

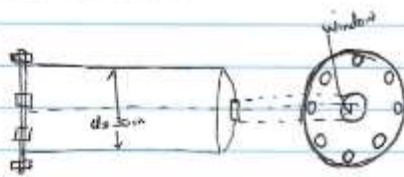
## **Homework 2.1**

This week was a short week because one of our class days was canceled due to a holiday, however we still learned a lot from the lecture we did have and the examples. We learned how fluids act as a force on flat surfaces, angled surfaces, and curved surfaces. The examples showed multiple ways and orientations of how fluids apply force to a surface or wall. We also learned how to find the center point of the moment of force acting on those surfaces. Another topic that was discussed this week was buoyancy. Part of this was learning the equations related to buoyancy and how to manipulate the equations to find information on the properties of the floating object and the fluid.

## Homework Problems

4.2

### Problem 4.2



Calculate the total force that must be resisted by the bolts in the flange.

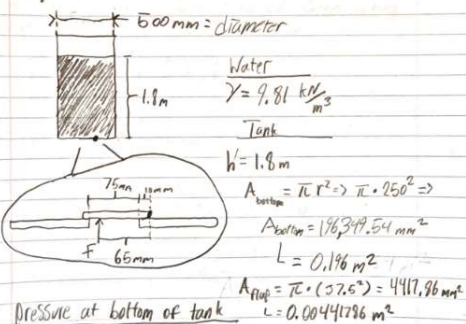
$$F = P \times A$$

$$= 14.4 \text{ in}^2 \times \frac{\pi (30 \text{ in})^2}{4}$$

$$F = 10,178.76 \text{ lb}$$

4.10

4-10) A simple shower for remote locations is designed with a cylindrical tank 500 mm in diameter and 1.8 m high. The water flows through a flapper valve in the bottom through a 75 mm opening. The flapper must be pushed upwards to open the valve. How much force is required to open the valve?



Pressure at bottom of tank

$$\{P = \gamma_{\text{water}} \cdot h\} \Rightarrow P_{\text{bot}} = 9.81 \frac{\text{kN}}{\text{m}^3} \cdot 1.8 \text{ m}$$

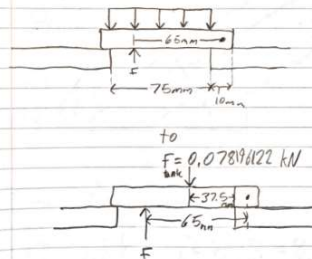
$$\Rightarrow P_{\text{bot}} = 17.7 \text{ kPa}$$

Force acting on unsupported portion of valve

$$\{F = P \cdot A\} \Rightarrow F = P_{\text{bot}} \cdot A_{\text{flap}} = 17.2 \frac{\text{kN}}{\text{m}^2} \cdot 0.00441786 \text{ m}^2$$

$$F_{\text{bot}} = 0.078196122 \text{ kN}$$

• Replace distributed load with singular force acting at the center of the flapper valve



Solve for moments about the pin of the valve

$$\sum M = 0 = (F_{\text{bot}} \cdot 37.5 \text{ mm}) - (F \cdot 65 \text{ mm})$$

$$\Rightarrow F = (0.078196122 \text{ kN} \cdot 37.5 \text{ mm}) / 65 \text{ mm}$$

$$F = 0.0451131479 \text{ kN} = 45.1 \text{ Newtons}$$

4.17

Problem 4.17

If the wall is 4m long:

- Calculate the total force due to the oil pressure
- Location of the center of pressure
- Show the resultant point force on the wall



Slanting depth  $L = \frac{h}{\sin \theta}$   
 $L = \frac{1.4}{\sin 45^\circ} = 1.98 \text{ m}$

$$F_R = \gamma (h_{ca}) A = (0.86 \times 9806 \text{ N/m}^3 \times 1.4 \text{ m}) \left( \frac{4 \text{ m}}{2} \right) (1.98 \text{ m})$$

$$F_R = 46772.51 \text{ N}$$

Location of center of pressure  
 $h_p = h/3 = 1.4 \text{ m} / 3$   $h_p = 0.47 \text{ m}$  from the bottom (vertical)

