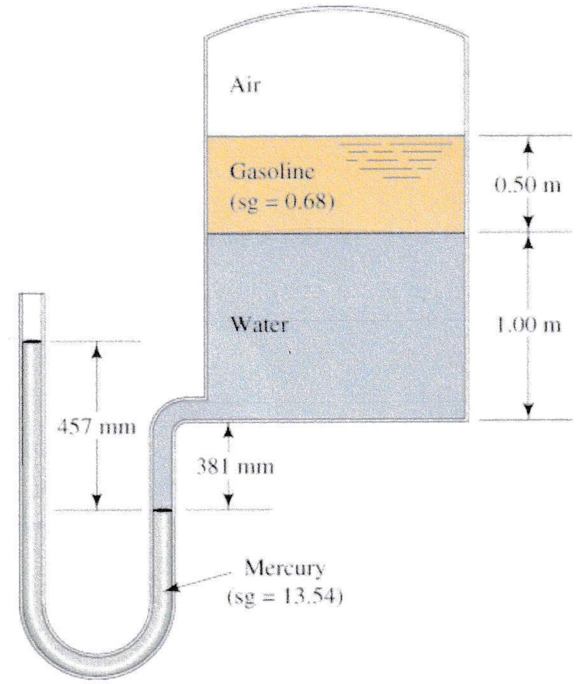
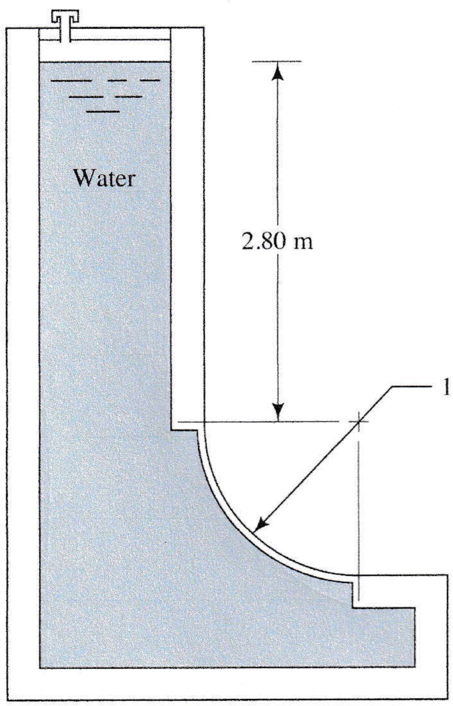
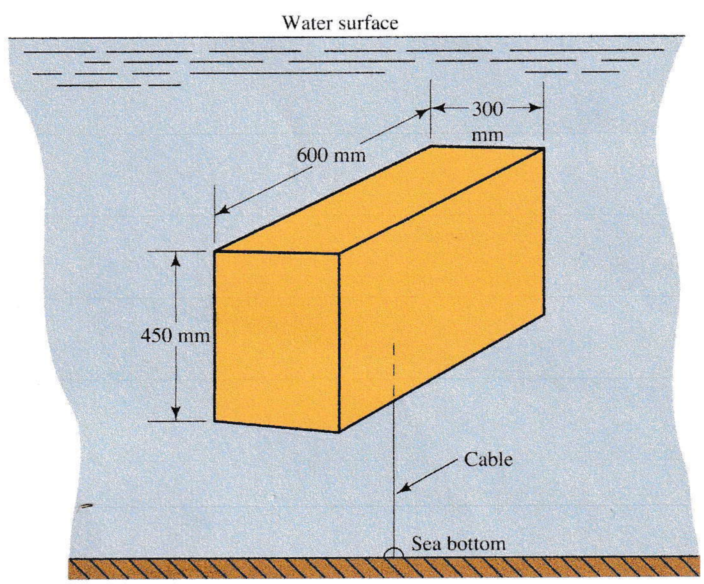
1) The set up in the right shows a closed tank that contains gasoline floating on water. Calculate the air pressure above the gasoline? If the mercury reading drops from 457 mm to zero, what is the new air pressure? Assume the closed tank is very large.





2) What should be the radius of the curved surface if we want to reduce the horizontal force to half of its original magnitude when R=1.2-m? In the new setup, what would the resultant force be? Also, find the resultan force direction and the location of the new vertical and horizontal forces.

3) The package shown in the figure on the right weighs 258 N and it is hold near the sea bottom by a cable. The sea water specific weight is $10.05 kN/m^3$. If the cable breaks, the package will move to the water surface. The package will float once it reaches the surface. The package will not be stable while floating in the orientation shown. It will rotate seeking for a more stable position. Prove that it will be stable in this other orientation.



