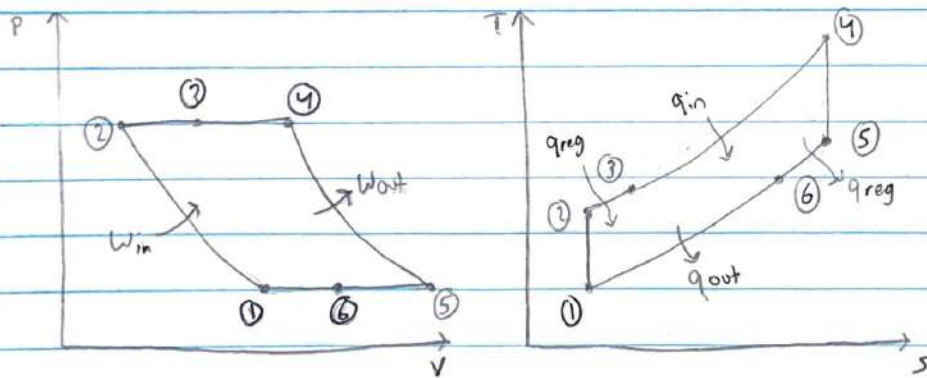
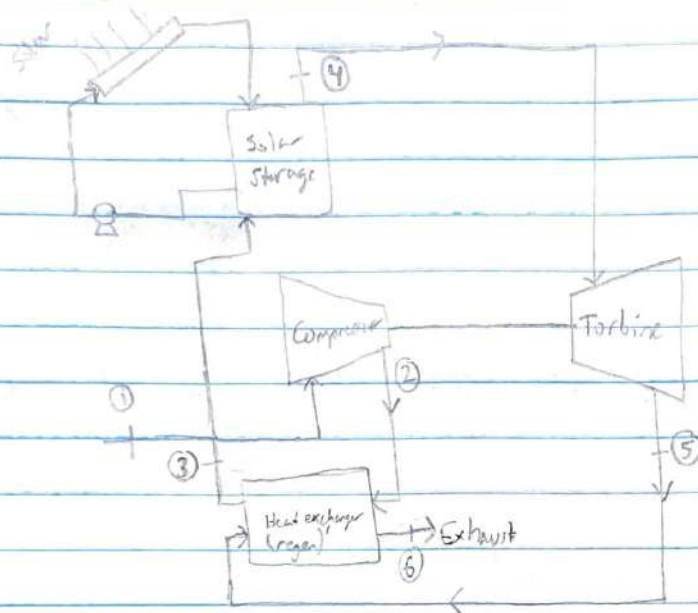


- 1.) Purpose: Determine thermal efficiency and effectiveness of the heat exchanger, and air mass flow in an ideal cold-air standard Brayton cycle.

Diagrams:



Sources: Cengel, Boles, and Kanoglu. Thermodynamics: An Engineering Approach, 9th Edition, McGraw Hill, 2020.

Design Considerations:

- Ideal cold-air standard
- No heat or friction losses
- Assume constant C_p , C_v at room temp. (300K)
- Counter flow through heat exchanger

Data and Variables:

$$C_p = 1.005 \text{ kJ/kg}\cdot\text{K} \quad C_v = 0.718 \text{ kJ/kg}\cdot\text{K}$$

$$k = 1.400 \quad R = 0.2870 \text{ kJ/kg}\cdot\text{K}$$

$$P_{\text{out}} = 500 \text{ kW}$$

$$P_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$P_2 = 5 \text{ bar} = 500 \text{ kPa}$$

$$T_1 = 310 \text{ K}$$

$$\eta_T = 0.8$$

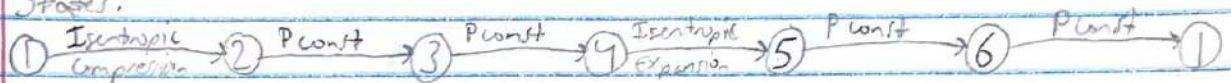
$$T_2 = 760 \text{ K}$$

$$T_4 = 1140 \text{ K}$$

Material: Air

Procedure: Find all states then solve each question.