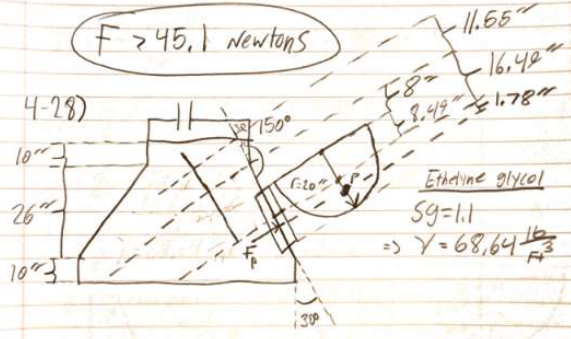
Measure along the wall,  $L_p = \frac{L}{3} = \frac{1.98 \text{ m}}{3} = 0.66 \text{ m}$

$L_p = 0.66 \text{ m}$  (from the bottom along the wall)

4.28

• Since the equation is balanced when force acting to open the flapper is 45.1 newtons, the force must be greater than that to actually open the flapper valve

$F > 45.1 \text{ newtons}$



Centroid of a half circle  
 $\{ \bar{y} = \frac{4r}{3\pi} \} \Rightarrow \bar{y} = \frac{(4 \cdot 20)}{(3 \cdot \pi)} = 8.49''$

Distance to Centroid  
 $L_c = 16.49'' + \left( 10'' \cdot \frac{\sin(90^\circ)}{\sin(60^\circ)} \right) = 28.04''$

Distance to Center of Pressure  
 $\{ L_p = L_c + \frac{I_c}{L_c \cdot A} \} \quad \{ I_c = \frac{\pi \cdot r^4}{4} \}$   
 $\Rightarrow I_c = \frac{\pi \cdot (10'')^4}{4} = 7853.98 \text{ in}^4$   
 $\Rightarrow L_p = 28.04'' + \left( \frac{7853.98 \text{ in}^4}{28.04'' \cdot \left( \frac{\pi \cdot 10'^2}{2} \right)} \right)$   
 $L_p = 28.04'' + 1.78'' = 29.82''$

$h_p = L_p \sin(\theta) = 29.82'' \sin(30^\circ)$   
 $\Rightarrow h_p = 14.91''$

$A_{flap} = 157.08 \text{ in}^2$

Solving for force acting at the Center of Pressure  
 $\{ P_p = h_p \cdot \gamma \} \quad \{ F = P \cdot A_{flap} \}$   
 $P_p = 14.91'' \cdot 0.036 \frac{\text{lb}}{\text{in}^2} = 0.53676 \frac{\text{lb}}{\text{in}^2} \text{ (PSI)}$   
 $F = 0.53676 \frac{\text{lb}}{\text{in}^2} \cdot 157.08 \text{ in}^2 = 84.31 \text{ lbs}$

4.41

Problem 4.41

or Compute the net force on the gate due to the fluid on each side. & compute the force on the hinge & on the support.

Force due to the water:  $F_w = \gamma_w(h_w)A$

$$= 9810 \text{ N/m}^3 \times 2.5 \text{ m} \times (2.0 \text{ m} \times 0.6 \text{ m})$$

$$F_w = 18378.5 \text{ N}$$

Force due to the oil:  $F_o = \gamma_o(h_o)A$

$$= (9810 \times 0.90) \left(\frac{2}{2}\right) (2.0 \text{ m} \times 0.6 \text{ m})$$

