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Homework 1.4

Problem 9-88

An aircraft engine operates on a simple ideal Brayton cycle with a pressure ratio of 10. Heat is added to the cycle at a rate of 500 kW; air passes through the engine at a rate of 1 kg/s; and the air at the beginning of the compression is at 70 kPa and 0°C. Determine the power produced by this engine and its thermal efficiency. Use constant specific heats at room temperature.

Given:

- Ratio = 10
- $Q_{in} = 500 \text{ kW}$
- Flow Rate (m) = 1 kg/s
- $P_1 = 70 \text{ kPa}$
- $T_1 = 0^\circ\text{C}$
- Use Textbook Appendix to find Air Properties at Room Temperature:
 - Specific heat ratio (γ) = 1.4

To Find Thermal Efficiency (Part (a)):

Use:

- $n = 1 - (1 / \text{Ratio}^{(\gamma - 1/\gamma)})$
- $n = 1 - (1 / 10^{(1.4 - 1 / 1.4)})$
- $n = 0.482 = 48.2\%$

To Find Power Produced (Part (b)):

Use:

- $n = W / Q_{in}$

Or

- $W = n * Q_{in}$
- $W = 0.482 * 500$
- $W = 241 \text{ kW}$

Problem 9-99

A gas turbine for an automobile is designed with a regenerator. Air enters the compressor of this engine at 100 kPa and 30°C. The compressor pressure ratio is 10; the maximum cycle temperature is 800°C; and the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator. Assuming both the compressor and the turbine to be isentropic, determine the rates of heat addition and rejection for this cycle when it produces 115 kW. Use constant specific heats at room temperature.

Given:

- Ratio = 10
- $P_1 = 100 \text{ kPa}$
- $T_1 = 30^\circ\text{C} = 303.15 \text{ K}$
- $T_4 (\text{Maximum Temp.}) = 800^\circ\text{C} = 1073.15 \text{ K}$

- Power (P) = 115 kW
- Use Textbook Appendix to find Air Properties at Room Temperature:
 - Specific heat ratio (γ) = 1.4
 - Specific heat constant pressure (C_p) = 1.005 kJ/kg

To Find Temperature at State 2:

Use:

- $T_2 = T_1 * (\text{Ratio})^{(\gamma - 1/\gamma)}$
- $T_2 = 303.15 * (10^{(1.4 - 1/1.4)})$
- $T_2 = 585.29 \text{ K}$

To Find Temperature at State 5:

Use:

- $T_4 / T_5 = \text{Ratio}^{(\gamma - 1/\gamma)}$

Or

- $T_5 = T_4 / (\text{Ratio})^{(\gamma - 1/\gamma)}$
- $T_5 = 1073.15 / (10^{(1.4 - 1/1.4)})$
- $T_5 = 555.84 \text{ K}$

To Find Temperature at State 3:

Problem States that “the cold air stream leaves the regenerator 10°C cooler than the hot air stream at the inlet of the regenerator”

Use:

- $T_3 = T_5 - 10$
- $T_3 = 555.84 - 10$
- $T_3 = 545.84 \text{ K}$

To Find Temperature at State 6:

Use:

- $T_5 - T_6 = T_3 - T_2$

Or

- $T_6 = T_5 - (T_3 - T_2)$
- $T_6 = 555.84 - (545.84 - 585.29)$
- $T_6 = 595.29 \text{ K}$

To Find the Net Work Output:

Use:

- $W_{\text{net}} = C_p * (T_4 - T_5) - C_p * (T_2 - T_1)$
- $W_{\text{net}} = 1.005 * (1073.15 - 555.84) - 1.005 * (585.29 - 303.15)$
- $W_{\text{net}} = 236.35 \text{ kJ/kg}$

To Find the Flow Rate (m):

Use:

- $P = m * W_{\text{net}}$

Or

- $m = P / W_{\text{net}}$
- $m = 115 / 236.35$
- $m = 0.487 \text{ kg/s}$

To Find Heat Added to the System (Part (a)):

