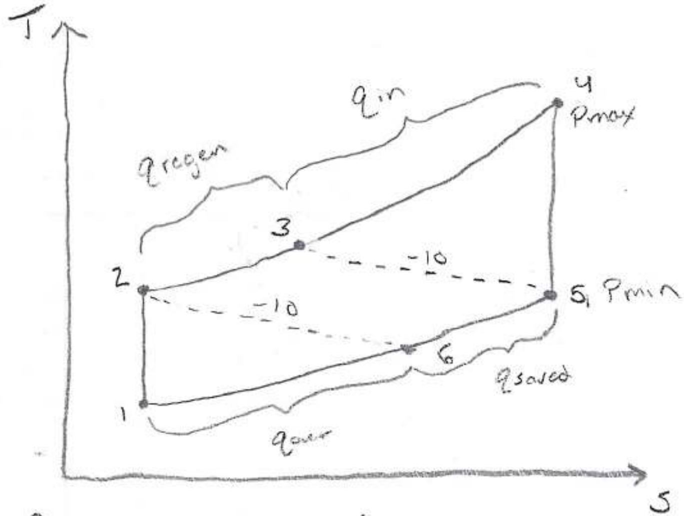
Problem 9-99

A gas Turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa a 30°C. The compressor pressure ratio is 10; the maximum cycle temperature is 800°C; and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and redection for this cycle when it produces 115 kW. (use constant specific heats at room temp)



Given:

$T_1 = 30°C = 303\,K$

$P_1 = 100\ kpa$

$r_p = 10$

$T_4 = 800°C = 1073\,K$

$T_5 - T_3 = 10°C$

$T_6 - T_2 = 10°C$

— Process 1→2) Isentropic

$T_1 = 303\ K \quad P_1 = 100\ Kpa \qquad \frac{P_2}{P_1} = r_p = 10$

$$\frac{T_2}{T_1} = (r_p)^{\frac{k-1}{k}} \qquad T_2 = 303K\,(10)^{\frac{1.4-1}{1.4}}$$

$T_2 = 585\ K$

$$\frac{P_2}{P_1} = 10 \qquad P_2 = 100(10) = 1000\ kpa$$

— Process 4→5) Isentropic $\quad T_4 = 1073\ K$

$$\frac{T_4}{T_5} = \left(\frac{P_4}{P_5}\right)^{\frac{k-1}{k}} \qquad \frac{T_4}{(r_p)^{\frac{k-1}{k}}} = T_5 = \frac{1073K}{(10)^{\frac{1.4-1}{1.4}}} = 555.76K$$

$P_2 = P_4 = 1000\ kpa. \qquad P_1 = P_5 = 100\ kpa$

- Finding T_3 & T_6

$T_3 = T_5 - 10K = 555.76 - 10 = 545.76\ K$

$T_6 = 10K + T_2 = 585K + 10K = 595K$

- Finding Qin & Qout

$W_{net} = W_T - W_c$

$W_c = c_p(T_2 - T_1)\dot{m}$

$W_T = c_p(T_4 - T_5)\dot{m}$

$W_{net} = C_p(T_4 - T_5)\dot{m} - C_p(T_2 - T_1)\dot{m}$

$115\ kJ = C_p(1073K - 555.76)\dot{m} - c_p(585K - 303K)$

$115\ kJ = C_p \cdot \dot{m}\,(517.24\ K - 282\ K)$

$115\ \cancel{kJ} = 1.005\ \frac{\cancel{kJ}}{kg \cdot K} \cdot \dot{m}\,(235.24\ \cancel{K})$

$\dot{m} = 0.4864\ kg$

$Q_{in} = \dot{m} \cdot c_p(T_4 - T_3)$

$= 0.4864\ \cancel{kg} \cdot 1.005\ \frac{kJ}{\cancel{kg} \cdot K}(1073K - 545.76K)$

$Q_{in} = 257.7\ kJ = \boxed{257.7\ kW}$

$Q_{out} = \dot{m} \cdot c_p(T_6 - T_1)$

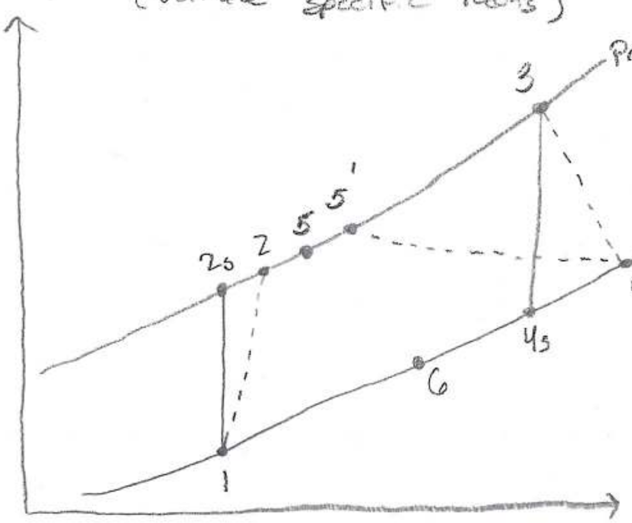
$= 0.4864\ \cancel{kg} \cdot 1.005\ \frac{kJ}{\cancel{kg} \cdot \cancel{K}}(595\cancel{K} - 303\cancel{K})$