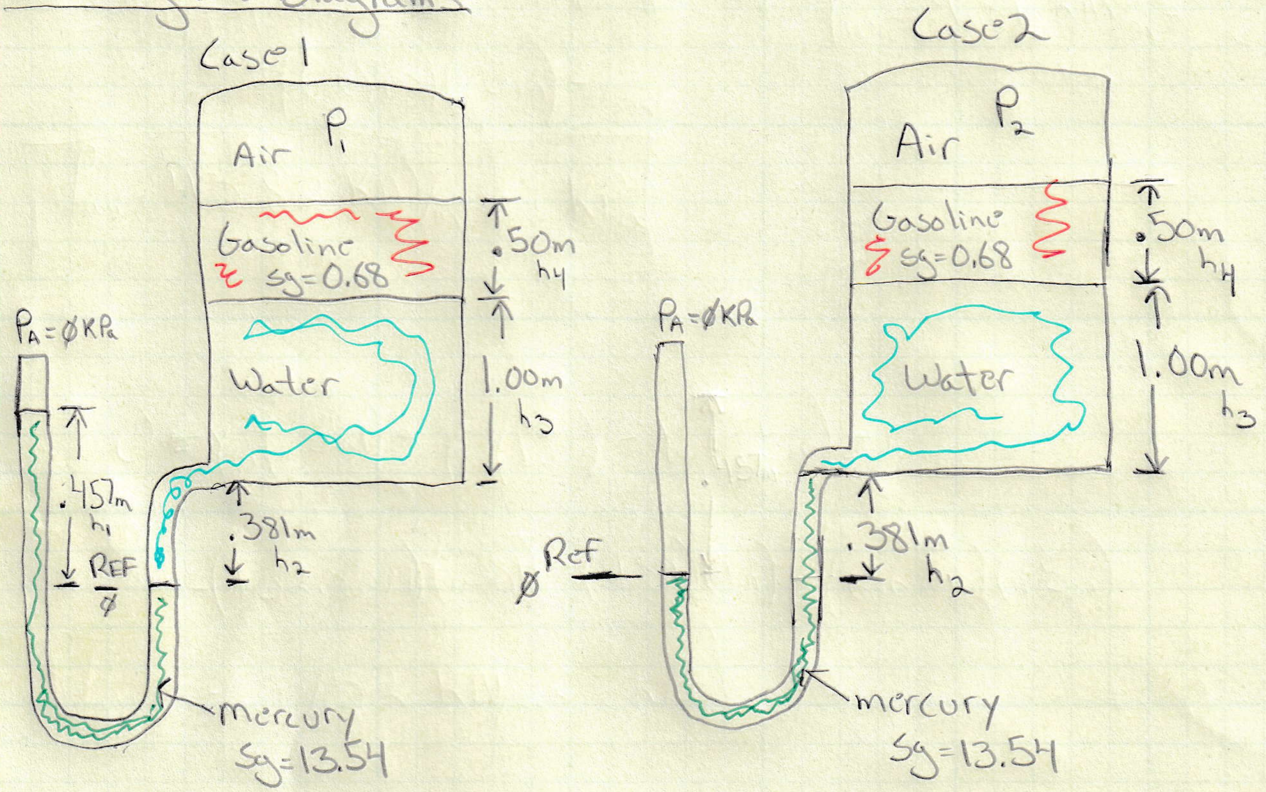
MET-330 | Exam #1 | Bryant Teagle

A

① Purpose

- Determine the air pressure above the gasoline.
- Determine th air pressure above the gasoline if level of the mercury drops from 457mm to Ømm

Drawing and Diagrams



Sources

- Mott, R., Untener, J.A., "Applied Fluid Mechanics", 7th edition Pearson Education, Inc. (2015)

Design Considerations

Based on the problem description, I assume the following:

1. Incompressible Fluids
2. Isothermal Problem $T = 25°C$
3. Constant Properties
4. Tank is large enough that the change in mercury height will have no discernible impact on the fluid levels inside the tank.
5. Fluids do not mix.

