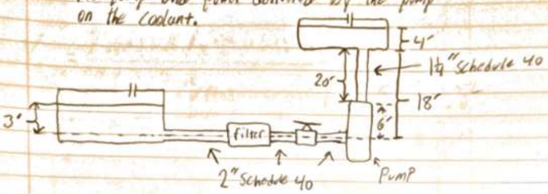
Homework 3.2

The problems discussed this week were a continuation of series and parallel piping systems. We continued our discussion on how to tackle the problems and ended up creating a new way to do them, dubbed “Kyle’s Method”. We also began talking about how to select pumps and why pump choice is an important factor in piping systems. One of the factors that was discussed was how the RPM of a pump affects the flowrate of a liquid.

Homework Problems

11.23

11-23) For the system below, Compute total head on the pump and power delivered by the pump on the coolant.



$L = 10'$

Pump

$Q = 30\ \frac{gal}{min} = 6930\ \frac{in^3}{min} = 115.5\ \frac{in^3}{s}$

$\Rightarrow Q = 0.07\ ft^3/s$

Coolant

$SG = 0.92 \Rightarrow \gamma = 57.4\ \frac{lb}{ft^3}$

$\eta = 3.6 \cdot 10^{-6}\ \frac{lb \cdot s}{ft^2}$

2" Schedule 40

$D_I = 2.067'' \Rightarrow A_I = 3.36\ in^2 = 0.0233\ ft^2$

$1\frac{1}{4}$ Schedule 40

$D_I = 1.38'' \Rightarrow A_I = 1.50\ in^2 = 0.01\ ft^2$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + P_{del} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L \Big\} \Big\{ Q = V \cdot A \Big\} \Big\{ h_L = f \cdot \frac{L}{D} \cdot \frac{V^2}{2g} \Big\}$$

$$V_{2'' pipe} = 0.07\ ft^3/s \,/\, 0.0233\ ft^2 = 3\ ft/s = V_1 \quad \Big\{ N_R = \frac{V \cdot D \cdot \rho}{\eta} \Big\}$$

$$V_{1\frac{1}{4}'' pipe} = 0.07\ ft^3/s \,/\, 0.01\ ft^2 = 7\ ft/s = V_2 \quad \Big\{ f = \frac{64}{N_R} \text{ if laminar} \Big\}$$

$$\rho = \gamma / g \Rightarrow 57.4\ lb/ft^3 \,/\, 32\ ft/s = 1.79\ \frac{lb \cdot s}{ft^4}$$

$$N_{R_{2''}} = \frac{3\ \frac{ft}{s} \cdot (2.067''/12\ \frac{inch}{ft}) \cdot 57.4\ \frac{lb}{ft^3}}{3.6 \cdot 10^{-5}\ \frac{lb}{ft \cdot s}} = 823929 \quad \Big\{ \frac{ft}{s} \cdot ft \cdot \frac{lb}{ft^3} \cdot \frac{ft}{lb \cdot s} \Big\}$$

$$N_{R_{1\frac{1}{4}''}} = \frac{7\ ft/s \cdot (1.38''/12\ \frac{inch}{ft}) \cdot 57.4\ \frac{lb}{ft^3}}{3.6 \cdot 10^{-5}\ lb/ft \cdot s} \Big\{ \frac{ft}{s} \cdot ft \cdot \frac{lb}{ft^3} \cdot \frac{ft \cdot s}{lb} \Big\} = 1283527.8$$

$\Rightarrow f_{2''} = 0.0185 \quad f_{1\frac{1}{4}''} = 0.016$

$$\Rightarrow h_{L_{2''}} = 0.0185 \cdot \left(\frac{120''}{2.067''}\right) \cdot \left(\frac{36\ \frac{in}{s}^2}{(2 \cdot 386\ \frac{in}{s^2})}\right)$$

$\Big\{ \frac{in^2}{s^2} \cdot \frac{s^2}{in} = in \Big\}$ $h_{L_{2''}} = 1.8'' = 0.15'$

$$\Rightarrow h_{L_{1\frac{1}{4}''}} = 0.016 \cdot \left(240''/1.38''\right) \cdot \left(84\ \frac{in}{s}^2 / 772\ \frac{in}{s^2}\right)$$

