A

(1) **Purpose:** Determine the flow rate at the exit of the system. Check if the maximum velocity is violated. If so, Provide suggestions to avoid it. Determine the pressure at the exit of Tee.

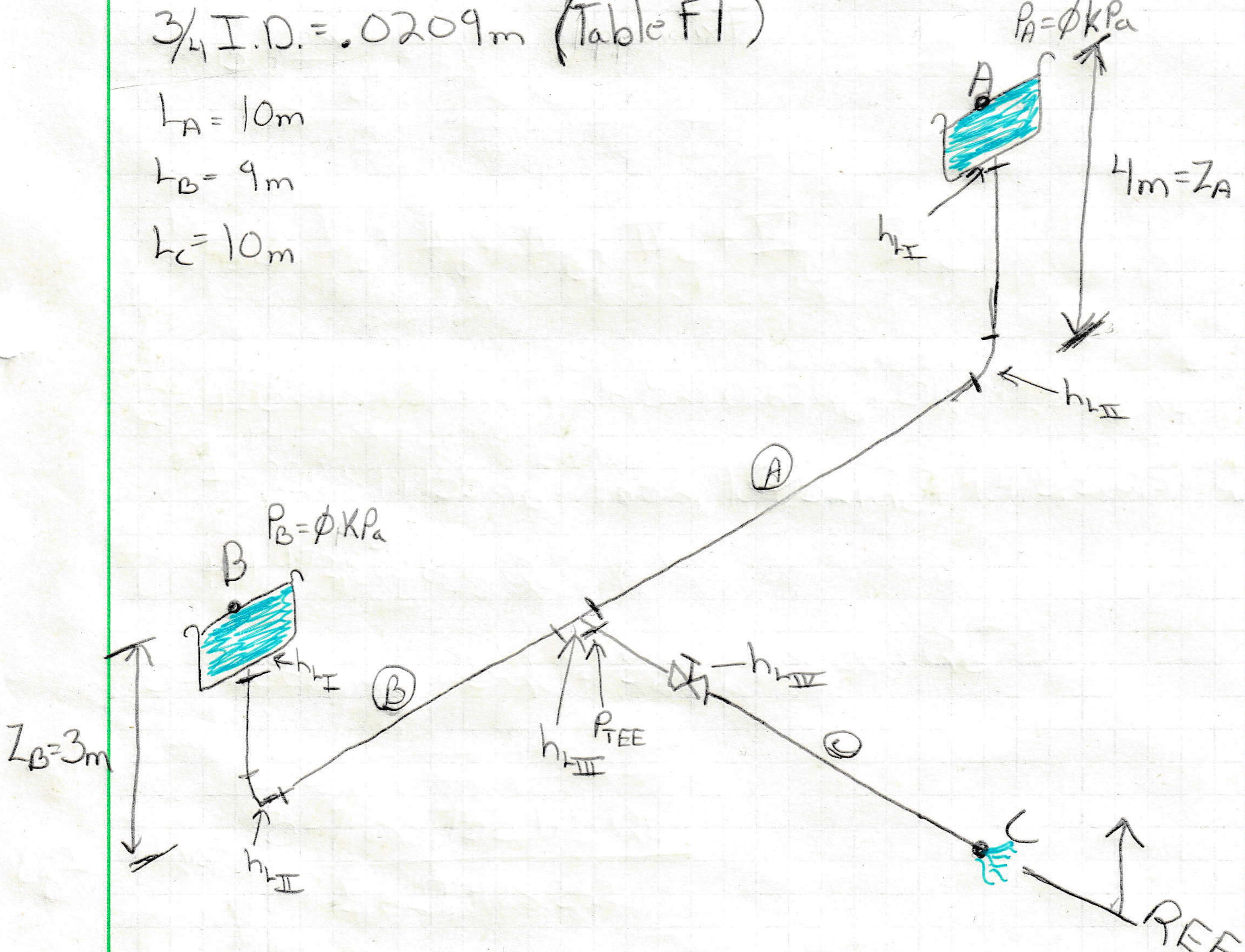
Diagram:

3/4 I.D. = .0209m (Table F1)

$L_A = 10m$

$L_B = 9m$

$L_C = 10m$



Sources: Mott and Untener. Applied Fluid Mechanics. 7th edition. Pearson 2015

Design Considerations:

1. Incompressible fluids
2. Steady State
3. Isothermal Process
4. Friction Factor is only a factor of relative roughness

Data and Variables:

Pipes are 3/4" I.D. schedule 40 Steel pipe

- Actual I.D. = 0.0209 m
- $\varepsilon = 4.6 \times 10^{-5}$ m
- γ_{water} @ 15°C = 9.81 kN/m^3
- $L_A = 10$ m $L_C = 10$ m

  $L_B = 9$ m

Materials:

Water in Steel Pipes

Procedure and Calculations:

- First, I will calculate all of the losses individually.
- I will then apply Bernoulli's from A to C and from B to C.
- Using the equations produced in the previous step and $Q_C = Q_A + Q_B$, I will create a spreadsheet to calculate an accurate Q_C through iterations.

(calculations are shown on pages D-G)

Eq. #1

$$Q_A = \sqrt{\frac{4 - 7{,}068{,}753.175 Q_C^2}{6{,}124{,}708.422}}$$

Eq. #2

$$Q_B = \sqrt{\frac{3 - 7{,}068{,}753.175 Q_C^2}{5{,}627{,}428.362}}$$

Eq. #3

$$Q_C = Q_A + Q_B$$

I - Entrance Loss

$$K = 0.5 \qquad h_{L_{Ent}} = 0.5\left(\frac{8Q^2}{\pi^2 g D^4}\right) = \frac{4Q^2}{g\pi^2 D^4}$$

$$h_{L_{Ent}} = 216{,}524.0262\, Q^2$$

II - Elbow

$$K = 30 \cdot f_T = (30)(.024) = 0.72$$

$$h_{L_{Elbow}} = 0.72\left(\frac{8Q^2}{g\pi^2 D^4}\right) = \frac{5.76\, Q^2}{g\pi^2 D^4}$$

$$h_{L_{Elbow}} = 311{,}794.5978\, Q^2$$

III - Tee

$$K = 60 \cdot f_T = (60)(.024) = 1.44$$

$$h_{L_{Tee}} = 1.44\left(\frac{8Q^2}{g\pi^2 D^4}\right) = \frac{11.52\, Q^2}{g\pi^2 D^4}$$

$$h_{L_{Tee}} = 623{,}589.1955\, Q^2$$

IV - ½ Closed Gate

$$K = 160 \cdot f_T = (160)(.024) = 3.84$$

$$h_{L_{gate}} = (3.84)\left(\frac{8 Q_c^2}{g\pi^2 D_c^4}\right) = \frac{30.72 Q_c^2}{g\pi^2 D^4}$$

$$h_{L_{gate}} = 1,662,404.521 Q_c^2$$

$$D/\varepsilon = \frac{.0209\text{m}}{4.6\times10^{-5}} = 454.3478$$

$$f = \frac{.25}{\left[\log\left(\frac{1}{3.7\frac{D}{\varepsilon}}\right)\right]^2}$$

$$f = .024$$

V - Pipe A

$$h_{L_A} = f \cdot \frac{L_A}{D} \cdot \frac{8 Q_A^2}{g\pi^2 D_A^4}$$

$$= .024\left(\frac{10}{.0209}\right) 433,048.0524 Q_A^2$$

$$h_{L_A} = 4,972,800.602 Q_A^2$$

VI - Pipe B

$$h_{L_B} = .024\left(\frac{9}{.0209}\right) 433,048.0524 Q_B^2$$

$$h_{L_B} = 4,475,520.542 Q_B^2$$

VII - Pipe C

$$h_{L_C} = .024\left(\frac{10}{.0209}\right) 433,048.0524 Q_C^2$$

$$h_{L_C} = 4,972,800.602 Q_C^2$$

Bernoulli $A \to C$

F

$$\cancel{\frac{P_A}{\gamma}}^{0} + \cancel{\frac{V_A^2}{2g}}^{0} + Z_A = \cancel{\frac{P_C}{\gamma}}^{0} + \frac{V_C^2}{2g} + \cancel{Z_C}^{0} + h_{L_{A-C}}$$

$$Z_A = \frac{V_C^2}{2g} + h_{L_{A-C}} \qquad * V = \frac{4Q}{\pi D^2}$$

$$Z_A = \frac{8Q_C^2}{g\pi^2 D^4} + h_{L_{A-C}} \qquad \frac{8Q_C^2}{(9.81)\pi^2(.0209)^4} = 433{,}048.0524 Q_C^2$$