Use:

- $Q_{\text{in}} = m * C_p * (T_4 - T_3)$
- $Q_{\text{in}} = 0.487 * 1.005 * (1073.15 - 545.84)$
- $Q_{\text{in}} = 258.1 \text{ kW}$

To Find Heat Rejected by the System (Part (b)):

Use:

- $Q_{\text{Out}} = m * C_p * (T_6 - T_1)$
- $Q_{\text{Out}} = 0.487 * 1.005 * (595.29 - 303.15)$
- $Q_{\text{Out}} = 142.98 \text{ kW}$

Problem 9-107

A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 and 1150 K. Assuming an isentropic efficiency of 75 percent for the compressor and 82 percent for the turbine and an effectiveness of 65 percent for the regenerator, determine

- (a) the air temperature at the turbine exit
- (b) the net work output, and
- (c) the thermal efficiency

Given:

- Ratio = 7
- T_1 (Minimum Temp.) = 310 K
- T_4 (Maximum Temp.) = 1150 K
- Compressor Efficiency (η_c) = 75 % = 0.75
- Turbine Efficiency (η_t) = 82 % = 0.82
- Regenerator Effectiveness (E) = 65% = 0.65
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 1 (310 K):
 - $h_1 = 310.24$ kJ/kg
 - $P_{r1} = 1.555$
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 4 (1150 K):
 - $h_4 = 1219.25$ kJ/kg
 - $P_{r4} = 200.15$
- Use Textbook Appendix to find Specific Enthalpy at State 2 when $P_{r2} = 10.89$
 - $h_{2s} = 541.26$
- Use Textbook Appendix to find Specific Enthalpy at State 5 when $P_{r5} = 28.59$
 - $h_{5s} = 711.8$

To Find Relative Pressure at State 2:

Use:

- $P_{r2} = P_{r1} * \text{Ratio}$
- $P_{r2} = 1.555 * 7$
- $P_{r2} = 10.89$

To Find Relative Pressure at State 5:

- $P_{r4} = P_{r5} * \text{Ratio}$

Or

- $P_{r5} = P_{r4} / \text{Ratio}$
- $P_{r5} = 200.15 / 7$
- $P_{r5} = 28.59$

To Find Enthalpy at State 2:

Use:

- $h_2 = h_1 + ((h_{2s} - h_1) / N_c)$
- $h_2 = 310.24 + ((541.26 - 310.24) / 0.75)$
- $h_2 = 618.27 \text{ kJ/kg}$

To Find Enthalpy at State 5:

Use:

- $h_5 = h_4 - N_t * (h_4 - h_{5s})$
- $h_5 = 1219.25 - 0.82 * (1219.25 - 711.8)$
- $h_5 = 803.14 \text{ kJ/kg}$

To Find the Temperature at State 5 (Part (a)):

Use linear interpolation based on Textbook Appendix Table A-17 values based on Enthalpy $h_5 = 803.14$:

- $T_5 = 780 + ((803.14 - 800.03) / (821.95 - 803.14)) * (800 - 780)$
- $T_5 = 783.31 \text{ K}$

To Find the Net Work (W_{net}) (Part (b)):

Use:

- $W_{\text{net}} = (h_4 - h_5) - (h_2 - h_1)$

- $W_{\text{net}} = (1219.25 - 803.14) - (618.27 - 310.24)$
- $W_{\text{net}} = 108.08 \text{ kJ/kg}$

To Find Enthalpy at State 3:

Use:

- $E = (h_3 - h_2) / (h_5 - h_2)$
- $0.65 = (h_3 - 618.27) / (803.14 - 618.27)$
- $h_3 = 738.44$

To Find Heat Added to the System:

Use:

- $Q_{\text{in}} = h_4 - h_3$
- $Q_{\text{in}} = 1219.25 - 738.44$
- $Q_{\text{in}} = 480.81 \text{ kJ/kg}$

To Find Thermal Efficiency (Part (c)):

Use:

- $N = W_{\text{net}} / Q_{\text{in}}$
- $N = 108.08 / 480.81$
- $N = 0.225 = 22.5\%$

Problem 9-119

Consider a regenerative gas-turbine power plant with two stages of compression and two stages of expansion. The overall pressure ratio of the cycle is 9. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Accounting for the variation of specific heats with temperature, determine the minimum mass flow rate of air needed to develop a net power output of 110 MW.

