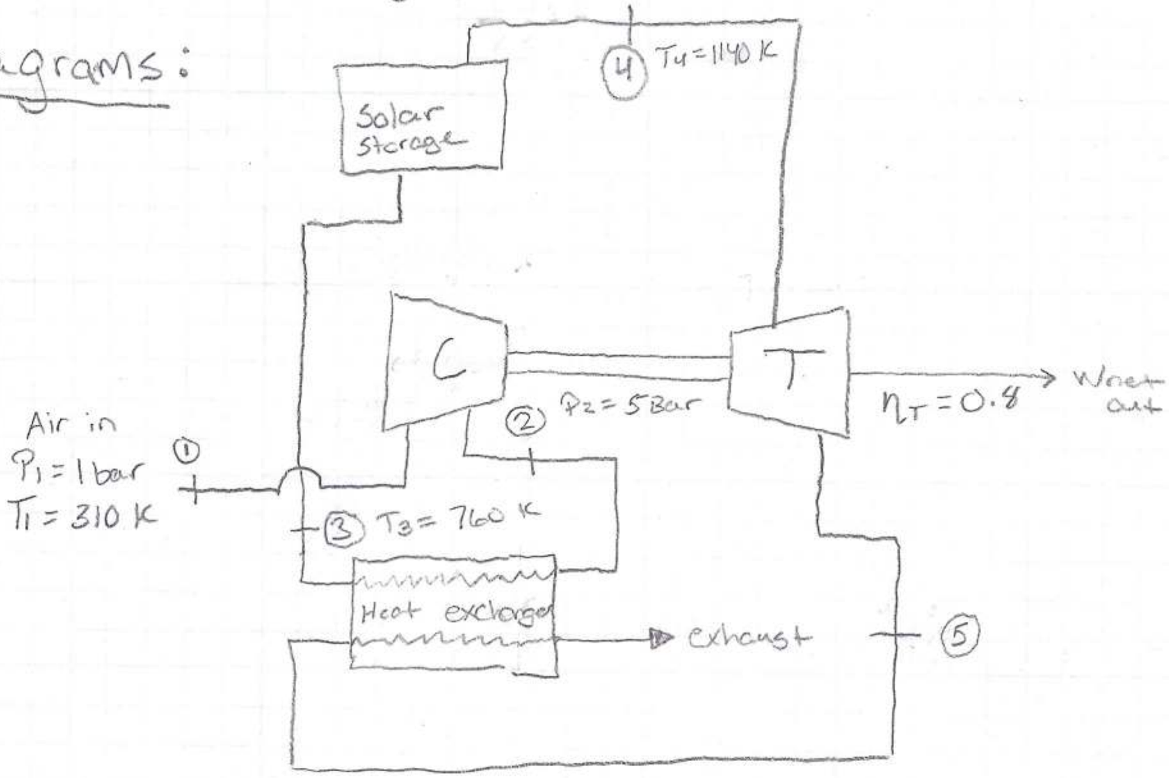
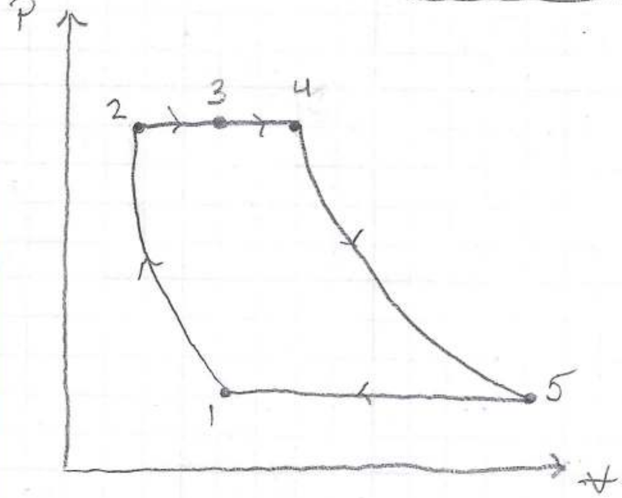
Rhiley Silvis | Test 1 | MET 350