Data and Variables

- $\gamma_{water} = 9.78 \ KN/m^3$
- $\gamma_{gasoline} = 6.67 \ KN/m^3$
- $\gamma_{mercury} = 132.8 \ KN/m^3$
- $1 \ KN/m^2 = 1 \ KPa$

Procedure

I will use manometry for the first case of mercury height = 457mm. I will solve for P_1, the Pressure above the gasoline. (see figure)

I will then use manometry for the case of mercury height = Ømm. I will solve for P_2, the pressure above the gasoline. It is important to note that the size of the tank is Very Large in comparison to the tube. There will be no discernible difference in the Fluid levels inside the tank in Case 1 or Case 2.

C

Calculations

For mercury height = 457 mm case:

$$P_A + \gamma_{mercury} h_1 - \gamma_{water} h_2 - \gamma_{water} h_3 - \gamma_{gasoline} h_4 = P_1$$

$$\phi \, KN/m^2 + (132.8\ KN/m^3)(.457 m) - (9.78\ KN/m^3)(.381 m) -$$

$$(9.78\ KN/m^3)(1.00 m) - (6.67\ KN/m^3)(.50 m) = P_1$$

$$\boxed{P_1 = 43.85\ KN/m^2 \text{ or } 43.85\ KPa}$$

For mercury height = ϕ mm case:

$$P_A - \gamma_{mercury} h_2 - \gamma_{water} h_3 - \gamma_{gasoline} h_4 = P_2$$

$$\phi \, KN/m^2 - (132.8\ KN/m^3)(.381 m) - (9.78\ KN/m^3)(1.00 m) - (6.67\ KN/m^3)(.50)_m = P_2$$

$$\boxed{P_2 = -63.71\ KN/m^2 \text{ or } -63.71\ KPa}$$

Summary

The pressure of the air above the gasoline in case 1 is:

$$\boxed{P_1 = 43.85\ KPa}$$

The pressure of the air above the gasoline in Case 2 is:

$$\boxed{P_2 = -63.71\ KPa}$$

D

Materials

Water
gasoline
mercury
air

Analysis

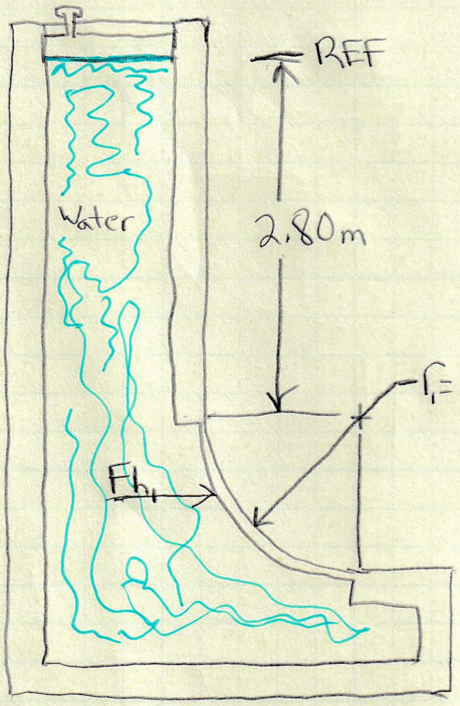
Case 1 of this probem is simple manometry. Case 2 requires the assumption that the tank is large enough so that the change in mercury height will have no discernible effect on the level of fluids in the tank. Case 1 requires a positive pressure, P_1 to hold the mercury height at 457mm. Case 2 requires a negative pressure or a vacuum at P_2 to draw the mercury height down to Ømm or REF.

② E

Purpose

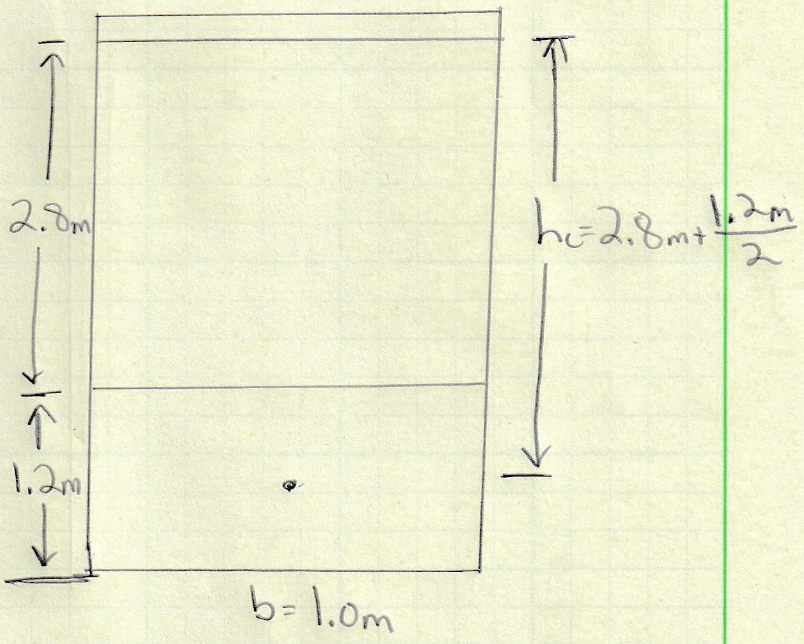
Determine the radius of the curved surface required to reduce the horizontal force to half of its original magnitude when r=1.2m. Determine the new resultant force and its direction. Determine the location of the new vertical and horizontal forces.