States:



$$\begin{array}{cccccc} P_1 = 100 \text{ kPa} & P_2 = 500 \text{ kPa} & P_3 = 500 \text{ kPa} & P_4 = 500 \text{ kPa} & P_5 = 100 \text{ kPa} & P_6 = 100 \text{ kPa} \\ T_1 = 310 \text{ K} & T_2 = 760 \text{ K} & T_3 = 760 \text{ K} & T_4 = 1140 \text{ K} & T_5 = 803.8 \text{ K} & T_6 = 534.8 \text{ K} \end{array}$$

Find T_2 :

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}$$

$$T_2 = 310 \left(\frac{500}{100} \right)^{\frac{1.4-1}{1.4}}$$

Find T_5 :

$$\text{Ideal: } \frac{T_{5a}}{T_4} = \left(\frac{P_5}{P_4} \right)^{\frac{k-1}{k}}$$

$$T_{5a} = 1140 \left(\frac{100}{500} \right)^{\frac{1.4-1}{1.4}}$$

$$T_{5a} = 719.8 \text{ K}$$

Find T_6 :

$$\text{1st law}$$

$$h_5 - h_6 = h_2 - h_1$$

$$c_p(T_5 - T_6) = c_p(T_2 - T_1)$$

$$(803.8 - T_6) = 760 - 760$$

$$T_6 = 534.8 \text{ K}$$

$$\text{Actual: } \eta_T = \frac{T_4 - T_5}{T_4 - T_{5a}}$$

$$0.8 = \frac{1140 - T_5}{1140 - 719.8}$$

$$T_5 = 803.8 \text{ K}$$

$$W_{net} = W_{in} - W_{out}$$

$$= c_p(T_2 - T_1) - c_p(T_4 - T_5)$$

$$= 1.005(491 - 310) - 1.005(1140 - 803.8)$$

$$= 155.9 \text{ kJ/kg}$$

$$Q_{in} = c_p(T_4 - T_3)$$

$$= 1.005(1140 - 760)$$

$$= 381.9 \text{ kJ/kg}$$

$$a.) \quad \eta = \frac{W_{net}}{Q_{in}} = \frac{155.9}{381.9} = .41 = \boxed{41\%}$$

$$\epsilon = \frac{T_3 - T_2}{T_5 - T_2} = \frac{760 - 491}{803.8 - 491} = .86 = \boxed{86\%}$$

$$b.) \quad W = \dot{m} W_{net}$$

$$500 = \dot{m} (155.9)$$

$$\dot{m} = 3.21 \text{ kg/s}$$

$$c.) \quad \epsilon = 100\% \Rightarrow T_5 = T_3 = 760 \text{ K}$$

$$\epsilon = \frac{T_3 - T_2}{T_5 - T_2}$$

$$\eta = \frac{W_{net}}{Q_{in}} = \frac{c_p(T_2 - T_1) - c_p(T_4 - T_5)}{c_p(T_4 - T_3)} = \frac{1.005(491 - 310) - 1.005(1140 - 760)}{1.005(1140 - 760)} = \frac{199.99}{381.9}$$