Given:

- Ratio = 9
- $T_1 = 300 \text{ K}$
- $T_5 = 1200 \text{ K}$
- $W = 110 \text{ MW} = 110000 \text{ kW}$
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 1 (300 K):
 - $h_1 = 300.19 \text{ kJ/kg}$
 - $P_{r1} = 1.386$
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 5 (1200 K):
 - $h_5 = 1277.79 \text{ kJ/kg}$
 - $P_{r5} = 238$
- Since the Overall Pressure Ratio is 9:
 - $(P_2 / P_1) = (P_4 / P_3) = 9^{0.5} = 3$

And

- $(P_5 / P_6) = (P_7 / P_8) = 9^{0.5} = 3$

To Find Relative Pressure at State 2:

Use:

- $P_{r2} = (P_2 / P_1) * P_{r1}$
- $P_{r2} = 3 * 1.386$
- $P_{r2} = 4.16$

To Find the Specific Enthalpy at State 2:

Use linear interpolation based on Textbook Appendix Table A-17 values based on Relative Pressure $P_{r2} = 4.16$:

- $((h_2 - 411.12) / (421.26 - 411.12)) = ((4.16 - 4.153) / (4.22 - 4.153))$
- $h_2 = 412.18$

To Find Relative Pressure at State 6:

Use:

- $P_{r6} = (P_6 / P_5) * P_{r5}$
- $P_{r6} = (1 / 3) * 238$
- $P_{r6} = 79.33$

To Find the Specific Enthalpy at State 6:

Use linear interpolation based on Textbook Appendix Table A-17 values based on Relative Pressure $P_{r5} = 79.33$:

- $((h_6 - 932.93) / (955.38 - 932.93)) = ((79.33 - 75.29) / (82.05 - 75.29))$
- $h_6 = 946.35$

To Find Work during the Two Compression Stages (W_c):

Use:

- $W_c = 2 * (h_2 - h_1)$
- $W_c = 2 * (412.18 - 300.19)$
- $W_c = 223.98 \text{ kJ/kg}$

To Find Work during the Two Turbine Stages (W_t):

Use:

- $W_t = 2 * (h_5 - h_6)$
- $W_t = 2 * (1277.79 - 946.35)$
- $W_t = 662.88 \text{ kJ/kg}$

To Find the Net Work (W_{net}):

Use:

- $W_{net} = W_t - W_c$
- $W_{net} = 662.88 - 223.98$
- $W_{net} = 438.9 \text{ kJ/kg}$

To Find the Mass Flow Rate:

Use:

- $W = m * W_{net}$

Or

- $m = W / W_{\text{net}}$
- $m = 110000 / 438.9$
- $m = 250.6 \text{ kg/s}$

Problem 9-121

Consider an ideal gas-turbine cycle with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and turbine is 3. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. Determine the back work ratio and the thermal efficiency of the cycle, assuming

- (a) no regenerator is used and
 - (b) a regenerator with 75 percent effectiveness is used.
- Use variable specific heats.