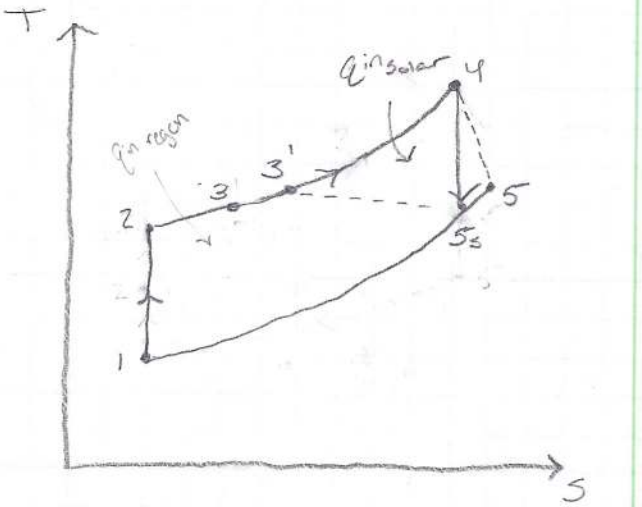
Problem 1

Purpose: Given the Brayton cycle with solar energy added, Determine the Thermal effiency and effectiviness of the heat exchanger. Also find the mass air flow rate to reach a net power output of 500 kW. Lastly, Determine the thermal efficency if the heat exchanger was operating at 100% effectiveness.

Diagrams:







1→2) Isentropic compression

2→3) P = const, q_{in}

3→4) P = const, q_{in} solar

4→5) Isentropic expansion

Sources:
Cengel & Boles & Kanoglu
Thermodynamics An engineering approach. 9th edition Mcraw Hill. 2019

Design Coniderations:

I assume the following:

1) Air behaves as an Ideal gas.
2) Cp & Cv are constant at 300 K
3) Isentropic effieney of turbine is 0.8
4) There are no heat losses in transfer of air Through the system
5) There are no mechanical losses due to bearings ect.
6) From table A-2 $c_p = 1.005$ $c_v = 0.718$
   $k = c_p/c_v = 1.4$

Data and Variables:

$P_1 = 1$ bar $= 100$ kpa

$T_1 = 310$ K

$P_2 = 5$ bar $= 500$ kpa

$T_3 = 760$ K

$T_4 = 1140$ K

$\eta_T = 0.8$

$W_{net} = 500$ kW

Materials:

Air as an Ieal gas

Procedue & Calculations:

I will First find all values of air for the various stages.

— Process 1 → 2) Isentropic compression

$T_1 = 310\ K$ $\quad$ $P_2 = 500\ kpa$

$P_1 = 100\ kpa$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \qquad T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = 310\ K\left(\frac{500\ kpa}{100\ kpa}\right)^{\frac{1.4-1}{1.4}} = 490.98\ K$$

— Process 2 → 3) P = const, q_{in}, Heat exchanger

$P_2 = 500\ kpa$ $\quad$ $T_3 = 760\ K$

$T_2 = 490.98\ K$ $\quad$ $P_3 = 500\ kpa$

$$q_{in} = c_p(T_3 - T_2) = 1.005(760\ K - 490.98\ K)$$

q_{in} (Heat exchanger) = 270.36 kJ/kg

— Process 3 → 4) P = const, $q_{in\ solar}$, Solar

$P_3 = 500\ kpa$ $\quad$ $T_4 = 1140\ K$

$T_3 = 760\ K$ $\quad$ $P_4 = 500\ kpa$

$$q_{in\ solar} = c_p(T_4 - T_3) = 1.005(1140\ K - 760\ K)$$

$q_{in\ solar} = 381.9$ kJ/kg

— Process 4 → 5) Isentropic expansion

$$\frac{T_4}{T_{5_s}} = \left(\frac{P_4}{P_5}\right)^{\frac{k-1}{k}} \qquad \frac{P_4}{P_5} = r_p = \frac{P_{max}}{P_{min}} = \frac{500\ kpa}{100\ kpa} = 5$$

$$T_{5_s} = \frac{T_4}{(r_p)^{\frac{k-1}{k}}} = \frac{1140}{(5)^{\frac{1.4-1}{1.4}}} = 720.78\ K$$

$$\eta_T = \frac{T_4 - T_5}{T_4 - T_{5_s}} \qquad T_5 = T_4 - \eta_T (T_4 - T_{5_s})$$

$$T_5 = 1140 - 0.8(1140 - 720.78) = 804.62 \text{ K}$$

- I will now use values from Processes to solve questions

a) Thermal efficiency and the heat exchanger effectiveness

$$\eta_{Th} = \frac{w_{net}}{q_{in_{solar}}}$$

$$w_T = c_p(T_4 - T_5) = 1.005(1140 - 804.62) = 335.38 \text{ kJ/kg}$$

$$w_C = c_p(T_2 - T_1) = 1.005(490.98 - 310) = 181.88 \text{ kJ/kg}$$

$$w_{net} = w_T - w_C = 335.38 - 181.88 = 153.5 \text{ kJ/kg}$$

$$\eta_{Th} = \frac{153.5}{381.9} = \boxed{40.19\%}$$

$$\varepsilon = \frac{T_3 - T_2}{T_5 - T_2} = \frac{760 - 490.98}{804.62 - 490.98} = \boxed{0.858}$$