$$F_o = 10594.8 \text{ N}$$

Moment about H:  $F_w\left(\frac{2}{3}\right) - F_o(2.0) - F_h(2.0) = 0$

$$F_h = \frac{18378.5 \left(\frac{2}{3}\right) - 10594.8 \times 2.0}{2.0}$$

$$F_h = 2951.76 \text{ N}$$

Moment about the support:  $F_h(2.0) + F_o(2.0) - F_w(2.0) = 0$

$$F_h = \frac{18378.5 \times 2.0 - 10594.8 \times 2.0}{2.0}$$

$$F_h = 4850.6 \text{ N}$$

4.54

4.54) Surface length =  $60'' = w$

Area:  $S_G = 0.78$

$\gamma_o = 49.3 \text{ lb/ft}^3 = 0.02853 \text{ lb/in}^3$

$A = 3392.92 \text{ in}^2$

Find  $F_w, F_o, F_h, F_R$

Length of Surface(s) and Area (A) of Surface

$$\{A_{\text{face}} = S \cdot w\} \text{ \& \; } \{S = \pi r^2\}$$

$$S = \pi \cdot 18'' = 56.54''$$

$$A = 56.54'' \cdot 60'' = 3392.92 \text{ in}^2$$

Centroid of Semi-circle and depth of Centroid

$$\bar{X} = \frac{4r}{3\pi} = 7.64''$$

$$\Rightarrow h_c = 48'' + 18'' = 66''$$

Depth of Center of Pressure

$$\{h_{cp} = \left(\frac{I_{xc}}{A \cdot h_c}\right) + h_c\} \text{ \& \; } \{I_c = \frac{\pi \cdot r^4}{12}\}$$

$$I_c = \frac{\pi \cdot 18^4}{12} = 90371.33 \text{ in}^4$$

Depth of Center of Pressure & force acting at it

$$\left(\frac{90371.33 \text{ in}^4}{66'' \cdot 3392.92 \text{ in}^2}\right) = 4.04''$$

$$\Rightarrow 4.04'' + 66'' = 70.04'' = h_p$$

Force due to water:  $F_w = \gamma_w \cdot h_w \cdot A$

$$F_w = 6279.46 \text{ lbs}$$

Force due to oil:  $F_o = \gamma_o \cdot h_o \cdot A$

$$F_o = 6835.21 \text{ lbs}$$

Resultant force:  $F_R = \sqrt{F_w^2 + F_o^2}$

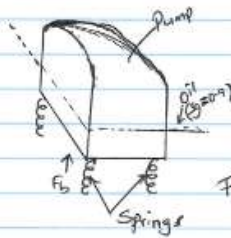
$$F_R = 871.2 \text{ lbs}$$

Angle:  $\theta = \tan^{-1}\left(\frac{F_o}{F_w}\right)$

$$\theta = \tan^{-1}\left(\frac{6835.21}{6279.46}\right) = 7.32^\circ$$

5.8

Problem 5.8



Pump partially submerged in oil and supported by springs. If the net weight of the pump is 14.6 lb and the submerged volume is 40 in<sup>3</sup>, calculate the supporting force exerted by the springs.

(upward)  
Force exerted by the displaced fluid  
 $F_b = \gamma_f V_f$

$$= (0.9 \times 62.4 \frac{\text{lb}}{\text{ft}^3}) (40 \text{ in}^3) (\frac{1 \text{ ft}^3}{12^3 \text{ in}^3})$$

$$= 1.3 \text{ lb}$$

Supporting force = Weight of the pump - Buoyancy force

$$= 14.6 \text{ lb} - 1.3 \text{ lb}$$

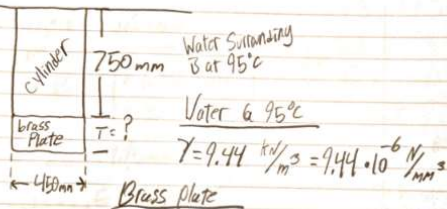
$$= \underline{\underline{13.3 \text{ lb}}}$$

5.24

6-24) A brass weight is to be attached to the cylinder as seen below. The cylinder will be completely submerged and neutrally buoyant in water at 95°C. The brass is to be a cylinder with the same diameter as the original cylinder shown below.

What is the required thickness of the brass?