$h_{L_{1\frac{1}{4}''}} = 25.4'' = 2.11'$

$\Rightarrow h_{L_{tot}} = 27.2'' = 2.27'$ $\quad \{ h_{L_{tot}} = P_{del} \}$ $\Rightarrow \boxed{P_{del} = 27.2'' = 2.27'}$ = Power delivered

$P_1 = 57.4\ \frac{lb}{ft^3} \cdot 3\ ft = 172.2\ lb/ft^2 = 1.2\ PSI\ (\frac{lb}{in^2})$

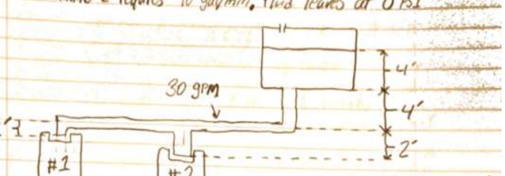
$P_2 = 57.4\ \frac{lb}{ft^3} \cdot 22\ ft = 1262.8\ lb/ft^2 = 8.77\ PSI\ (\frac{lb}{in^2})$

$\Rightarrow$ total head at pump $= \frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + P_{del}$

$$\Rightarrow \frac{172.2\ \frac{lb}{ft^2}}{57.4\ \frac{lb}{ft^3}} + \frac{3\ \frac{ft}{s}^2}{(2 \cdot 32\ \frac{ft}{s^2})} + 3\ ft + 2.27\ ft = \boxed{8.41\ ft}$$ = head at inlet of pump

11.24

11-24) For the system in the previous problem, specify the size pipes necessary to return coolant to the machines. Machine 2 requires 20 gal/min & machine 2 requires 10 gal/min. Fluid leaves at 0 PSI



$Q = 30\ \frac{gal}{min} = 6930\ \frac{in^3}{min} = 115.5\ \frac{in^3}{s} = 0.07\ \frac{ft^3}{s} = 4.2\ \frac{ft^3}{min}$

- Velocity must remain constant, therefore area of the pipes must change to achieve desired flow rates

$\Rightarrow 0.07\ \frac{ft^3}{s} = 3\ \frac{ft}{s} \cdot A \Rightarrow A = 0.0233\ ft^2 \Rightarrow$ 2" Schedule 40

$20\ \frac{gal}{min} = 4620\ \frac{in^3}{min} = 77\ \frac{in^3}{s} = 0.045\ \frac{ft^3}{s}$

$\Rightarrow 0.045\ \frac{ft^3}{s} = 3\ \frac{ft}{s} \cdot A \Rightarrow A = 0.015\ ft^2 \Rightarrow 1\frac{1}{2}''$ Schedule 40 pipe

$10\ \frac{gal}{min} = 2310\ \frac{in^3}{min} = 38.5\ \frac{in^3}{s} = 0.0222\ \frac{ft^3}{s}$

$\Rightarrow 0.0222\ \frac{ft^3}{s} = 3\ \frac{ft}{s} \cdot A \Rightarrow A = 0.0074 \Rightarrow 1''$ Schedule 40 pipe

Main pipe: 2" schedule 40
to #1 : $1\frac{1}{2}$" Schedule 40
to #2 : 1" schedule 40

12.3

12.3

$$\frac{D}{\varepsilon} = \frac{0.0525\ m}{4.6\times10^{-5}\ m} = 1.141\times10^{3}$$

MOODY'S $\rightarrow \approx 0.019$ for f_2 (and f_1)

$$\left[571.43 f_a + 0.7\right]\left(\frac{V_a^2}{2g}\right) = \left[1142.86 f_b + 4.66\right]\left(\frac{V_b^2}{2g}\right)$$

$V_b = 0.64\,V_a$ $\quad A = 2.168\times10^{-3}\ m^2$ $\quad 850\ L/min = 0.0141\ m^3/s$

$$Q_T = Q_a + Q_b \rightarrow Q = A_a V_a + A_b V_b$$

$$0.0141 = (2.168\times10^{-3})V_a + (2.168\times10^{-3})(0.64 V_a)$$

$$V_a = 3.97\ m/s$$

$$V_b = 0.64(3.97) = 2.54\ m/s$$

$$N_{R} = \frac{V_b D_b}{\nu} \rightarrow \frac{(2.54)(0.0525)}{1.30\times10^{-6}} = 102576.9 \xrightarrow{D/\varepsilon} f_b = 0.021$$