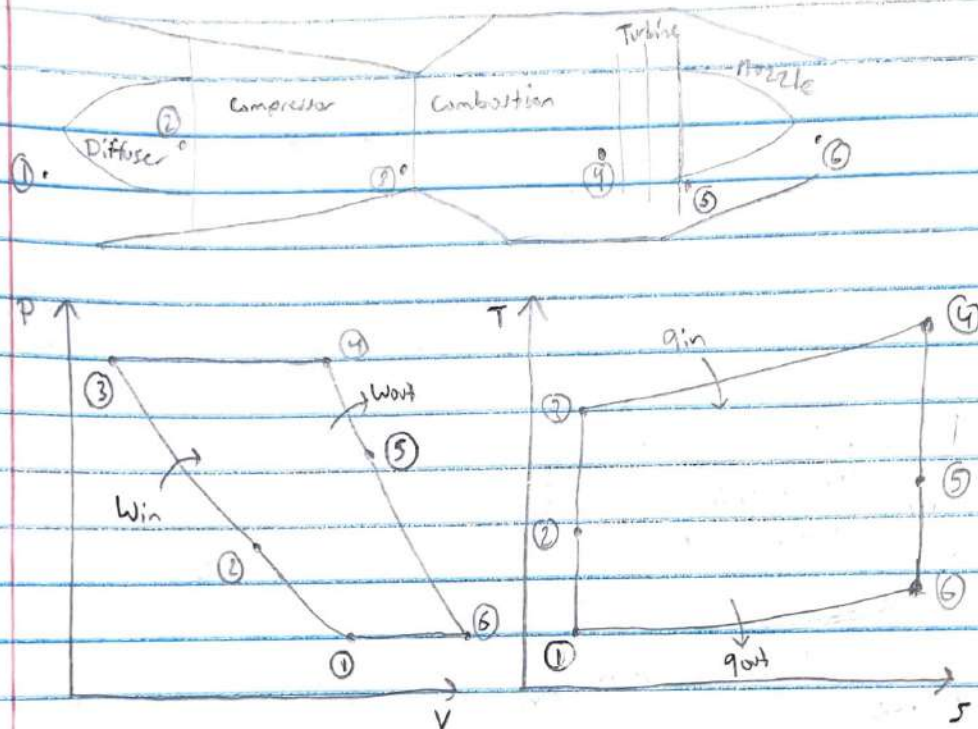
$$\eta = .52 = \boxed{52\%}$$

Summary: The brayton cycle consists of isentropic compression and expansion, with heat addition in the form of solar energy and then heat rejection after turbine expansion.

Analysis: After finding the pressure and temperature for each state, the net work, heat in, and mass flow rate were calculated. It was shown that the overall efficiency was 41%, with 86% effectiveness of the heat exchanger, and mass flow rate of 3.21 kg/s. When the heat exchanger was operating at 100% effectiveness it was shown that the efficiency increased from 41% to 52%.

2.) Purpose: Determine pressure of combustion gases at turbine exit, velocity of gas at nozzle exit, and engine thrust for a turbojet aircraft.

Diagram:



Sources: Cengel, Boles, and Kanoglu. Thermodynamics: An Engineering Approach, 9th Edition. McGraw Hill, 2020

Design Considerations:

- Assume Ideal gas (air)

Data and Variables:

$$V_1 = 900 \text{ km/h} = 250 \text{ m/s}$$

$$T_1 = -35^\circ\text{C} = 238 \text{ K}$$

$$P_1 = 40 \text{ kPa}$$

$$T_4 = 950^\circ\text{C} = 1223 \text{ K}$$

$$W_f = 500 \text{ kJ/kg}$$

$$\eta_c = 80\% = 0.80$$

$$\eta_T = 90\% = 0.90$$

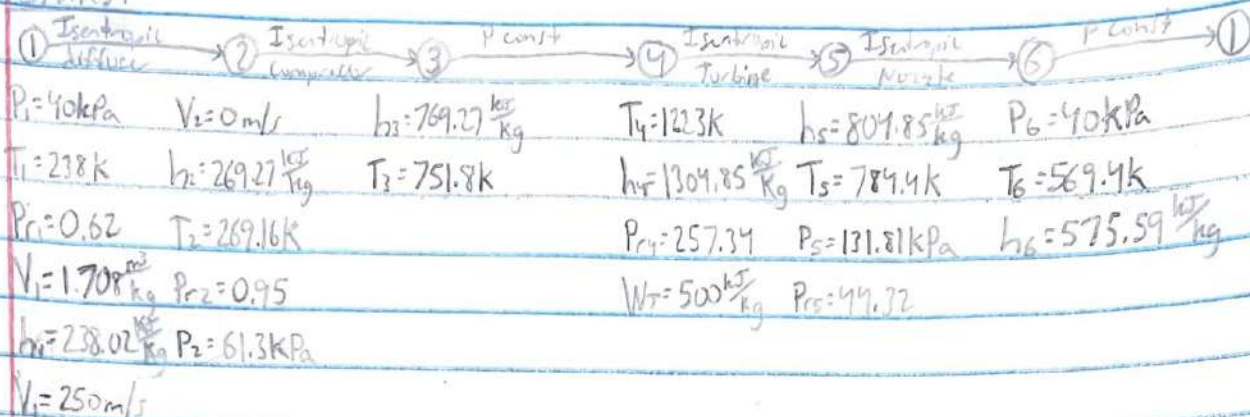
$$C_p, C_v: \text{variable}$$

$$D = 1.6 \text{ m}$$

Materials: Air

Procedure: Find all states then solve each question. Interpolation was completed using a calculator.