Drawings and Diagrams

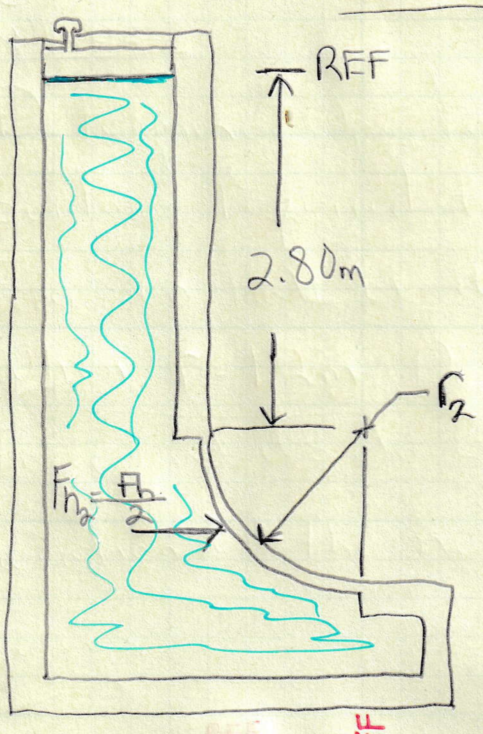


Case 1

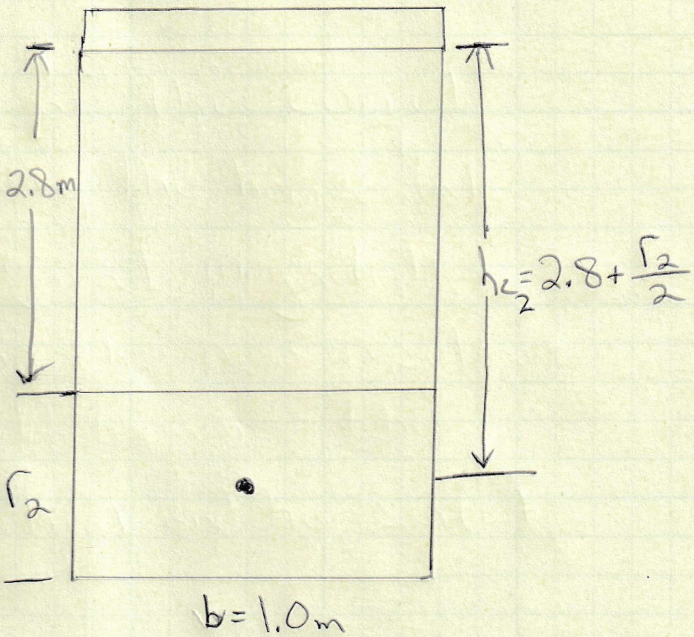
Projected Area

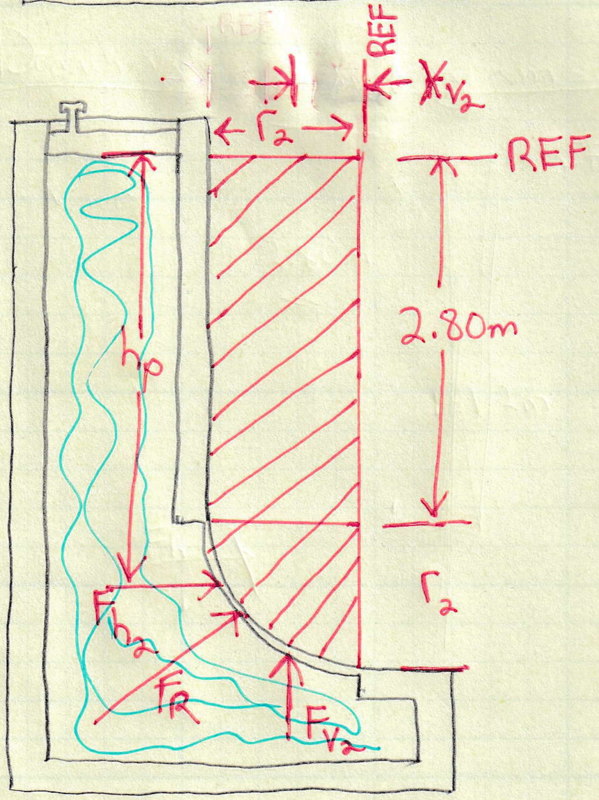


Case 2



Projected Area





Assumed

Sources

Mott, R., Untener, J.A., "Applied Fluid Mechanics," 7th edition
Pearson Education, Inc. (2015)

Design Considerations

Based on the problem description, I assume the following:

1. Incompressible Fluids
2. Isothermal Process: $T = 15°C$
3. Constant Properties
4. Width of tank, $b = 1m$

Data and Variables

$\gamma_{water} = 9.81\ kN/m^3$ $F_{h_2} = F_{h_1}/2$

$r_1 = 1.2m$

Materials

water

Procedure and Calculations

- In Case 1, I am only concerned with the magnitude of the horizontal Force, F_{h_1}. Using the projected area of Case 1 (see Figure) I will use the following equation.

$$F_h = \gamma \cdot h_c \cdot A$$

Transformed using terms of r:

$$F_h = \gamma_{water} \cdot \left(2.8m + \frac{r}{2}\right) \cdot (r \cdot 1m)$$

$$F_h = 9.81\ KN/m^3 \left(2.8m + \frac{r_m}{2}\right) \cdot (r_m \cdot 1m)$$

$$F_h = 27.468r + 4.905r^2 \leftarrow Eq.\ 1$$

$$F_{h_1} = 27.468(1.2m) + 4.905(1.2m)^2$$

$$F_{h_1} = 40.0248\ KN$$

Using F_{h_1}, F_{h_2} can be calculated.

$$F_{h_2} = \frac{F_{h_1}}{2}$$

$$F_{h_2} = \frac{40.0248\ KN}{2}$$

$$* F_{h_2} = 20.0124\ KN$$