$$N_R = \frac{V_a D_a}{\nu} = \frac{(3.97)(0.0525)}{1.30\times10^{-6}} = 160326.9 \xrightarrow{D/\varepsilon} f_a = 0.021$$

smaller THAN ASSUMED

$$\left[571.43(0.021) + 0.7\right]\frac{V_a^2}{2g} = \left[1142.86(0.021) + 6.66\right]\frac{V_b^2}{2g}$$

$$V_b = 0.644\,V_a$$

$$0.0141 = (2.168\times10^{-3})V_a + (2.168\times10^{-3})(0.644 V_a)$$

$$V_a = 3.96\ m/s$$

$$V_b = 0.644(3.96) = 2.55\ m/s$$

$$Q_a = A_a V_a = (2.168\times10^{-3})\,3.96 = 0.008585\ m^3/s = 515.15\ L/min$$

$$Q_b = A_b V_b = (2.168\times10^{-3})\,2.55 = 0.0055284\ m^3/s = 331.7\ L/min$$

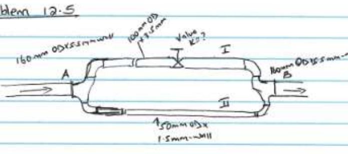
$$h_{L_a} = \left[571.43(0.021) + 0.7\right]\frac{3.96^2}{2g} = 10.15\ m$$

$$\Delta p = \gamma h_L$$

$$9.81\ kN/m^3\,(10.15\ m) = 99.57\ kPa$$

12.5

Problem 12.5



Desired flow rate through the branches = 500 L/min = 0.008 m^3/s

$L = 30\,m$

Water at $10^\circ C$

$\gamma = 9.81\,kN/m^3$

$\nu = 1.30\times10^{-6}\,m^2/s$

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B} + h_{L A-B}$$

$$\frac{P_A}{\gamma} - \frac{P_B}{\gamma} = h_{L A-B} \Rightarrow \frac{\Delta P}{\gamma} = h_{L A-B\,I}$$

$$\Rightarrow \frac{\Delta P}{\gamma} = h_{L A-B\,II}$$

$$h_{L AB\,I} = 2\,Tee_T + h_{L pipe\,I} + 2h_{L elb\,I} + h_{L valve\,I}$$

$$h_{L AB\,II} = 2\,Tee_T + h_{L pipe\,II} + 2h_{L elb\,II}$$

For the pipe network, $h_{L AB\,I} = h_{L AB\,II}$

$$\cancel{2\,Tee_T} + h_{L pipe\,I} + 2h_{L elb\,I} + h_{L valve\,I} = \cancel{2\,Tee_T} + h_{L pipe\,II} + 2h_{L elb\,II}$$

$$LHS = h_{L pipe\,I} + 2h_{L elb\,I} + h_{L valve\,I}$$

$$= f_I\frac{L_I}{D_I}\frac{V_I^2}{2g} + 2K_{elb}\frac{V_I^2}{2g} + K_{valve}\frac{V_I^2}{2g} \qquad V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$LHS = f_I\frac{L_I}{D_I}\frac{8}{g\pi^2 D_I^4}Q_I^2 + 2K_{elb}\frac{8}{g\pi^2 D_I^4}Q_I^2 + K_{valve}\frac{8}{g\pi^2 D_I^4}Q_I^2$$

$$= \left(f_I\frac{L_I}{D_I} + 2K_{elb} + K_{valve}\right)\frac{8}{g\pi^2 D_I^4}Q_I^2$$

$$\text{Similarly, } RHS = \left(f_{II}\frac{L_{II}}{D_{II}} + 2K_{elb}\right)\frac{8}{g\pi^2 D_{II}^4}Q_{II}^2$$

Combining the two equations $\qquad Q_I = Q_{II}$, $L_I = L_{II}$

$$\left(f_I\frac{L_I}{D_I} + 2K_{elb} + K_{valve}\right)\cancel{\frac{8}{g\pi^2}}\frac{1}{D_I^4}\cancel{Q_I^2} = \left(f_{II}\frac{L_{II}}{D_{II}} + 2K_{elb}\right)\cancel{\frac{8}{g\pi^2}}\frac{1}{D_{II}^4}\cancel{Q_{II}^2}$$