$Q_{out} = 142.74\ kJ = \boxed{142.74\ kW}$

Problem 9-107

A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 and 1150 K. Assuming and Isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine and an effectiveness of 65 percent for the regenerator, determine (a) the temperature at the turbine exit, (b) the network output, and (c) the thermal efficiency.

(Varible specific heats)



Given:-

$r_p = 7$

$T_1 = 310\,K$

$T_3 = 1150\,k$

$\eta_c = 0.75$

$\eta_T = 0.82$

$\varepsilon = 0.65$

Process 1→2) Isentropic

$T_1 = 310\,K$

$P_{r_1} = 1.5546$

$h_1 = 310.24\ KJ/Kg$

$$\frac{P_{r2s}}{P_{r1}} = \frac{P_2}{P_1} = r_p$$

$$P_{r2s} = P_{r1}(r_p) = 1.5546(7) = 10.8822$$

P_r	
10.37	533.98
10.8822	h_{2s}
11.10	544.35

$$\frac{10.8822 - 10.37}{11.10 - 10.37} = \frac{h_{2s} - 533.98}{544.35 - 533.98}$$

$$h_{2s} = 541.26\ KJ/kg$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} \qquad h_2 = \frac{h_{2s} - h_1}{\eta_c} + h_1 = \frac{541.26 - 310.24}{0.75} + 310.24$$

$$h_2 = 618.27\ KJ/kg$$

– Process 3 → 4) Isentropic

$T_3 = 1150\text{ K}$

T	Pr
1140	193.1
1150	P_{r3}
1160	207.2

$$\frac{1150 - 1140}{1160 - 1140} = \frac{P_{r3} - 193.1}{207.2 - 193.1}$$

$$P_{r3} = 200.15$$

T	h
1140	1207.57
1150	h_3
1160	1230.92

$$\frac{1150 - 1140}{1160 - 1140} = \frac{h_3 - 1207.57}{1230.92 - 1207.57}$$

$$h_3 = 1219.25 \text{ kJ/kg}$$

$$\frac{P_3}{P_{4s}} = \frac{P_{r3}}{P_{r4s}} = r_p \qquad P_{r4s} = \frac{P_{r3}}{r_p} = \frac{200.15}{7} = 28.59$$

Pr	h
27.29	702.52
28.59	h_{4s}
28.80	713.27

$$\frac{28.59 - 27.29}{28.80 - 27.29} = \frac{h_4 - 702.52}{713.27 - 702.52}$$

$$h_{4s} = 711.77 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \qquad h_4 = \eta_T(h_3 - h_{4s}) - h_3 \cdot -1$$

$$h_4 = -(0.82(1219.25 - 711.77) - 1219.25) = 803.12 \text{ kJ/kg}$$

– Process 4 → 5) $\varepsilon = 0.65$

$$\varepsilon = \frac{q_{regen}}{q_{regen\,max}} = \frac{h_5 - h_2}{h_4 - h_2} \qquad \varepsilon(h_4 - h_2) + h_2 = h_5$$

$$h_5 = 0.65(803.12 - 618.27) + 618.27 = 738.42 \text{ kJ/kg}$$

- Calculating Questions)

a) T at turbine exit

T	h
780	800.03
T_4	803.12
800	821.95

$$\frac{T_4 - 780}{800 - 780} = \frac{803.12 - 800.03}{821.95 - 800.03}$$

$$T_4 = \boxed{782.81\ K}$$

b) W_{net}

$$W_{net} = W_T - W_C = (h_3 - h_4) - (h_2 - h_1)$$

$$W_{net} = (1219,25 - 803,12) - (618.27 - 310.24) = \boxed{108.1\ \text{kJ/kg}}$$