States:



Find V_1

$$V_1 = \frac{1}{\rho}$$

$$V_1 = \frac{RT_1}{P_1}$$

$$P_1$$

$$\rho = \frac{1}{V_1}$$

$$1.708$$

$$V_1 = \frac{287(238)}{40}$$

$$40$$

$$V_1 = 1.708 \frac{\text{m}^3}{\text{kg}}$$

Find P_{r1}

$$T$$

$$P_r$$

$$230$$

$$0.5477$$

$$238$$

$$P_{r1}$$

$$240$$

$$0.6355$$

Find h_1

$$T$$

$$h$$

$$230$$

$$230.02$$

$$238$$

$$h_1$$

$$240$$

$$240.02$$

Find h_2

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_2 = 238.02 \frac{\text{kJ}}{\text{kg}} + \frac{(250 \text{ m/s})^2}{2 \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$

$$h_2 = 269.27 \text{ kJ/kg}$$

Find T_2

$$T$$

$$h$$

$$260$$

$$260.09$$

$$T_2$$

$$269.27$$

$$270$$

$$270.11$$

Find P_{r2}

$$T$$

$$P_r$$

$$260$$

$$0.8405$$

$$260$$

$$P_{r2}$$

$$270$$

$$0.9590$$

Find P_2

$$P_2 = P_{r2}$$

$$P_1$$

$$P_{r1}$$

$$P_2 = 40 / 0.95$$

$$0.62$$

$$P_2 = 61.3 \text{ kPa}$$

Find h_5

$$W_T = h_4 - h_5$$

$$500 = 1304.85 - h_5$$

$$h_5 = 804.85 \text{ kJ/kg}$$

Find T_5

$$T$$

$$h$$

$$780$$

$$800.03$$

$$T_5$$

$$804.85$$

$$800$$

$$821.95$$

Find h_4

$$T$$

$$h$$

$$1220$$

$$1301.31$$

$$1223$$

$$h_4$$

$$1240$$

$$1324.93$$

Find P_{r4}

$$T$$

$$P_r$$

$$1220$$

$$254.7$$

$$1223$$

$$P_{r4}$$

$$1240$$

$$272.3$$

Find h_3

$$W_c = h_3 - h_2$$

$$500 = h_3 - 269.27$$

$$h_3 = 769.27 \text{ kJ/kg}$$

Find h_{3a}

$$\eta_c = h_{3a} - h_2$$

$$h_3 - h_2$$

$$0.80 = h_{3a} - 269.27$$

$$769.27 - 269.27$$

$$h_{3a} = 669.27 \text{ kJ/kg}$$

Find T_3

T	h
750	767.29
T_3	769.27
760	778.18

Find h_{5a}

$$\eta_T = \frac{h_{5a} - h_5}{h_5 - h_4}$$

$$0.90 = \frac{h_{5a} - 1309.85}{809.85 - 1309.85}$$

$$h_{5a} = 859.85 \text{ kJ/kg}$$

Find P_5

$$V_5 = \frac{RT_5}{P_5}$$

$$.5855 = \frac{.287 \cdot 784.9}{P_5}$$

$$P_5 = 131.81 \text{ kPa}$$

a.) $P_5 = 131.81 \text{ kPa}$

Find P_{r5}

T	P_r
780	43.35
789.9	P_{r5}
800	47.75

Find P_{r6}

$$P_6 = P_{r6}$$

$$P_5 = P_{r5}$$

$$P_{r6} = 49.32 / 40$$

$$(131.81)$$

$$P_{r6} = 13.45$$

Find T_6

T	P_r
560	12.66
T_6	13.45
570	17.50

Find h_6

T	h
560	565.17
569.9	h_6
570	575.59

b.) $\Delta h = h_4 - h_{5a}$

$$\Delta h = (1309.85 \text{ kJ/kg} - 859.85 \text{ kJ/kg}) \left(\frac{1000 \text{ J}}{1 \text{ kJ}} \right)$$

$$\Delta h = 450000 \text{ J/kg}$$

$$V_{exit} = \sqrt{2(\Delta h)}$$

$$V_{exit} = \sqrt{2(450000)}$$

$$V_{exit} = 948.68 \text{ m/s}$$

$$c) F = \dot{m}(V_{exit} - V_{inlet})$$

$$\dot{m} = \rho VA \quad A = \pi r^2 = \pi (0.8m)^2 = 2.01m^2$$

$$\dot{m} = (0.5855)(250)(2.01)$$

$$\dot{m} = 294.21 \text{ kg/s}$$

$$F = 294.21 (948.68 \text{ m/s} - 250 \text{ m/s})$$

$$F = 205,558.64 \text{ N}$$

Summary: The ideal jet-propulsion cycle is similar to the brayton cycle except that the gases are not expanded to ambient pressure in the turbine. The change in velocity gives you force which is the thrust acting on the aircraft.

Analysis: After finding the properties for each state and using the mass flow rate, it was calculated that the velocity at the nozzle exit is 948.68 m/s and the net thrust is 205,558.64 N.