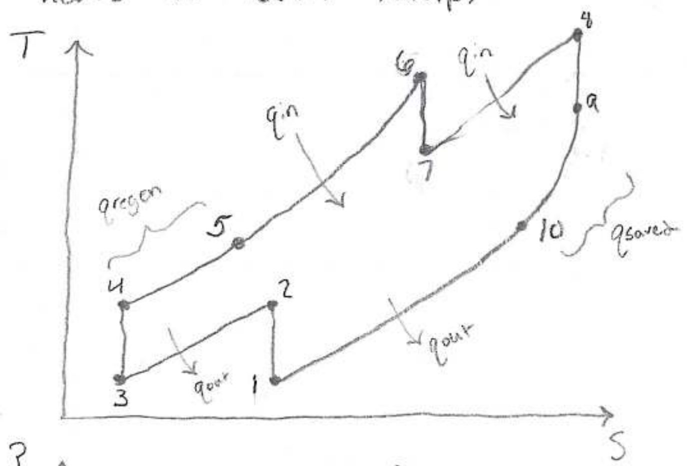
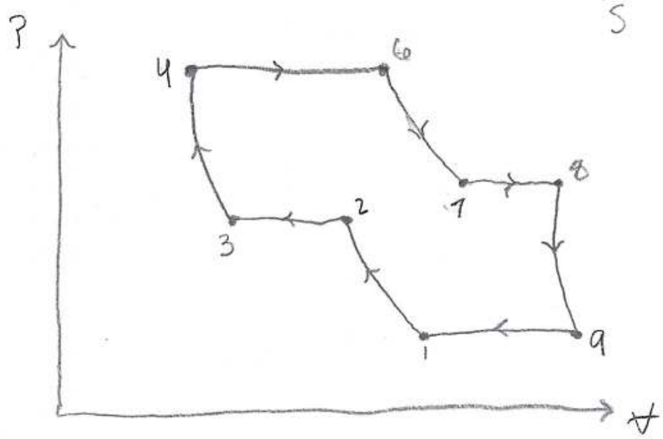
Rhiley Silvis | HW 1.5 | MET 350

Problem 9-123

Air enters a gas turbine with two stages of compression and two stages of expansion at 100 KPa and 17°c. This system uses a regenerator as well as reheating and intercooling. The pressure ratio across each compressor is 4; 300 kJ/kg of heat are added to the air in each combustion chamber; and the regenerator operates perfectly while increasing the temperature of the cold air by 20°c. Determine the systems thermal efficiency. Assume Isentropic operations for all compressor and turbine stages and use constant specific heats at room temp.





Given:

$T_1 = T_3 = 17^{\circ c} = 290\ K$

$P_1 = 100\ kpa$

$r_p = 4$

$q_{in_{5\to6}} = q_{in_{7\to8}} = 300\ kJ/kg$

$T_5 = T_4 + 20^{\circ c}$

$k = 1.4$

$c_p = 1.005$

$c_v = 0.718$

— Values for T_1 & T_3)

$T_1 = T_3 = 290\ K$

$P_1 = 100\ kpa$

— Values for T_2 & T_4) $1 \to 2$ & $3 \to 4$ Isentropic

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad \frac{P_2}{P_1} = r_p$$

$$T_2 = T_1 (r_p)^{\frac{k-1}{k}} = 290(4)^{\frac{1.4-1}{1.4}} = 430.94\ K$$

- Values for T_5)

$T_5 = T_4 + 20 = 430.94 + 20 = 450.94\ K$

- Process 5→6) contant P

$q_{in} = C_p (T_6 - T_5)$

$300\ kJ/kg = 1.005\ \frac{kJ}{kg \cdot K} (T_6 - 450.94K)$

$\frac{300\ kJ/kg}{1.005\ \frac{kJ}{kg \cdot K}} = T_6 - 450.94K$

$T_6 = 298.507\ K + 450.94\ K = 749.45\ K$

- Process (6→7)

$\frac{T_6}{T_7} = \left(\frac{P_6}{P_7}\right)^{\frac{K-1}{K}} \qquad \frac{P_6}{P_7} = r_p$

$\frac{T_6}{(r_p)^{\frac{K-1}{K}}} = T_7 = \frac{749.45}{(4)^{\frac{1.4-1}{1.4}}} = 504.34\ K$