b) The mass air flow rate in kg/s for a net power output of 500 kW

$$500 \text{ kW} = 500 \text{ kJ/s}$$

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{500 \text{ kJ/s}}{153.5 \text{ kJ/kg}} = \boxed{3.26 \text{ kg/s}}$$

c) Thermal efficiency if heat exchanger were operating at 100% effectiveness

$$q_{regen\,max} = c_p(T_5 - T_2) = 1.005(804.62 - 490.98) = 315.21$$

$$q_{regen\,max} - q_{actual} = 315.21 \text{ kJ/kg} - 270.36 \text{ kJ/kg} = 44.85 \text{ kJ/kg}$$

$$q_{in_{solar}} - 44.85 \text{ kJ/kg} = 381.9 - 44.85 = 337.05 \text{ kJ/kg}$$

$$\eta_{Th} = \frac{153.5}{337.05} = \boxed{45.54\%}$$

Summary:

The Brayton cycle with solor energy added appears to be relativly efficent compared to most other cycles. Having a heat exchanger equiped, especially one with 85% effectivness helps immensly. While Theroetically immpossible, the 100% effective heat regenerator raised the efficency by 5%. If one could aquire such a regenorator, It would probably be very expensive and not be able to outweigh the cost of energy without it.

Analysis:

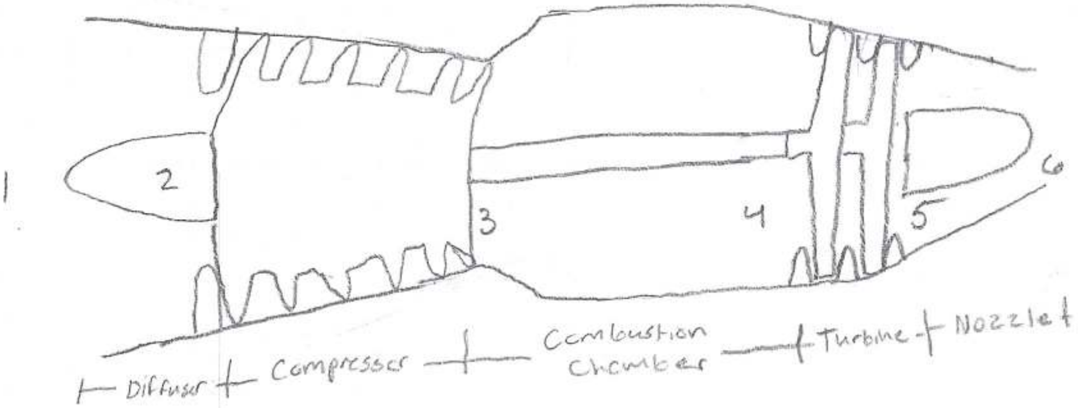
This system operates under the assumption it will be provided 381.9 KJ/kg energy from the solar storage. Therefore the solar panels need to averge that or greater. If the solar storage were to run low the efficency of this system would greatly diminish. So this needs to be somewhere where the amount of daylight is pretty standard, or a back up energy source would need to be considered.