c) η_{Th}

$$\eta_{Th} = \frac{W_{net}}{q_{in}} \qquad q_{in} = h_3 - h_5$$

$$\eta_{Th} = \frac{108.1}{(1219.25 - 738.42)} = \boxed{22.5\ \%}$$

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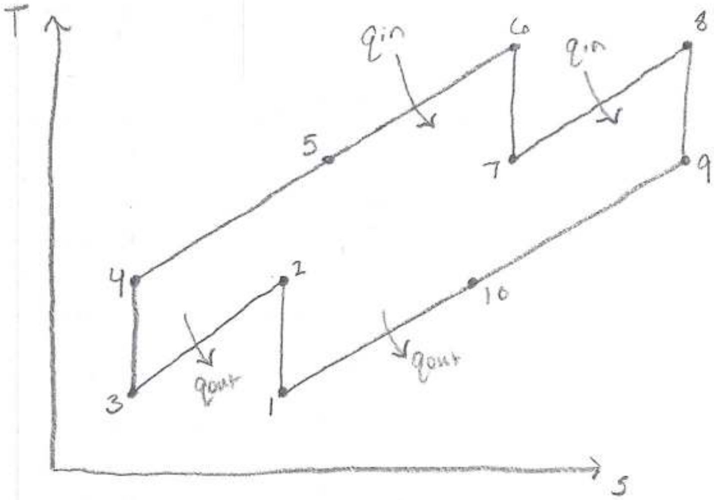
Problem 9-119

Consider a regenerative gas-turbine power plant with two stages of compression and two stages of expansion. The overall pressure ratio of the cycle is 9. The air enters each stage of the compressor at 300K and each stage of the turbine at 1200K. Accounting for the variation of specific heats with temperature determine the minimum mass flow rate of the air needed to develop a net power of 110 MW



Given:

$r_p = 8$

$T_1 = T_3 = 300K$

$T_6 = T_8 = 1200K$

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{9} = 3 \qquad \frac{P_6}{P_7} = \frac{P_8}{P_9} = \sqrt{9} = 3$$

- Values for T_1 & T_3

$T_1 = T_3 = 300K$

$h_1 = h_3 = 300.19$ KJ/kg

$P_{r1} = 1.3860$

- Values for T_2 & T_4

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} \qquad P_{r2} = P_{r1}\left(\frac{P_2}{P_1}\right) \qquad \frac{P_2}{P_1} = 3$$

$$P_{r2} = 1.3860(3) = 4.158$$

P_r	T
4.153	410
4.158	T_2
4.522	420

$$\frac{4.158 - 4.153}{4.522 - 4.153} = \frac{T_2 - 410}{420 - 410}$$

$$T_2 = T_4 = 410.13\ K$$

P_r	h
4,153	411,12
4.158	h_2
4,522	421,26

$$\frac{4{,}158 - 4.153}{4{,}522 - 4.153} = \frac{h_2 - 411{,}12}{421{,}26 - 411{,}12}$$