Given:

- Ratio = 3
- $T_1 = 300 \text{ K}$
- $T_5 = 1200 \text{ K}$

- Regenerator Effectiveness (E) = 75% = 0.75
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 1 (300 K):
 - $h_1 = 300.19 \text{ kJ/kg}$
 - $P_{r1} = 1.386$
- Use Textbook Appendix and Linear Interpolation based on Relative Pressure $P_{r2} = 4.16$:
 - $h_2 = h_4 = 411.26$
- Use Textbook Appendix to find Enthalpy and Relative Pressure at State 5 (1200 K)
 - $h_5 = h_7 = 1277.79 \text{ kJ/kg}$
 - $P_{r5} = 238$
- Use Textbook Appendix and Linear Interpolation based on Relative Pressure $P_{r6} = 79.33$:
 - $h_6 = h_8 = 946.36 \text{ kJ/kg}$
- Since the Overall Pressure Ratio is 3:
 - $(P_2 / P_1) = (P_4 / P_3) = 3$

And

- $(P_5 / P_6) = (P_7 / P_8) = 3$

To Find Relative Pressure at State 2:

Use:

- $P_{r2} = (P_2 / P_1) * P_{r1}$
- $P_{r2} = 3 * 1.386$
- $P_{r2} = 4.16$

To Find Relative Pressure at State 6:

- $P_{r6} = (P_6 / P_5) * P_{r5}$
- $P_{r6} = (1 / 3) * 238$
- $P_{r6} = 79.33$

To Find Work during the Two Compression Stages (W_c):

Use:

- $W_c = 2 * (h_2 - h_1)$
- $W_c = 2 * (411.26 - 300.19)$
- $W_c = 222.14 \text{ kJ/kg}$

To Find Work during the Two Turbine Stages (W_t):

Use:

- $W_t = 2 * (h_5 - h_6)$
- $W_t = 2 * (1277.79 - 946.36)$
- $W_t = 662.86 \text{ kJ/kg}$

To Find the Back Work Ratio (R_{bw}) (Part (a)):

Use:

- $R_{bw} = W_c / W_t$
- $R_{bw} = 222.14 / 662.86$
- $R_{bw} = 0.335 = 33.5\%$

To Find the Heat Added to the System:

Use:

- $Q_{in} = (h_5 - h_4) + (h_7 - h_6)$
- $Q_{in} = (1277.79 - 411.26) + (1277.79 - 946.36)$
- $Q_{in} = 1179.96 \text{ kJ/kg}$

To Find the Net Work (W_{net}):

Use:

- $W_{net} = W_t - W_c$
- $W_{net} = 662.86 - 222.14$
- $W_{net} = 440.72 \text{ kJ/kg}$

To Find the Thermal Efficiency (Part (a)):

Use:

- $\eta = W_{net} / Q_{in}$
- $\eta = 440.72 / 1179.96$
- $\eta = 0.374 = 37.4\%$

To Find the Back Work Ratio with Regenerator (R_{bwrg}) (Part (b)):

Use:

- $R_{bwrg} = W_c / W_t$
- $R_{bwrg} = 222.14 / 662.86$
- $R_{bwrg} = 0.335 = 33.5\%$

To Find the Heat Added to the System with Regenerator (Q_{inrg}):

Use:

- $Q_{\text{inrg}} = E * (h_8 - h_4)$
- $Q_{\text{inrg}} = 0.75 * (946.36 - 411.26)$
- $Q_{\text{inrg}} = 401.33 \text{ kJ/kg}$

To Find the Net Heat Additions to the Systems (Q_{Net}):

Use:

- $Q_{\text{Net}} = Q_{\text{in}} - Q_{\text{inrg}}$
- $Q_{\text{Net}} = 1179.96 - 401.33$
- $Q_{\text{Net}} = 778.63$

To Find the Net Work with Regenerator (W_{netrg}):

Use:

- $W_{\text{netrg}} = W_t - W_c$
- $W_{\text{netrg}} = 662.86 - 222.14$
- $W_{\text{netrg}} = 440.72 \text{ kJ/kg}$

To Find the Thermal Efficiency with Regenerator (N_{regen}) (Part (b)):

Use:

- $N_{\text{regen}} = W_{\text{netrg}} / Q_{\text{Net}}$
- $N_{\text{regen}} = 440.72 / 778.63$
- $N_{\text{regen}} = 0.566 = 56.6\%$