Water Surface



Brass plate  
 $\gamma_{\text{brass}} = 84.0 \frac{\text{kN}}{\text{m}^3} = 8.4 \cdot 10^{-5} \frac{\text{N}}{\text{mm}^3}$

$$A = \pi \cdot r^2 = \pi \cdot (225)^2 = 159043.13 \text{ mm}^2$$

$$V_{\text{cyl}} = A \cdot T = (159043.13 \cdot T) \text{ mm}^3$$

$$V_{\text{brass}} = (8.4 \cdot 10^{-5}) \cdot (159043.13 \cdot T)$$

$$\{W_{\text{brass}} = F_b\} \Leftrightarrow \{F_b = \gamma_w \cdot V_{\text{displaced}}\}$$

$$\Rightarrow (\gamma_{\text{brass}} \cdot V_{\text{brass}}) = (\gamma_{\text{water}} \cdot V_{\text{displaced}})$$

$$8.4 \cdot 10^{-5} \frac{\text{N}}{\text{mm}^3} \cdot (159043.13 \text{ mm}^2 \cdot t)$$

$$= 9.44 \cdot 10^{-6} \frac{\text{N}}{\text{mm}^3} \cdot (159043.13 \text{ mm}^2 \cdot (750 + t))$$

$$\Rightarrow (13.36 \frac{\text{N}}{\text{mm}} \cdot t \text{ mm}) = (1.501 \frac{\text{N}}{\text{mm}} \cdot (750 + t))$$

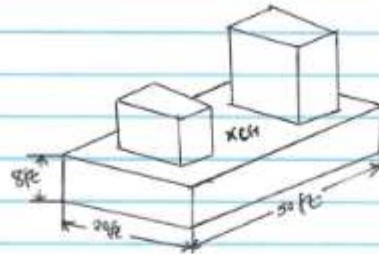
$$13.36 \cdot t = 1126.03 + (1.501 \cdot t)$$

$$\Rightarrow 11.859 \cdot t = 1126.03$$

$$\Rightarrow t = \frac{1126.03}{11.859} = \boxed{94.95 \text{ mm}}$$

$$13.36 \frac{\text{N}}{\text{mm}} \cdot 94.95 \text{ mm} = 1.501 \frac{\text{N}}{\text{mm}} \cdot (844.95 \text{ mm})$$

$$1268.6 = 1268.3$$

Problem 5.41

Total weight = 450,000 lb

CG is only of platform 8 ft from the bottom

Will the platform be stable? in seawater?

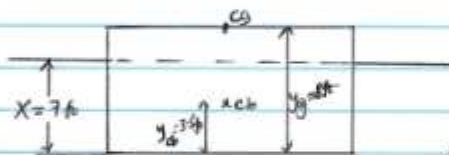
$$\gamma = 1.030$$

$$F_b = \gamma_v V_f = W$$

$$62.4 \times 1.03$$

$$(62.4 \text{ lb/ft}^3 \times 1.03) \times (20 \text{ ft} \times 50 \text{ ft} \times h) = 450,000 \text{ lb}$$

$$h = 7.00 \text{ ft.}$$



$$L = 50 \text{ ft}$$

$$B = 20 \text{ ft}$$

$$X = 7 \text{ ft.}$$

$$MB = \frac{3}{8} V_d$$

$$V_d = (50 \times 20 \times 7) \text{ ft}^3 = 7000 \text{ ft}^3$$

$$I = \frac{LB^3}{12} = \frac{50 \text{ ft} \times (20 \text{ ft})^3}{12} = \frac{(100,000)}{3} \text{ ft}^4$$

$$MB = \frac{100,000 \text{ ft}^4}{3} \div 7000 \text{ ft}^3 = 4.76 \text{ ft.}$$

$$y_{mc} = y_{cb} + MB$$

$$= 3.5 \text{ ft} + 4.76 \text{ ft}$$

$$\underline{y_{mc} = 8.26 \text{ ft.}}$$

$$y_{mc} = 8.26 \text{ ft.}$$

$$y_{cg} = 8 \text{ ft}$$

In this case  $y_{mc} > y_{cg}$ The platform is stable

5.61

5-61) A boat is shown below. Its geometry at the water line is the same as the top surface. The hull is solid.

Is the boat stable?