- Using equation 1, I can now solve for r_2 using F_{h_2}.

$$F_{h_2} = 27.468 r_2 + 4.905 r_2^2$$

$$\downarrow$$

$$20.0124 = 27.468 r_2 + 4.905 r_2^2$$

$$\downarrow$$

$$4.905 r_2^2 + 27.468 r_2 - 20.0124 = \emptyset$$

$$\boxed{r_2 = 0.653\,m}$$

- Using the "Assumed" figure in Case 2, which is an imaginary column of water, I can solve for the magnitude of the new vertical force, F_{V_2}.

$$F_{V_2} = \gamma_{water}\left(V_{rectangle} + V_{1/4\,circle}\right)$$

$$F_{V_2} = 9.81\,KN/m^3\left[(2.8m \cdot 0.653m) + \left(\frac{\pi(0.653m)^2}{4}\right)\right] \cdot 1m$$

$$\underline{F_{V_2} = 21.223\,KN}$$

- I can now find the location of the new horizontal Force, h_p using the equation: (See Projected Area Case 2)

$$h_p = h_c + \frac{I_c}{h_c \cdot A}$$

↓

$$h_p = \left(2.8 + \frac{r_2}{2}\right)_m + \left(\frac{(1 \cdot r_2^3)/12 \ m^3}{\left(2.8 + \frac{r_2}{2}\right)_m \cdot (r_2 \cdot 1)_m}\right)$$

↓

$$h_p = \left(2.8 + \frac{r_2}{2}\right)_m + \frac{r_2^2 \ m^2}{12\left(2.8 + \frac{r_2}{2}\right)_m}$$ ← In terms of r

$$h_p = 3.127_m + 0.0114 m$$

$$\boxed{h_p = 3.138 m}$$ - Position of new horizontal Force

- I will solve for the location of the new vertical Force, X_{v_2} using the equation:

$$X_v = \frac{A_{rectangle} \cdot X_{rectangle} + A_{1/4 circle} \cdot X_{1/4 circle}}{A}$$