$$h_{L_{A-C}} = \mathrm{I}_A + \mathrm{II}_A + \mathrm{III}_A + \mathrm{IV}_C + \mathrm{V}_A + \mathrm{VII}_C$$

$$4 = 433{,}048.0524 Q_C^2 + 216{,}524.0262 Q_A^2 + 311{,}794.5978 Q_A^2$$
$$+ 623{,}589.1955 Q_A^2 + 1{,}662{,}904.521 Q_C^2 + 4{,}972{,}800.602 Q_A^2$$
$$+ 4{,}972{,}800.602 Q_C^2$$

$$4 = 7{,}068{,}753.175 Q_C^2 + 6{,}124{,}708.422 Q_A^2$$

Eq I $$Q_A = \sqrt{\frac{4 - 7{,}068{,}753.175 Q_C^2}{6{,}124{,}708.422}}$$

Bernoulli B→C

$$\frac{\cancel{P_B}}{\cancel{\gamma}} + \frac{\cancel{V_B^2}}{\cancel{2g}} + Z_B = \frac{\cancel{P_C}}{\cancel{\gamma}} + \frac{V_C^2}{2g} + \cancel{Z_C} + h_{r_{B-C}}$$

$$Z_B = \frac{V_C^2}{2g} + h_{r_{B-C}}$$

$$\downarrow$$

$$Z_B = \frac{8Q_C^2}{g\pi^2 D^4} + h_{r_{B-C}}$$

$$h_{r_{B-C}} = \overline{I}_B + \overline{II}_B + \overline{III}_B + \overline{IV}_C + \overline{VI}_B + \overline{VII}_C$$

$$3 = 433{,}048.0524\,Q_C^2 + 216{,}524.0262\,Q_B^2 + 311{,}794.5978\,Q_B^2$$
$$+ 623{,}589.1955\,Q_B^2 + 1{,}662{,}904.521\,Q_C^2 + 4{,}475{,}520.542\,Q_B^2$$
$$+ 4{,}972{,}800.602\,Q_C^2$$

$$3 = 7{,}068{,}753.175\,Q_C^2 + 5{,}627{,}428.362\,Q_B^2$$

Eq 2 $$Q_B = \sqrt{\frac{3 - 7{,}068{,}753.175\,Q_C^2}{5{,}627{,}428.362}}$$

Eq 3 $$Q_C = Q_A + Q_B$$

H

Exam 2 - Problem 1

DATA

Da=	0.0209	m	Aa=	0.0003437	m^2
Db=	0.0209	m	Ab=	0.0003437	m^2
Dc=	0.0209	m	Ac=	0.0003437	m^2
La=	10	m	Za=	4	m
Lb=	9	m	Zb=	3	m
Lc=	10	m	Zc=	0	m
ε=	0.000046				
D/ε=	454.3478261	m			
f=	0.024	(Same for all)			
$f_{t=}$	0.024				
K_{ent}=	0.5				
$K_{elbow=}$	30				
$K_{tee=}$	60				
$K_{1/2gate=}$	160				
gravity=	9.81	m/s^2			

Itereration	Q_c	Q_a	Qb	New Q_c	% Error	V_c	3m/s Violation
1	0.0005	0.000603786	0.000468051	0.001071838	-114.37%	3.118527	VIOLATES
2	0.0006	0.000487445	0.000284426	0.000771871	-28.65%	2.245771	OK
3	0.00065	0.000406779	4.88917E-05	0.000455671	29.90%	1.325781	OK
4	0.00062	0.000457648	0.000224162	0.000681811	-9.97%	1.983738	OK
5	0.00063	0.000441605	0.000185869	0.000627474	0.40%	1.825645	OK
6	0.00062963	**0.000442214**	**0.000187437**	**0.000629651**	**0.00%**	1.831978	OK

Qa=	0.000442214	Va=	1.286627812
Qb=	0.000187437	Vb=	0.545350593
Qc=	**0.00062965**	Vc=	1.831978404

Procedure and Calculations Cont.:

- I will now apply Bernoulli's again to determine the pressure at the exit of the Tee. $P_{Tee_C} = P_{Tee_A} = P_{Tee_B}$

$$\frac{\cancel{P_A}}{\cancel{\gamma}} + \cancel{\frac{V_A^2}{2g}} + Z_A = \frac{P_{Tee}}{\gamma} + \frac{V_c^2}{2g} + \cancel{Z_c} + h_{L_{A-TEE}}$$

$$4m = \frac{P_{Tee}}{\gamma} + \frac{V_c^2}{2g} + h_{L_{A-Tee}}$$

$$P_{Tee_A} = \gamma\left[4m - \frac{V_c^2}{2g} - h_{L_{A-Tee}}\right]$$

$$P_{Tee_A} = \gamma\left[4m - \frac{V_c^2}{2g} - \frac{V_a^2}{2g}\left(.5 + .72 + 1.44 + f\frac{L_A}{D}\right)\right]$$

$$P_{Tee_A} = 9.81\,KN/m^2\left[4m - .171057m - 1.1933m\right]$$

$$\underline{P_{Tee_A} = 25.855\,KN/m^2 \text{ or } KPa}$$

$$P_{Tee_B} = \gamma\left[3m - \frac{V_c^2}{2g} - \frac{V_B^2}{2a}\left(.5 + .72 + 1.44 + f\frac{L_B}{D}\right)\right]$$

$$P_{Tee_B} = 9.81\,KN/m^2\left[3m - .171057m - .19698m\right]$$

$$\underline{P_{Tee_B} = 25.8119\,KN/m^2 \text{ or } KPa}$$

$P_{TeeA} = P_{TeeB}$
by
0.167%

Summary:

- Flow at Outlet of System = $\boxed{Q_c = 6.2965 \times 10^{-4}}$
- The Velocity criterion is $\boxed{\text{NOT}}$ violated
- The Pressure at the Exit of the Tee is = $\boxed{25.8\ \text{KPa}}$

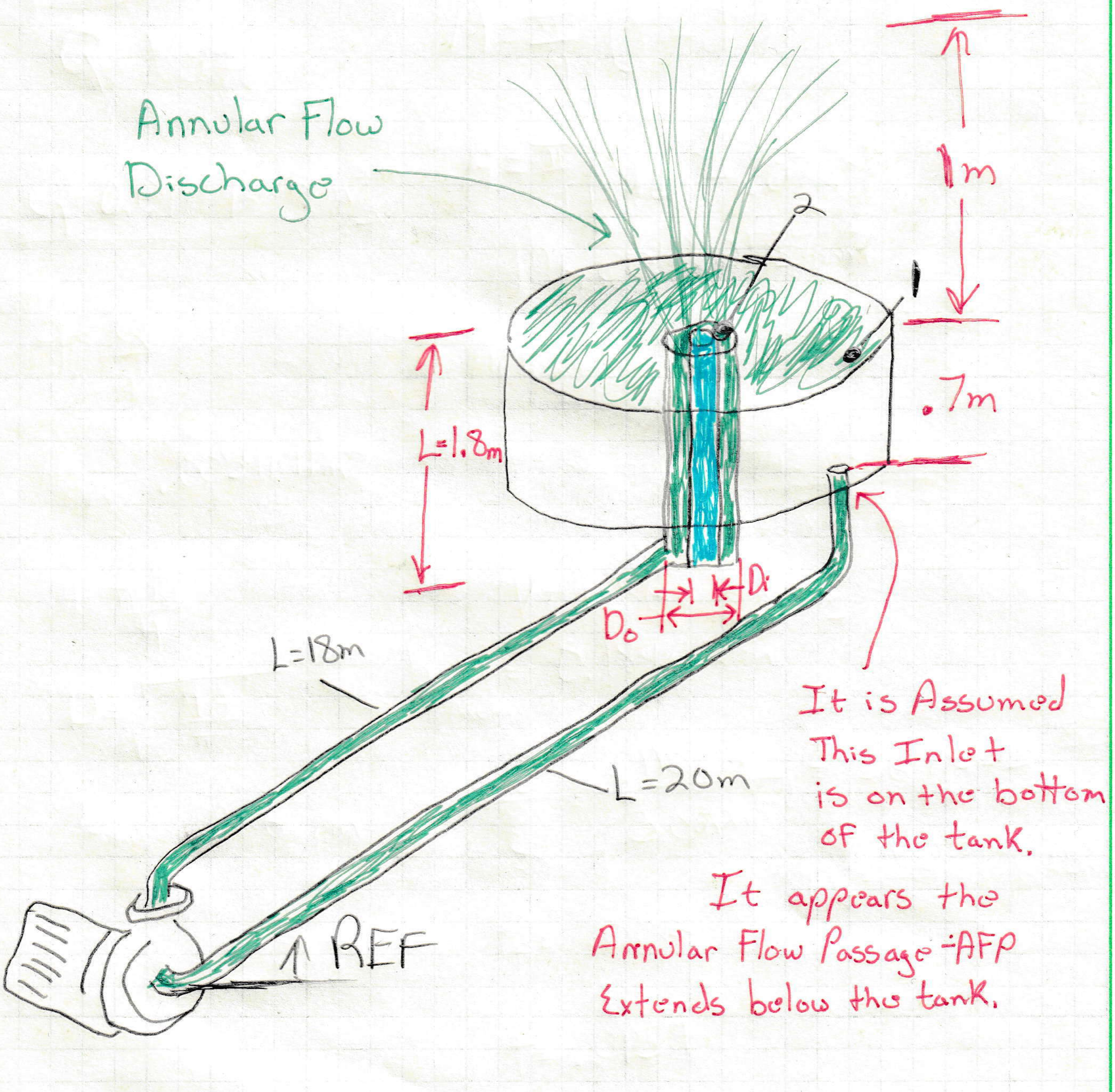
Analysis:

- No Violation of V_{max} was found
- If there were a Violation of V_{max}, further closing of the gate would increase the loss's and decrease Q_c and also V_c - as: $V_c = \frac{Q_c}{A_c}$

K

(2) Purpose: Determine the pump power required.
Determine the electrical power requirements.

Diagram:



Sources:
Mott and Untener. Applied Fluid Mechanics 7th edition. Pearson 2015

Design Considerations:

1. Incompressible Fluids
2. Steady State
3. Isothermal Process

Data and Variables:

- Pipes are to be PVC
- Water is 15°C
  $\nu = 1.15 \times 10^{-6}\ m^2/s$
  $\gamma = 9.81\ KN/m^3$
- $\varepsilon = 3.0 \times 10^{-7}\ m$
- Expansion into Anular Flow Passage $K = 2$
- Anular Flow Passage
  $D_o = 10cm$
  $D_i = 7cm$
- Pipes
  $L_I = 20m$
  $L_O = 18m$
- Fountain Height $= 1m$

Materials:

- Water
- PVC Pipes
- pump

Procedure and Calculations:

- First I will determine the Velocity at the Exit of the Fountain using $V=\sqrt{2gh}$

$$V=\sqrt{2(9.81\,m/s^2)(1\,m)}$$

$$V_{Fexit}=4.429\,m/s$$

- I will now find the flow Area of the Anular Flow Passage, using $A=\frac{\pi}{4}(D^2-d^2)$

$$A=\frac{\pi}{4}(.1m^2-.07m^2)$$

$$A_{AFP}=.0040055\,m^2$$

- Now, Using V_{Fexit} and A_{AFP}, I will solve for Q.

$$Q=VA$$

$$Q=(4.429\,m/s)(.0040055\,m^2)$$

$$Q=.01774036\,m^3/s$$

- I will now Solve for the diameter of the Pipes using $V_{crit} = 3 m/s$ and $A = \frac{Q}{V}$ and $D = \sqrt{\frac{4A}{\pi}}$

$$A = \frac{.01774036 m^3/s}{3 m/s} \qquad D = \sqrt{\frac{4(.005913 m^2)}{\pi}}$$

$$A = .005913 m^2 \qquad D = 0.08677 m$$

- I will now use Table G-3 to choose the appropriate PVC Pipe.

Pipe to be used: O.D = .125 m

I.D. = .1102 m

$A = 9.538 \times 10^{-3} m^2$

- Now, with the Pipe I.D. fixed, I will calculate the true velocity in the pipe.

$$V = \frac{Q}{A}$$

$$V = \frac{0.01774036 m^3/s}{9.538 \times 10^{-3} m^2}$$

$$V = 1.86 m/s$$

- I will now Solve for $\frac{D}{\varepsilon}$, R_e, F_T, and F.

$$\frac{D}{\varepsilon} = \frac{.1102\,m}{3.0\times10^{-7}} = 367,333.33$$

$$R_e = \frac{VD}{\nu} = \frac{(1.86\,m/s)(.1102\,m)}{1.15\times10^{-6}\,m^2/s} = 178236.5217$$

$$F_T = \frac{.25}{\left[\log\left(\frac{1}{3.7\,D/\varepsilon}\right)\right]^2} = 0.0066$$

$$F = \frac{.25}{\left[\log\left(\frac{1}{3.7\,D/\varepsilon} + \frac{5.74}{R_e^{.9}}\right)\right]^2} = 0.0159$$

- I will now calculate all of the Losses seperately.