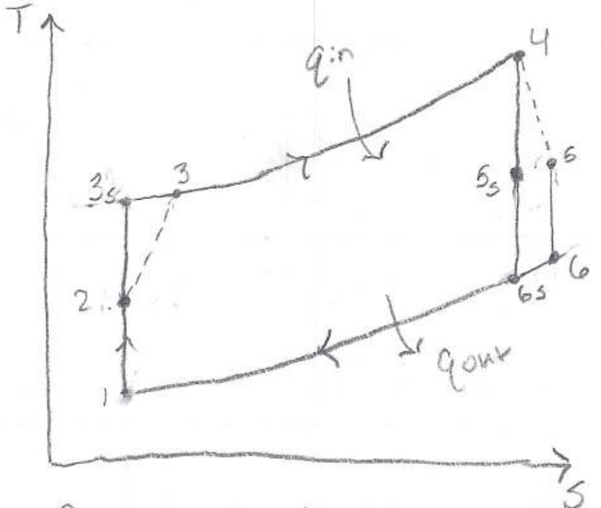
Problem 2

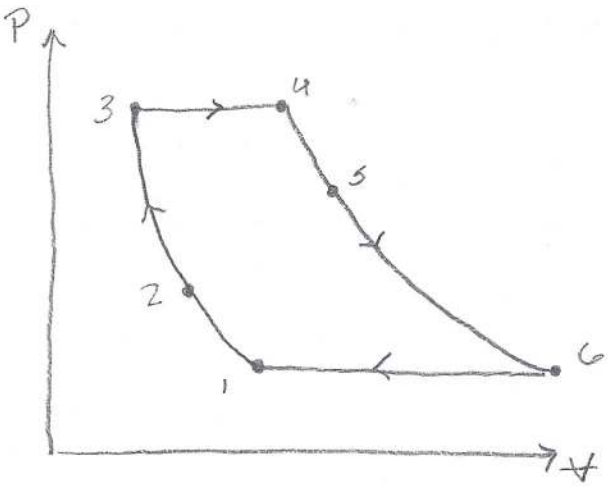
Purpose:

Given the set of conditions, I must find the pressure of the combustion gasses at the turbine exit. Aswell as the velocity of the gasses at the nozzle exit, And lastly the thrust produced if the diffuser inlet is 1.6m.

Diagrams:







Process 1→2) Diffuser, Isentropic compression

Process 2→3) Isentropic compression, Wcin

Process 3→4) Isobaric heat addition

Process 4→5) Turbine, Isentropic expansion

Process 5→6) Nozzle, Isentropic expansion

Sources: Cengel & Boles & Kanoglu
Thermodynamics An Engineering Approach.
9th edition Mcgraw Hill, 2019

Design considerations:

I assume the following:

1) Air behaves as an Ideal gas
2) Variable specific heats
3) There are no heat losses due to mechanical components

Data & Variables:

$V_{aircraft} = 900 \ km/h$
$T_1 = -35^{\circ C} = 238.15 \ K$
$P_1 = 40 \ kpa$
$T_4 = 950^{\circ C} = 1223.15 \ K$

$w_T = 500 \ kJ/kg$
$\eta_c = 0.8$
$\eta_T = 0.9$

Materials:

Air as an Ideal gas

Procedure & Calculations:

I first need to find the states of air at each process.

– Finding Stage 1 states

$T_1 = 238.15\ K$

$P_1 = 40\ Kpa$

T	h
230	230.02
238.15	h_1
240	240.02

$$\frac{238.15 - 230}{240 - 230} = \frac{h_1 - 230.02}{240.02 - 230.02}$$

$h_1 = 238.17\ KJ/kg$

T	P_r
230	0.5477
238.15	P_{r_1}
240	0.6355

$$\frac{238.15 - 230}{240 - 230} = \frac{P_{r_1} - 0.5477}{0.6355 - 0.5477}$$

$P_{r_1} = 0.6193$

– Process 1 → 2) Isentropic Compression

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2} \qquad V_2 = 0$$

$$V_1 = 900\ km/h \left(\frac{1\ m/s}{3.6\ km/h}\right) = 250\ m/s$$

$$h_2 = h_1 + \frac{V_1^2}{2} = 238.17\ KJ/kg + \frac{(250\ m/s)^2}{2}\left(\frac{1\ KJ/kg}{1000\ m^2/s^2}\right)$$

$$h_2 = 238.17\ KJ/kg + 31.25\ KJ/kg = 269.42\ KJ/kg$$

– Process 2 → 3) Isentropic compression

$W_c = \frac{W_s}{0.8} \qquad W_s = 500\ KJ/kg$

$$W_c = \frac{500\ KJ/kg}{0.8} = 625\ KJ/kg$$

$W_c = h_3 - h_2 \qquad h_3 = W_c + h_2$

$$h_3 = 625\ KJ/kg + 269.42\ KJ/kg = 894.42\ KJ/kg$$

- States of air at Stage 4

$T_4 = 1223.15\,K$

T	h
1220	1301,31
1223,15	h_4
1240	1324,93

$$\frac{1223.15 - 1220}{1240 - 1220} = \frac{h_4 - 1301.31}{1324.93 - 1301.31}$$

$h_4 = 1305.03\ kJ/kg$

- Process 4→5) Turbine

$W_T = 500\ kJ/kg = h_4 - h_5$

$h_5 = h_4 - W_T = 1305.03\ kJ/kg - 500\ kJ/kg$

$h_5 = 805.03\ kJ/kg$

- Process 5→6) Nozzle

$$\frac{P_{r3s}}{P_{r1}} = \frac{P_{r4}}{P_{r6s}} = r_p = \frac{P_{max}}{P_{min}}$$