A floating body is stable if meta-center is above the center of gravity, and can be quantified using MB

$\{MB = I / V_d\}$

Volume displaced

Rectangle:  $0.9m \Rightarrow A_{rectangle} = 2.16m^2$

Triangle:  $0.6m \Rightarrow A_{triangle} = \frac{0.6 \cdot 2.4}{2} = 0.72m^2$

$\Rightarrow$  Cross Sectional Area Submerged  $= A_D + A_\Delta = 2.16 + 0.72 = 2.88m^2$

$\Rightarrow$  Volume displaced  $= 2.88m^2 \cdot 5.5m = 15.84m^3$

Calculating Center of gravity

Rectangle:  $\bar{x} = 1.2m, \bar{y} = 0.6m, A = 2.88m^2$

Triangle:  $\bar{x} = 1.2m, \bar{y} = 0.2m + 1.2 = 1.4m, A = 0.72m^2$

|           | $\bar{x}$ | $\bar{y}$ | A                  | $\bar{x}A$          | $\bar{y}A$          |
|-----------|-----------|-----------|--------------------|---------------------|---------------------|
| Rectangle | 1.2m      | 0.6m      | 2.88m <sup>2</sup> | 3.456m <sup>3</sup> | 1.728m <sup>3</sup> |
| Triangle  | 1.2m      | 1.4m      | 0.72m <sup>2</sup> | 0.864m <sup>3</sup> | 1.008m <sup>3</sup> |
| Composite |           |           | 3.6m <sup>2</sup>  | 4.32m <sup>3</sup>  | 2.736m <sup>3</sup> |

$\Rightarrow \bar{X} = \frac{4.32m^3}{3.6m^2} = 1.2m, \bar{Y} = \frac{2.736m^3}{3.6m^2} = 0.76m$

depth from water level to center of gravity

$0.76m - 0.3m = 0.46m$

buoyancy force acting on the boat

$\{F_b = \gamma_{water} \cdot V_d\} \Rightarrow F_b = 9.44 \frac{kN}{m^3} \cdot 15.84m^3$

$\Rightarrow F_b = 149.53kN \Rightarrow W = 149.53kN$

Calculating for center of buoyancy

Rectangle:  $0.9m \Rightarrow \bar{x} = 1.2m, \bar{y} = 0.45m, Area = 2.16m^2$

Triangle:  $0.6m \Rightarrow \bar{x} = 1.2m, \bar{y} = \frac{1}{3} = 0.2m + 0.9 = 1.1m, Area = 0.72m^2$

|           | $\bar{x}$ | $\bar{y}$ | A                  | $\bar{x}A$          | $\bar{y}A$          |
|-----------|-----------|-----------|--------------------|---------------------|---------------------|
| Rectangle | 1.2m      | 0.45m     | 2.16m <sup>2</sup> | 2.592m <sup>3</sup> | 0.972m <sup>3</sup> |
| Triangle  | 1.2m      | 1.1m      | 0.72m <sup>2</sup> | 0.864m <sup>3</sup> | 0.792m <sup>3</sup> |
| Composite |           |           | 2.88m <sup>2</sup> | 3.456m <sup>3</sup> | 1.764m <sup>3</sup> |

$\bar{X}_{Composite} = \frac{3.456m^3}{2.88m^2} = 1.2m$

$\bar{Y}_{Composite} = \frac{1.764m^3}{2.88m^2} = 0.6125m$

Calculating Composite moment of inertia

$\{I = I_{rec} + I_{tri}\} \left\{ I_{rec} = \frac{L \cdot w^3}{12} \right\} \left\{ I_{tri} = \frac{L \cdot (\frac{w}{2})^3}{12} \right\}$

$L = 5.5m, w = 2.4m$

$I_{rec} = \frac{(5.5) \cdot (2.4)^3}{12} = 6.336m^4$

$I_{tri} = \frac{5.5 \cdot (1.2)^3}{12} = 0.792m^4$

$\Rightarrow I = 7.128m^4$

Metacenter depth

$\{MB = I / V_d\} I = 7.128m^4, V_d = 15.84m^3$

$\Rightarrow MB = 7.128m^4 / 15.84m^3 = 0.45m$

The meta-center is 0.1m above the center of gravity, therefore the boat is stable