- Process 7→8) second combustion chamber ; 300 kJ/kg constant P

$\frac{300\ kJ/kg}{1.005\ \frac{kJ}{kg \cdot K}} = T_8 - T_7$

$T_8 = 298.507 + 504.34 = 802.85\ K$

- Process 8→9) Isentropic exansion

$\frac{T_8}{T_9} = \left(\frac{P_8}{P_9}\right)^{\frac{K-1}{K}} \qquad \frac{P_8}{P_9} = r_p$

$\frac{T_8}{r_p^{\frac{K-1}{K}}} = T_9 = \frac{802.85\ K}{(4)^{\frac{1.4-1}{1}}} = 540.28\ K$

— Calculating η_{Th})

$q_{in} = 300 \ kJ/kg \cdot 2 = 600 \ kJ/kg$

$q_{out} = c_p (T_{10} - T_1) + c_p (T_2 - T_3)$

$T_{10} = T_9 - 20 = 540.28 - 20 = 540.28 - 20 = 520.28$

$q_{out} = 1.005 (520.28 - 290) + 1.005 (430.94 - 290)$

$q_{out} = 373.08 \ kJ/kg$

$\eta_{Th} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{373.08}{600} = \boxed{37.82 \%}$

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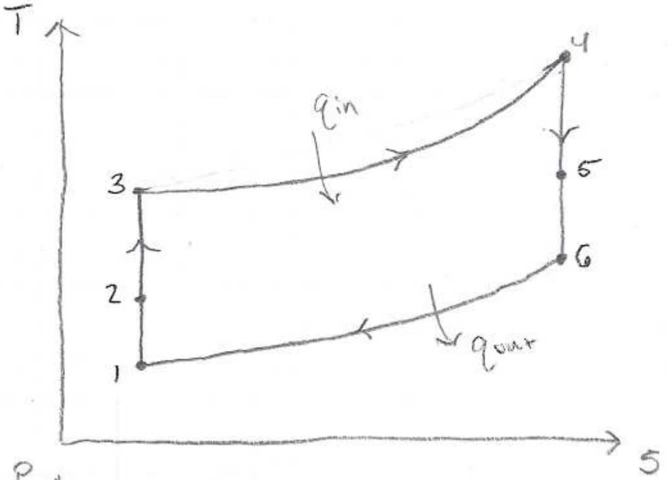
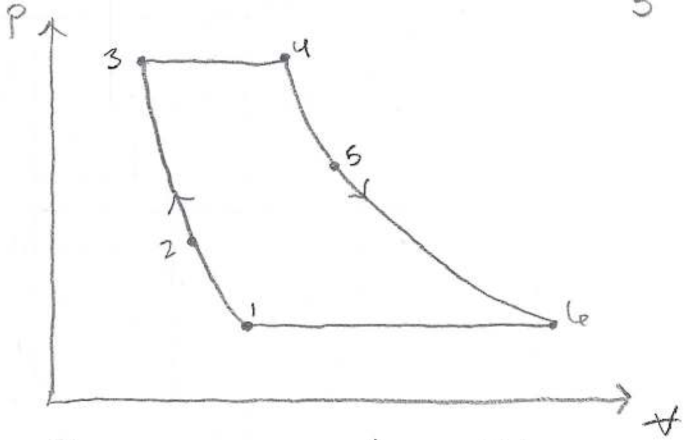
Problem 9-129

A Turbo-Jet is flying with a velocity of 900 ft/s at an altitude of 20,000 ft, where ambient conditions are 7 psia and 10°F. The pressure ratio across the compressor is 13, and the temperature at turbine inlet is 2400 R. Assuming ideal operations for all components and constant specific heats for room temp, determin (a) the pressure at the turbine exit, (b) the velocity of the exhaust gases, and (c) the propulsive efficiency.





Given:

$V_1 = 900$ ft/s

$h = 20{,}000$ ft

$T_1 = 10°F = 470$ R

$P_1 = 7$ psia

$r_p = 13$

$T_4 = 2400$ R

$k = 1.4$

$c_p = 0.240$ BTU/lbm·R

$c_v = 0.171$ BTU/lbm·R

– Process 1 → 2) Diffuser Isentropic compression

$$T_2 = T_1 + \frac{V_1^2}{2c_p} = 470R + \frac{(900 \text{ ft/s})^2}{2 \cdot 0.240 \frac{Btu}{lbm \cdot R}} \left(\frac{1 \text{ Btu/lbm}}{25037 \text{ ft}^2/\text{s}^2}\right)$$