$$h_2 = h_4 = 411.26 \ \text{KJ/kg}$$

- Values for T_6 & T_8

$T_6 = T_8 = 1200\text{K}$

$h_6 = h_8 = 1277.79 \ \text{KJ/kg}$

$P_{r6} = 238.0$

- Values for T_7 & T_9

$$\frac{P_{r6}}{P_{r7}} = \frac{P_6}{P_7} = r_p$$

$$P_{r7} = \frac{P_{r6}}{r_p} = \frac{238}{3} = 79.33$$

P_r	T
75.29	900
79.33	T_7
82.05	920

$$\frac{79.33 - 75.29}{82.05 - 75.29} = \frac{T_7 - 900}{920 - 900}$$

$$T_7 = T_9 = 911.95\text{K}$$

P_r	h
75.29	932.93
79.33	h_7
82.05	955.38

$$\frac{79.33 - 75.29}{82.05 - 75.29} = \frac{h_7 - 932.93}{955.38 - 932.93}$$

$$h_7 = h_9 = 946.35 \ \text{KJ/kg}$$

- Calculating Questions

$$W_{net} = W_T - W_c = (2(h_6 - h_7)) - (2(h_4 - h_3)$$

$$W_{net} = (2(1277.79 - 946.35)) - (2(411.26 - 300.19))$$

$$W_{net} = 440.74 \ \text{KJ/kg}$$

$\dot{W} = 110 \text{ MW} = 110{,}000 \text{ kJ/s}$

$\dot{W} = \dot{m} \, w_{net}$

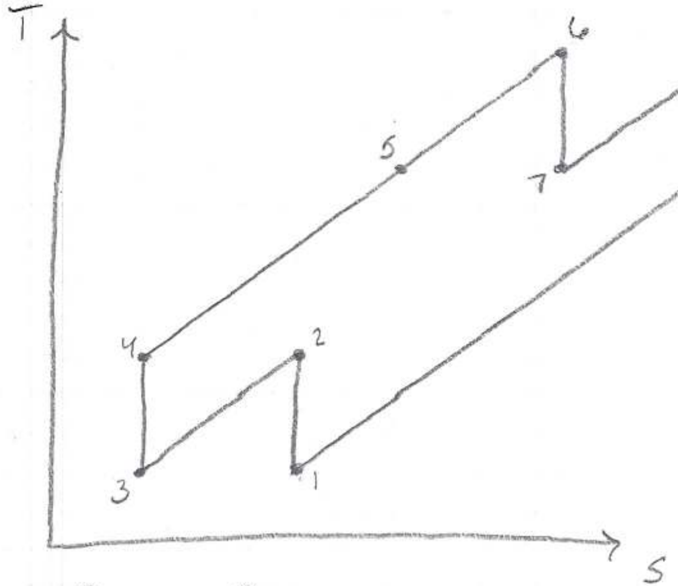
$110{,}000 \text{ kJ/s} = \dot{m} \, (440.74 \text{ kJ/kg})$

$\boxed{\dot{m} = 249.58 \text{ kg/s}}$

Problem 9-121

Consider an Ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and turbine is 3. The air enters each stage of the compressor at 300K and each stage of the turbine at 1200K. Determine the back work ratio and thermal efficiency of the cycle assuming (a) no regenorator is used and (b) a regenorator with 75% effectivness is used.

(Variable Specific heats)



Given:

$r_p = 3$

$T_1 = T_3 = 300\,K$

$T_6 = T_8 = 1200\,K$

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = 3$$

$$\frac{P_6}{P_7} = \frac{P_8}{P_9} = 3$$

Values for T_1 & T_3

$T_1 = T_3 = 300\,K$

$h_1 = h_3 = 300.19 \ \text{kJ/kg}$

$P_{r_1} = 1.3860$

- Values for T_2 & T_4

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} = r_p \qquad P_{r2} = P_{r1}(r_p)$$

$$P_{r2} = (1.3860)(3) = 4.158$$

P_r	h
4.153	411.12
4.158	h_2
4.522	421.26

$$\frac{4.158 - 4.153}{4.522 - 4.153} = \frac{h_2 - 411.12}{421.26 - 411.12}$$

$$h_2 = h_4 = 411.26 \text{ KJ/kg}$$

- Values for T_6 & T_8

$T_6 = T_8 = 1200\text{K}$

$h_6 = h_8 = 1277.79 \text{ KJ/kg}$

$P_{r6} = 238.0$

- Values for T_7 & T_9

$$\frac{P_{r6}}{P_{r7}} = \frac{P_6}{P_7} = r_p \qquad P_7 = \frac{P_{r6}}{r_p} = \frac{238}{3} = 79.33$$

P_r	h
75.29	932.93
79.33	h_7
82.05	955.38

$$\frac{79.33 - 75.29}{82.05 - 75.29} = \frac{h_7 - 932.93}{955.38 - 932.93}$$

$$h_7 = h_9 = 946.35 \text{ KJ/kg}$$

- Calculating Back work (No Regen)

$$r_{BW} = \frac{W_C}{W_T} = \frac{(2(h_4 - h_3))}{(2(h_6 - h_7))}$$

$$\frac{(2(411.26 - 300.19))}{(2(1277.79 - 946.35))} = \boxed{0.335}$$

- Calculating Thermal efficiency (No Regen)

$$\eta_{Th} = \frac{W_{net}}{q_{in}}$$

$W_{net} = W_T - W_c = (2(h_6 - h_7)) - (2(h_4 - h_3))$

$W_{net} = (2(1277.79 - 946.35)) - (2(411.26 - 300.19))$

$W_{net} = 440.74$ kJ/kg

$q_{in} = (h_6 - h_4) + (h_8 - h_7)$

$q_{in} = (1277.99 - 411.26) + (1277.79 - 946.35) = 1197.97 \frac{kJ}{kg}$

$$\eta_{Th} = \frac{440.74}{1197.97} = \boxed{36.8\%}$$

Calculating Backwork (Regen Added)

No change to turbine or compressor

$\boxed{r_{BW} = 0.335}$

Calculating Thermal efficiency (Regen Added)

$q_{regen} = 0.75(h_9 - h_4) = 0.75(946.35 - 411.26)$

$q_{regen} = 401.32$ kJ/kg

$q_{in} = q_{in_{old}} - q_{regen} = 1197.97 - 401.32$

$q_{in} = 796.65$ kJ/kg

$$\eta_{Th} = \frac{440.74}{796.65} = \boxed{55.3\%}$$