Procedure and Calculations

K

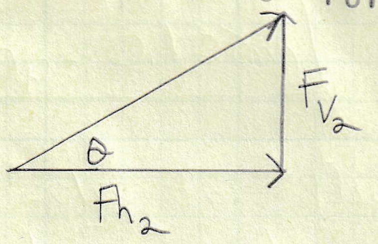
$$X_{V_2} = \frac{(2.8_m \cdot 0.653)_{m^2}\left(\frac{0.653_m}{2}\right)_m + \left(\frac{\pi \cdot 0.653^2_m}{4}\right)_{m^2}(.424 \cdot 0.653)_m}{(2.8 \cdot 0.653)_{m^2} + \left(\frac{\pi \cdot 0.653^2}{4}\right)_{m^2}}$$

↓

$$X_{V_2} = \frac{.597\,m^3 + 0.0927\,m^3}{2.163\,m^2}$$

$$\boxed{X_{V_2} = .319\,m}$$ - Position of new vertical force

- Using Vector Analysis, I can Solve for the New resultant Force, F_R, and its direction.



$$F_R = \sqrt{F_{h_2}^2 + F_{V_2}^2}$$

$$F_R = \sqrt{(20.0124)^2 + (21.223)^2}\ KN$$

$$\boxed{F_R = 29.170\ KN}$$ - Resultant Force Magnitude

$$\theta = \tan^{-1}\left(\frac{21.223\,KN}{20.0124\,KN}\right)$$

$$\boxed{\theta = 46.682^\circ}$$ - Direction of Resultant Force

Summary

- The radius required to reduce the horizontal-force to half of its original magnitude is $r_2 = 0.653\,m$
- The new resultant Force, $F_R = 29.170\ KN$
  It is perpendicular to the curved Surface at an angle, $\theta = 46.682°$ from the horizontal-bottom of the tank.
- The location of the new horizontal Force, $h_p = 3.138\,m$ below the surface of the water.

  The location of the new vertical Force, $X_{V_2} = .319\,m$ to the left of the vertical reference in the "Assumed" figure of Case 2.

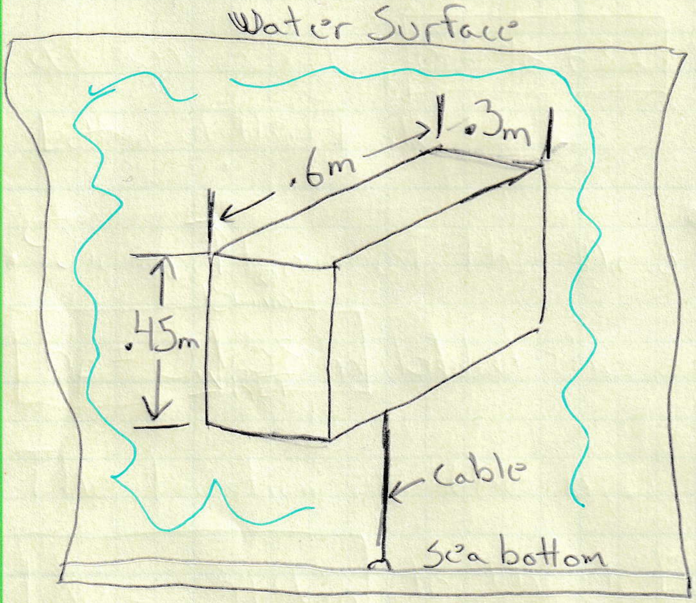
Analysis

The new radius is larger than half of the original radius in order to reduce the horizontal Force by half of its original magnitude. Therefore, it can be stated that the radius of the curved surface does not share a linear relationship with the horizontal Force Component.