$T_2 = 537.4$ R

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} \cdot P_1 = P_2$$

$$P_2 = \left(\frac{537.4}{470}\right)^{\frac{1.4}{1.4-1}} \cdot 7 = 11.19 \text{ psia}$$

— Process 2 → 3) Isentropic Comprsion , $W_{c,in}$

$$\frac{P_3}{P_2} = r_p \qquad P_3 = P_2 \cdot r_p = 11.19 \cdot 13 = 145.47 \text{ psia}$$

$$\frac{T_3}{T_2} = \left(\frac{P_3}{P_2}\right)^{\frac{k-1}{k}} \qquad T_3 = 537.4(13)^{\frac{1.4-1}{1.4}} = 1118.32 \text{ K}$$

— Process 3 → 4) Isobaric heat addition

$T_4 = 2400 \text{ R} \qquad P_4 = P_3 = 145.47 \text{ psia}$

— Process 4 → 5) Turbine, Isentropic expansion

$W_c = W_T$

$c_p(T_3 - T_2) = c_p(T_4 - T_5)$

$T_5 = T_4 - (T_3 - T_2) = 2400 - (1118.32 - 537.4)$

$T_5 = 1819.08 \text{ R}$

$$\frac{P_5}{P_4} = \left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}}$$

$$P_5 = P_4\left(\frac{T_5}{T_4}\right)^{\frac{k}{k-1}} = 145.47\left(\frac{1819.08}{2400}\right)^{\frac{1.4}{1.4-1}} = 55.15 \text{ psia}$$

— Process 5 → 6) Nozzle, Isentropic expansion

$$\frac{T_6}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} \qquad P_6 = P_1$$

$$T_6 = 1819.08\left(\frac{7}{55.15}\right)^{\frac{1.4-1}{1.4}} = 1008.61 \text{ K}$$

— Calculating Questions

a) pressure at turbine exit

$$\boxed{P_5 = 55.15 \text{ psia}}$$

b) the velocity of exhaust gases

From Chapter 9 page 513

$$V_6 = \sqrt{2c_p(T_5 - T_6)}$$

$$V_6 = \sqrt{2 \cdot 0.24 \left(\frac{Btu}{lbm \cdot R}\right)\left(\frac{25037 \frac{ft^2}{s^2}}{1 \frac{Btu}{lbm}}\right) \cdot (1819.08 - 1008.61) R}$$

$$\boxed{V_6 = 3120.9 \text{ ft/s}}$$

c) Propulsive effiency

$$W_p = (V_{exit} - V_{inlet}) \cdot V_{aircraft}$$

$$= (3120.9 - 900) \cdot 900 \left(\frac{1 Btu/lbm}{25037 ft^2/s^2}\right)$$

$$W_p = 79.83 \frac{Btu}{lbm}$$

$$Q_{in} = c_p(T_4 - T_3) = 0.24 \frac{Btu}{lbm \cdot R}(2400 - 1118.32) R$$

$$Q_{in} = 307.6 \frac{Btu}{lbm}$$

$$\eta_p = \frac{79.83}{307.6} = \boxed{25.9\%}$$

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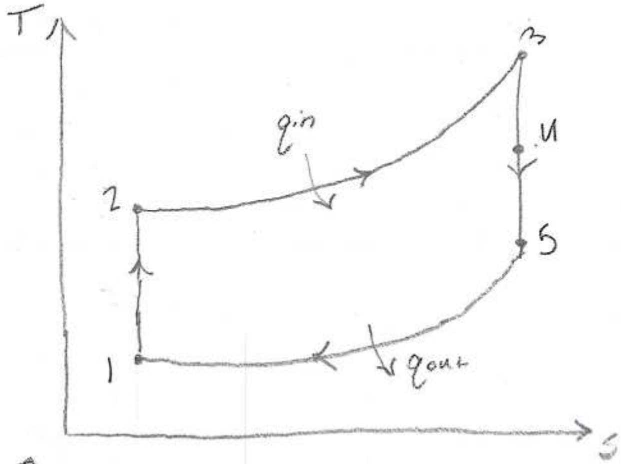
Problem 9-135

Consider an aircraft powered by a turbojet engine that has a pressure ratio of 9. The aircraft is stationary on the ground, held in position by its breaks. The ambient air is at 7°C and 95 kPa and enters the engine at a rate of 20 kg/s. The Jet fuel has a heating value of 42,700 kJ/kg and it is burned completly at a rate of 0.5 kg/s. Neglecting the effect of the diffuser and disregarding the slight increase in mass at the engine exit as well as the inefficiencies of the engine components, determine the force that must be applied on the brakes to hold the plane stationary.



