

- a. Matrix M3 is created based on  $f_3(f_2(f_1(k),p),k)=p$ ; for all  $k, p$  with  $1 \leq k, p \leq N$  resulting in a 5x5 matrix where each row and column represents the integers 1, 2, 3, 4, and 5 satisfying the specified condition.

$$f_1(1) = 5, f_1(2) = 4, f_1(3) = 2, f_1(4) = 3, \text{ and } f_1(5) = 5$$

M2:

- $f_2(1,1) = 5, f_2(1,2) = 2, f_2(1,3) = 3, f_2(1,4) = 4, \text{ and } f_2(1,5) = 1$
- $f_2(2,1) = 4, f_2(2,2) = 2, f_2(2,3) = 5, f_2(2,4) = 1, \text{ and } f_2(2,5) = 3$
- $f_2(3,1) = 1, f_2(3,2) = 3, f_2(3,3) = 2, f_2(3,4) = 4, \text{ and } f_2(3,5) = 5$
- $f_2(4,1) = 3, f_2(4,2) = 1, f_2(4,3) = 4, f_2(4,4) = 2, \text{ and } f_2(4,5) = 5$
- $f_2(5,1) = 2, f_2(5,2) = 5, f_2(5,3) = 3, f_2(5,4) = 4, \text{ and } f_2(5,5) = 1$

M3:

5	2	4	1	5
1	4	2	3	2
3	1	5	2	3
4	3	1	4	4
2	5	3	5	1

- b. M1 takes "k" as an input and displays the "x" as output. Then M2 takes "x" and "p" as the input and displays the "z" as output. M3 takes "z" and "k" as the input and displays the "p" as the output. User1 picks a random private key "k" and then gets "x" by mapping "k" in M1 and then sends the obtained public key "x" to User2. User2 gets the cipher text "z" by encrypting "p" by using "x" with M2. The receiver User1 decrypts "z" by using "k" with and M3.