~~From the excel document~~

$$\left(f_I\frac{L_I}{D_I} + 2K_{elb} + K_{valve}\right)\frac{1}{D_I^4} = \left(f_{II}\frac{L_{II}}{D_{II}} + 2K_{elb}\right)\frac{1}{D_{II}^4}$$

From the excel document:

$$(6.6926 + 0.9995 + K_{valve})\frac{1}{(0.093)^4} = (13.5353 + 1.17198)\frac{1}{(0.047)^4}$$

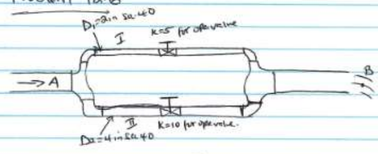
$$(7.69208 + K_{valve})\frac{1}{(0.093)^4} = 3026287.1$$

$$7.69208 + K = 226.382$$

$$K_{valve} = 218.69\,M$$

12.6

Problem 12.6



$P_A = 20$ psig

$K_{elb} = 0.9$

Neglect - Tee losses

- Pipe losses

water $\gamma = 62.4$ lb/ft³

$D_1 = 0.1723$ ft

$D_2 = 0.3355$ ft

$$\frac{P_A}{\gamma} + \cancel{\frac{V_A^2}{2g}} + \cancel{z_A} = \cancel{\frac{P_B}{\gamma}}^{0} + \cancel{\frac{V_B^2}{2g}} + \cancel{z_B} + h_{L_{A-B}}$$

$$\frac{P_A}{\gamma} = h_{L_{A-B}} \Rightarrow \frac{P_A}{\gamma} = h_{L_{A-BI}}$$

$$\Rightarrow \frac{P_A}{\gamma} = h_{L_{A-BII}}$$

~~$h_{LA-B} = 2k_{elb}$~~

$$h_{L_{A-BI}} = 2h_{L_{elb_I}} + h_{L_{valve_I}} = 2K_{elb}\frac{V_I^2}{2g} + K_{valve}\frac{V_I^2}{2g}$$

$$V^2 = \frac{16Q^2}{\pi^2 D^4}$$

$$h_{L_{AB\,II}} = 2h_{elb_{II}} + h_{L_{valve_{II}}} = 2K_{elb}\frac{V_{II}^2}{2g} + K_{valve}\frac{V_{II}^2}{2g}$$

$$h_{L_{A-BI}} = (2K_{elb} + K_{valve})\frac{8}{g\pi^2 D_I^4}Q_I^2 = (2\times0.9+5)\frac{8}{g\pi^2 D_I^4}\cdot Q_I^2$$

$$h_{L_{A-BII}} = (2K_{elb} + K_{valve})\frac{8}{g\pi^2 D_{II}^4}Q_{II}^2 = (2\times0.9+10)\frac{8}{g\pi^2 D_{II}^4}Q_{II}^2$$

$$\frac{P_A}{\gamma} = \left(\frac{20\times144\ lb/ft^2}{62.4\ lb/ft^3}\right) = 46.15\ ft.$$

To evaluate for Q_I

$$46.15\ ft = (6.8)\cdot\frac{8}{g\pi^2(0.1723\ ft)^4}Q_I^2$$

$$Q_I = \sqrt{\frac{46.15\times g\times\pi^2\times(0.1723)^4}{6.8\times8}} = \underline{\underline{0.487\ ft^3/s}}$$

To evaluate for Q_{II}

$$46.15\ ft = (11.8)\cdot\frac{8}{g\pi^2(0.3355)^4}\cdot Q_{II}^2$$

$$Q_{II} = \sqrt{\frac{46.15\times g\times\pi^2\times0.3355^4}{11.8\times8}} = \underline{\underline{1.403\ ft^3/s}}$$

Total flow rate if:

a) Both valves open

$$Q_T = Q_I + Q_{II}$$

$$= 0.487 + 1.403$$

$$\underline{\underline{Q_T = 1.89\ ft^3/s}}$$

b) Valve II open (only)

$$Q_T = Q_{II}$$

$$Q_T = \underline{\underline{1.403\ ft^3/s}}$$

c) Valve I open (only)

$$Q_T = Q_I$$

$$\underline{\underline{Q_T = 0.487\ ft^3/s}}$$