Given:

$r_p = 9$

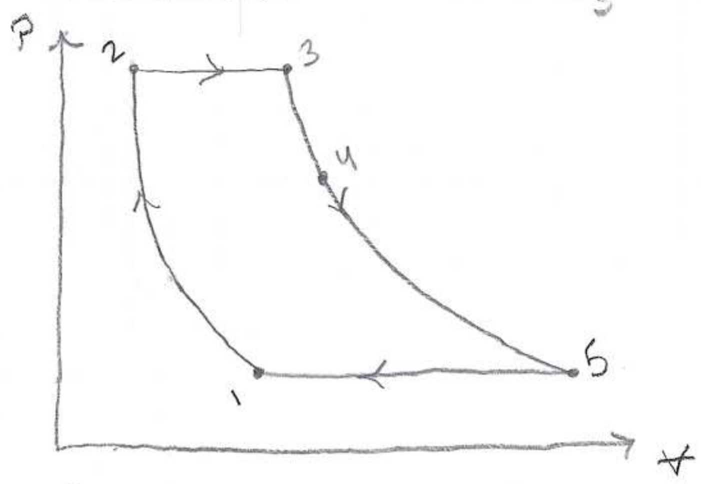
$T_1 = 7°C = 280\ K$

$P_1 = 95\ kPa$

$\dot{m}_{air} = 20\ kg/s$

$HV_F = 42,700\ kJ/kg$

$\dot{m}_f = 0.5\ kg/s$



Process $1 \to 2$) Isentropic compression

$T_1 = 280\ K$

$h_1 = 280.13\ kJ/kg$

$P_{r1} = 1.0889$

$$\frac{P_{r2}}{P_{r1}} = \frac{P_2}{P_1} = r_p \qquad P_{r2} = P_{r1}(r_p) = 1.0889(9)$$

$$P_{r2} = 9.8001$$

P_r	h
9.684	523.63
9.8001	h_2
10.37	533.98

$$\frac{9.8001 - 9.684}{10.37 - 9.684} = \frac{h_2 - 523.63}{533.98 - 523.63}$$

$$h_2 = 525.38 \text{ kJ/kg}$$

- Process $2 \rightarrow 3$

$$\dot{Q}_{in} = \dot{m}_a (h_3 - h_2) = \dot{m}_f HV_f \eta_{comb}$$

$$h_3 = \frac{\dot{m}_f HV_f \eta_{comb}}{\dot{m}_a} + h_2$$

$$= \frac{(0.5 \text{ kg/s})(42{,}700 \text{ kJ/kg})(1.0)}{20 \text{ kg/s}} + 525.38 \text{ kJ/kg}$$

$$h_3 = 1592.88 \text{ kJ/kg}$$

- Process $3 \rightarrow 4$

$$W_T = W_c \qquad (h_3 - h_4) = (h_2 - h_1)$$

$$h_4 = h_3 - (h_2 - h_1)$$

$$= 1592.88 - (525.38 - 280.13)$$

$$h_4 = 1347.63 \text{ kJ/kg}$$

- Process $4 \rightarrow 5$

$$\frac{P_{r3}}{P_{r5}} = \frac{P_3}{P_5} = r_p$$

h	P_r
1587.63	537.1
1592.88	P_{r3}
1611.79	568.8

$$\frac{1592.88 - 1587.63}{1611.79 - 1587.63} = \frac{P_{r3} - 537.1}{568.8 - 537.1}$$

$$P_{r3} = 543.98$$

$$\frac{P_{r3}}{r_p} = P_{r5} = \frac{543.98}{9} = 60.44$$

P_r	h
57.60	866.08
60.44	h_5
63.09	888.27

$$\frac{60.44 - 57.60}{63.09 - 57.60} = \frac{h_5 - 866.08}{888.27 - 866.08}$$

$$h_5 = 877.55$$

– Calculating Thrust)

$$V_5 = \sqrt{2(h_4 - h_5)}$$

$$= \sqrt{2(1347.63 - 877.55)\left(\frac{1000\ m^2/s^2}{1\ kJ/kg}\right)}$$

$$V_5 = 969.62\ m/s$$

$$F_{Thrust} = \dot{m}_a V_5 = \left(20\ \frac{kg}{s}\right)\left(969.62\ \frac{m}{s}\right)$$

$$= 19392.4\ \frac{kg \cdot m}{s^2} = N$$

$$= \boxed{19.39\ kN}$$