I. Entrance Loss into Pipe

$K = .5$

$$h_{L_{ent}} = .5\frac{V^2}{2g} = .5\left(\frac{1.86\,m/s^2}{2(9.81\,m/s^2)}\right)$$

$$h_{L_{ent}} = 0.08817\,m$$

II. Elbow

$$K = 30 f_T$$

$$h_{L_{Elbow}} = (30)(.0066)\left(\frac{1.86 m/s^2}{2(9.81 m/s^2)}\right)$$

$$h_{L_{Elbow}} = .0349 m$$

III. Expansion into AFP

$$K = 2$$

$$h_{L_{E_{AFP}}} = 2\left(\frac{1.86^2 m/s}{2(9.81 m/s^2)}\right)$$

$$h_{L_{E_{AFP}}} = .35266 m$$

IV Pipes

$$h_L = f \frac{L}{D} \frac{V^2}{2g}$$

$$h_L = (0.0159)\left(\frac{18+20 m}{.1102 m}\right)\left(\frac{1.86 m^2}{2(9.81 m/s^2)}\right)$$

$$h_L = 0.96678 m$$

V Loss in AFP

- I will solve for Wetted Perimiter and Hydraulic Radius of the Anular Flow Passage.

$$WP = \pi(D+d) \qquad r = \frac{A_{AFP}}{WP}$$

$$WP = \pi(.1+.07) \qquad r = \frac{.0040055\,m^2}{.534\,m}$$

$$WP = .534\,m \qquad r = .0075\,m$$

- I can now solve for D/ε and Re in order to solve for friction factor in the AFP.

$$D/\varepsilon = \frac{4r}{\varepsilon} = \frac{4(.0075\,m)}{3.0\times10^{-7}\,m} = 100{,}000$$

$$R_e = \frac{V(4r)}{\nu} = \frac{(4.429\,m/s)(4)(.0075\,m)}{1.15\times10^{-6}\,m^2/s} = 115{,}539.1304$$

$$f_{AFP} = .0174$$

- I can now solve for $h_{L_{AFP}}$ using; $f\frac{L}{4r}\frac{V^2}{2g}$

$$h_{L_{AFP}} = (.0174)\left(\frac{1.8\,m}{4(.0075)}\right)\left(\frac{4.429\,m/s^2}{2(9.81\,m/s^2)}\right)$$

$$h_{L_{AFP}} = 1.0438\,m$$

• I will now Apply Bernoulli's between the top of the water surface in the tank and the Exit of the AFP, using the pump as the reference.

$$h_A + \cancel{\frac{P_1}{\gamma}}^{0} + \cancel{\frac{V_1^2}{2g}}^{0} + \cancel{Z_1}^{1.8m} = \cancel{\frac{P_2}{\gamma}}^{0} + \frac{V_2^2}{2g} + \cancel{Z_2}^{1.8m} + h_{L_{1-2}}$$

$$h_A = \frac{(4.429\,m/s)^2}{2(9.81\,m/s^2)} + 0.08817m + 3(.0349m) + .35266m$$

$$+ 1.0438m + 0.96678m$$

$$h_A = 3.556m$$

• I will now solve for the power required to pump the water using; $P_A = \gamma Q h_A$

$$P_A = (9.81\,KN/m^3)(.01774036\,m^3/s)(3.556m)$$

$$\boxed{P_A = .61886\ KN{\cdot}m/s \text{ or } KW \quad \text{ALSO} = .8299\ HP}$$

• I will now Solve for the Electrical Power requirements using:

$$\frac{P_{out}}{Eff.} = P_{in}$$

$$\boxed{\frac{.8299\ HP}{.92} = .9021 \text{ Horse Power} \quad \text{or} \quad .6727\ KW}$$

Summary:

- Q required for fountain to reach 1m = $\boxed{0.01774036\ m^3/s}$
- Power required for the Flow Configuration = $\boxed{.8299\ HP}$
- Electrical Power requirements @ 92% eff. = $\boxed{.9021\ HP}$

Analysis:

Looking back at Torricelli's, the Key to fountain spray height is exit Velocity. This is Also seen in conservation of energy equations. This being the first problem with PVC, the losses were much smaller compared to steel pipes.