- Solving for pmax

$W_{cs} = 500\ kJ/kg$

$W_{cs} = h_{3s} - h_2 \qquad h_{3s} = 500\ kJ/kg + 238.17\ kJ/kg = 738.17\ kJ/kg$

h	Pr
734,82	32.02
738.17	P_{r3s}
745,62	33.72

$$\frac{738.17 - 734.82}{745.62 - 734.82} = \frac{P_{r3s} - 32.02}{33.72 - 32.02}$$

$P_{r3s} = 32.55$

$$\frac{P_{r3s}}{P_{r1}} = \frac{P_3}{P_1} \qquad P_3 = \frac{P_{r3s}}{P_{r1}} P_1 = \frac{32.55}{0.6193} \cdot 40\ kPa = 2102.37\ kPa$$

h	Pr
1301.31	254.7
1305.03	P_{r4}
1324.93	272.3

$$\frac{1305.03 - 1301.31}{1324.93 - 1301.31} = \frac{P_{r4} - 254.7}{272.3 - 254.7}$$

$P_{r4} = 257.47$

$$\frac{P_{r3s}}{P_{r1}} = \frac{P_{r4}}{P_{r6s}} \qquad P_{r6s} = \frac{P_{r4}}{r_p} = \frac{257.47}{52.56} = 4.9$$

$Pr_{6s} = 4.9$

P_r	h
4.522	421.26
4.9	h_{6s}
4.915	431.43

$$\frac{4.9 - 4.522}{4.915 - 4.522} = \frac{h_{6s} - 421.26}{431.43 - 421.26}$$

$$h_{6s} = 431.04 \text{ kJ/kg}$$

Assumption: Nozzle work is the same between an Isentropic turbine process and the turbine process after efficiency. $w_{Ns} = w_N$

$W_T = \eta_T W_{Ts}$ $\quad W_{Ts} = \frac{W_T}{\eta_T} = \frac{500}{0.9} = 555.55 \text{ kJ/kg}$

$$W_{Ns} = (h_4 - h_{6s}) - W_{Ts} = (1305.03 - 431.04) - 555.55$$

$$W_{Ns} = 318.44$$

$$W_{Ns} = w_N = h_5 - h_6 \qquad h_6 = h_5 - w_N$$

$$h_6 = 805.03 \text{ kJ/kg} - 318.44 \text{ kJ/kg} = 486.59 \text{ kJ/kg}$$

- Solving questions

a) The pressure of combustion gasses at turbine exit

$$P_4 = P_{max} = \boxed{2102.37 \text{ Kpa}}$$

b) The velocity of gasses at nozzle exit

$$h_6 + \frac{V_6^2}{2} = h_5 + \frac{V_5^2}{2} \qquad V_5 = 0$$

$$V_6 = \sqrt{2 \cdot h_5 - h_6} = \sqrt{2 \cdot (805.03 - 486.59) \text{ kJ/kg} \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}}\right)}$$

$$\boxed{V_6 = 798.05 \text{ m/s}}$$

c) the Thrust for this engine if difuser inlet diameter is 1.6 m

$$A = \pi \frac{d}{2}^2 = \pi \frac{1.6\,m}{2}^2 = 2.01\ m^2$$

$$F = \dot{m}(V_{exit} - V_{inlet})$$

$$F = \frac{V_1 A}{v_1}(V_{exit} - V_{inlet})$$

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287\left(\frac{kJ}{kg\cdot K}\right)\left(\frac{1\ kPa\cdot m^3}{1\ kJ}\right) 238.15\ K}{40\ kPa}$$

$$v_1 = 1.709\ m^3/kg$$

$$F = \frac{250\ m/s \cdot 2.01\ m^2}{1.709\ m^3/kg}(798.05 - 250)\ m/s$$

$$= 161144.017\ \frac{kg\cdot m}{s^2} = 161144.017\ N$$

$$\boxed{F = 161.14\ KN}$$

Summary:

Overall I believe all numbers came out reasonable. The only part im concerned about is the assumption I made for the Isentropic nozzle work and the actual nozzle work. The result of the velocity leaving the nozzle makes me think I should be close if anything.

Analysis:

The final thrust of this motor with a 1.6 m inlet seems pretty decent for a lightweight Jet. Of course in real life there would be alot more losses due to mechanical components and heat.