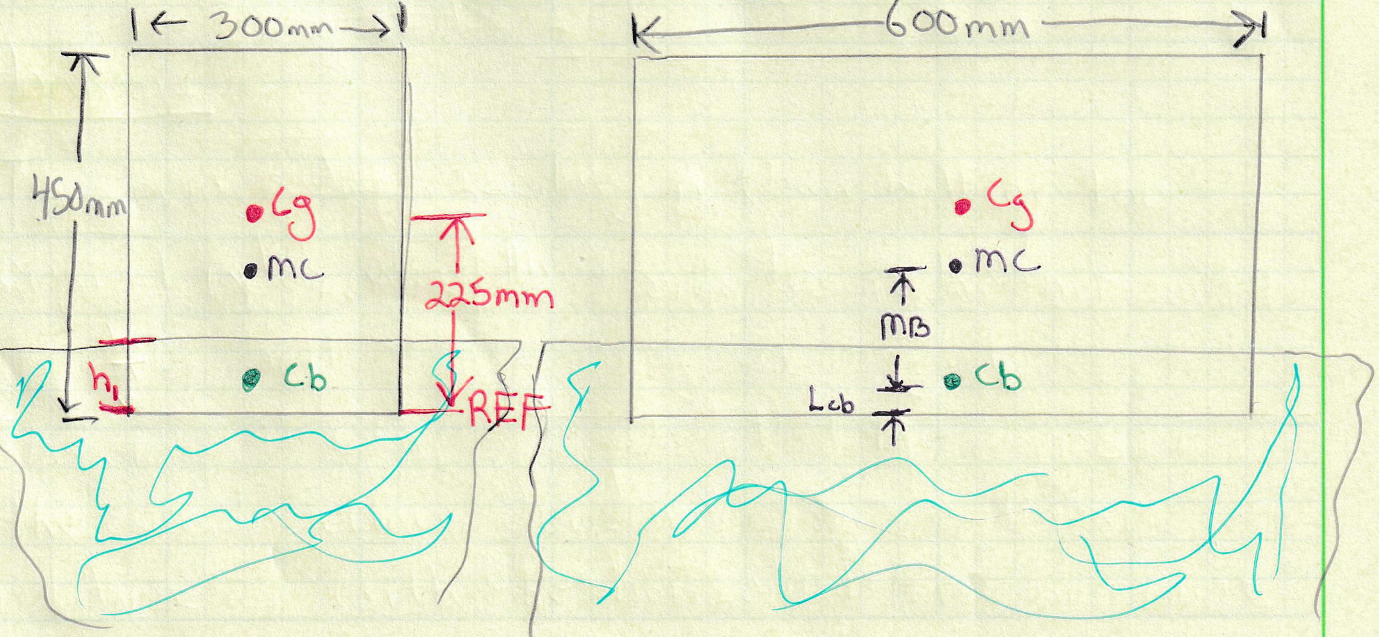
Purpose

Prove the package will be more stable in the orientation it will seek as it floats on the water surface, than the original orientation,

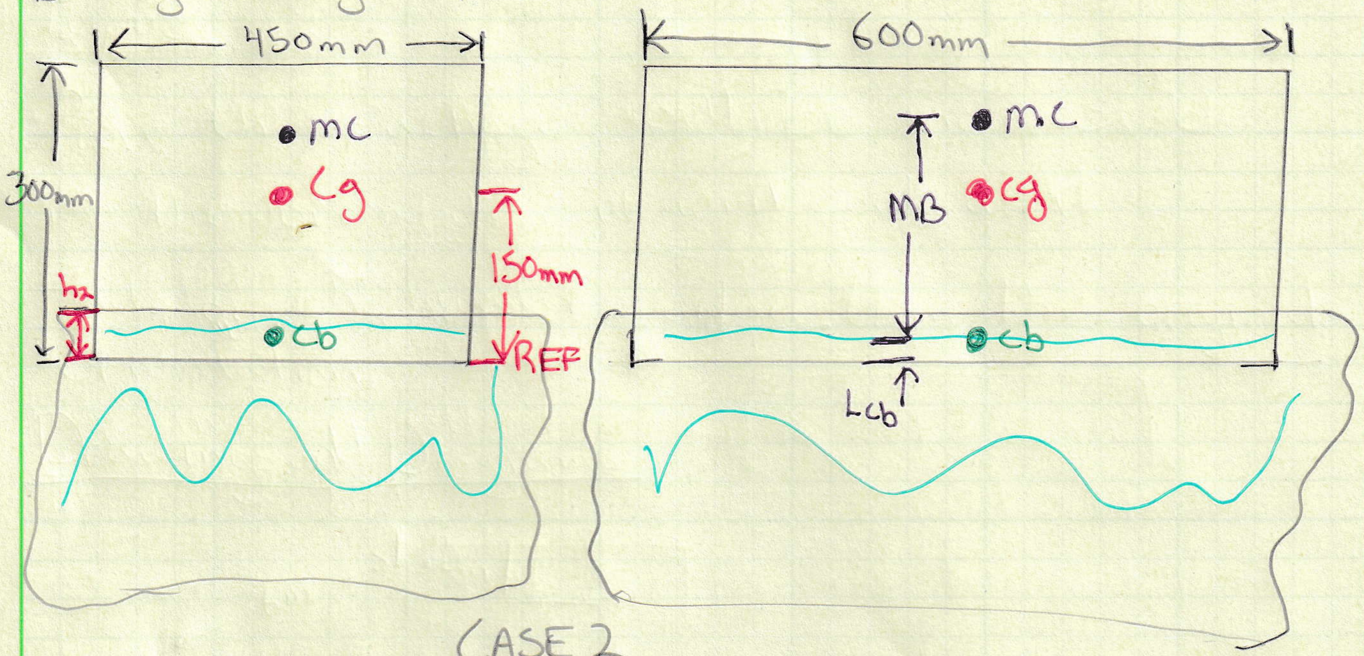
Drawings and Diagrams



CASE 1 = Original configuration



Drawing and Diagrams:



CASE 2

Sources

Mott, R., Untener, J.A., "Applied Fluid Mechanics", 7th edition
Pearson Education, Inc. (2015)

Design Considerations

Based on the problem description, I assume the following:

1. Incompressible Fluids
2. Isothermic Process
3. Constant Properties

Data and Variables

$\gamma_{seawater} = 10.05\ KN/m^3$

Weight of Package = 258 N

$F_b = W$

$M_C > C_g$ (Stability)

$M_C = L_{cb} + MB$

Procedure and Calculations

- I will first determine the Volume of water displaced by the package as it floats on the surface. Since we know $F_b = W$, I will solve using:

$$F_b = \gamma_{sw} V_d$$

$$V_d = \frac{.258\ KN}{10.05\ KN/m^3}$$

$$V_d = .02567\ m^3$$

- The Volume displaced will be the same for both Cases 1 and 2.

Case 1

- I can now solve for h_1, Submerged depth in Case 1

$$V_d = h_1 \cdot W_1 \cdot L_1$$

$$.02567 = h_1 \cdot (.30m) \cdot (.60m)$$

$$h_1 = .1426\ m$$

- I can now solve for L_{cb}, the position of the center of buoyancy from the REF → Bottom of Package.

$$L_{cb_1} = h_1/2$$

$$L_{cb_1} = \frac{.1426m}{2}$$

$$L_{cb_1} = .0713m$$

- Keeping in mind that $MC = L_{cb} + MB$, I will now solve for MB using the following equation:

$$MB = \frac{I_1}{V_d} \qquad I_1 = \frac{.6m \cdot (.3m)^3}{12}$$

$$MB_1 = \frac{0.00135m^4}{0.02567m^3} \qquad I_1 = 0.00135m^4$$

$$MB_1 = 0.0526m$$

- I will now determine MC_1

$$MC = L_{cb} + MB$$

$$MC_1 = .0713m + .0526m$$

$$MC_1 = 0.1239m$$

- I will calculate the Cg in Case 1

$$Cg_1 = \frac{.45m}{2}$$

$$Cg_1 = .225m$$

- In order for case 1 to be stable, MC must be greater or higher from REF than Cg.

$$MC > Cg = \text{Stability}$$

$$0.1239m \not> 0.225m$$ ← Case 1 is not stable ✗

Case 2

- I will now solve for h_2, submerged depth in Case 2 - as I did in Case 1.

$$V_d = h_2 \cdot W_2 \cdot L$$

$$.02567m^3 = h_2 \cdot (.45m) \cdot (.60m)$$

$$h_2 = .0951m$$

- I will now solve for Lcb_2 as I did in Case 1.

$$Lcb_2 = \frac{0.0951m}{2}$$

$$Lcb_2 = .0476m$$

 R

- I will now solve for MB_2:

$$MB_2 = \frac{I_2}{V_d} \qquad I_2 = \frac{(.6m)(.45m)^3}{12}$$

$$I_2 = .004556m^4$$

$$MB_2 = \frac{.004556m^4}{.02567m^3}$$

$$\underline{MB_2 = 0.1775m}$$

- I will now solve for MC_2

$$MC_2 = L_{cb_2} + MB_2$$

$$MC_2 = 0.0476m + 0.1775m$$

$$\underline{MC_2 = .2251m}$$

- I will now solve for Cg_2, the center of gravity of the package in Case 2.

$$Cg_2 = \frac{.30m}{2}$$

$$\underline{Cg_2 = .15m}$$

- In order for Case 2 to be stable, MC_2 must be higher from REF than Cg_2.

$MC_2 > Cg_2$ = Stability

.2251m > .15m ← Case 2 is Stable

Summary

Case 1: $MC_1 < Cg_1$ = instability
0.1239m < 0.225m

Case 2: $MC_2 > Cg_2$ = Stability
.2251m > .150m

Materials

Sea Water

Package

Analysis

Stability occurs when the metacenter of a floating object is Higher than its center of gravity. It can be stated that an object will tend to float in the orientation that provides lowest Cg to the Fluid.