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Homework 1.5

Problem 9-123

Air enters a gas turbine with two stages of compression and two stages of expansion at 100 kPa and 17°C. This system uses a regenerator as well as reheating and intercooling. The pressure ratio across each compressor is 4; 300 kJ/kg of heat are added to the air in each combustion chamber; and the regenerator operates perfectly while increasing the temperature of the cold air by 20°C.

Determine this system's thermal efficiency. Assume isentropic operations for all compressor and the turbine stages and use constant specific heats at room temperature.

Given:

- $P_1 = 100 \text{ kPa}$
- $T_1 = T_3 = 17^\circ\text{C} = 290.15 \text{ K}$
- $\text{Ratio} = 4$
- $Q_{\text{Add}} = 300 \text{ kJ/kg}$
- Use Textbook Appendix to find Air Properties at Room Temperature:
 - Specific heat ratio (γ) = 1.4
 - $C_p = 1.005 \text{ kJ/kg}\cdot\text{K}$
- $T_4 = T_2 = 431.16 \text{ K}$
- $T_5 = T_4 + 20$
- $T_{10} = T_9 - 20$

To Find the Temperature at State 2:

Use:

- $T_2 = T_1 * (\text{Ratio})^{(\gamma - 1/\gamma)}$
- $T_2 = 290.15 * (4^{(1.4-1/1.4)})$
- $T_2 = 431.16 \text{ K}$

To Find the Temperature at State 5:

Use:

- $T_5 = T_4 + 20$
- $T_5 = 431.16 + 20$
- $T_5 = 451.16 \text{ K}$

To Find the Temperature at State 6:

Use:

- $Q_{\text{Add}} = C_p * (T_6 - T_5)$

Or

- $T_6 = T_5 + (Q_{\text{Add}} / C_p)$
- $T_6 = 451.16 + (300 / 1.005)$
- $T_6 = 749.67 \text{ K}$

To Find the Temperature at State 7:

Use:

- $T_7 = T_6 * (1 / \text{Ratio}^{(y-1/y)})$
- $T_7 = 749.67 * (1 / 4^{(1.4-1/1.4)})$
- $T_7 = 504.49 \text{ K}$

To Find the Temperature at State 8:

Use:

- $Q_{\text{Add}} = C_p * (T_8 - T_7)$

Or

- $T_8 = T_7 + (Q_{\text{Add}} / C_p)$
- $T_8 = 504.49 + (300 / 1.005)$
- $T_8 = 802.97 \text{ K}$

To Find the Temperature at State 9:

Use:

- $T_9 = T_8 * (1 / \text{Ratio}^{(y-1/y)})$
- $T_9 = 802.97 * (1 / 4^{(1.4-1/1.4)})$
- $T_9 = 540.36 \text{ K}$

To Find the Temperature at State 10:

Use:

- $T_{10} = T_9 - 20$
- $T_{10} = 540.36 - 20$
- $T_{10} = 520.36 \text{ K}$

To Find the Head Added to the System:

Use:

- $Q_{in} = Q_{Add} * 2$
- $Q_{in} = 300 * 2$
- $Q_{in} = 600 \text{ kJ / kg}$

To Find the Head lost by the System:

Use:

- $Q_{Loss} = C_p * (T_{10} - T_1) + C_p * (T_2 - T_3)$
- $Q_{Loss} = 1.005 * (520.36 - 290.15) + 1.005 * (431.16 - 290.15)$
- $Q_{Loss} = 373.01 \text{ kJ / kg}$

To find the Thermal Efficiency:

Use:

- $N = Q_{Loss} / Q_{in}$
- $N = 373.01 / 600$
- $N = 0.622 = 62.2\%$

Problem 1-129

A turbojet is flying with a velocity of 900 ft/s at an altitude of 20,000 ft, where the ambient conditions are 7 psia and 10°F. The pressure ratio across the compressor is 13, and the temperature at the turbine inlet is 2400°R. Assuming ideal operation for all components and constant specific heats for air at room temperature, determine

- the pressure at the turbine exit
- the velocity of the exhaust gases
- the propulsive efficiency

Given:

- $V_1 = 900 \text{ ft / s}$
- Altitude (A) = 20,000 ft
- $P_1 = P_6 = 7 \text{ psia}$
- $T_1 = 10^\circ\text{F} = 469.67^\circ\text{R}$
- Ratio = 13
- $P_4 = P_3 = 145.73 \text{ psia}$
- $T_4 = 2400^\circ\text{R}$
- Use Textbook Appendix to find Air Properties at Room Temperature:
 - Specific heat ratio (γ) = 1.4
 - $C_p = 0.24 \text{ Btu / Lbm}^\circ\text{R}$

To Find the Temperature at State 2:

Use:

- $T_1 * C_p + (V_1^2 / 2) = T_2 * C_p$

Or

- $T_2 = T_1 + (V_1^2 / (2 * C_p))$
- $T_2 = 469.67 + (900^2 / (2 * 0.24))$
- $T_2 = 537.4^\circ\text{R}$

To Find the Pressure at State 2:

Use:

- $P_2 = P_1 * (T_2 / T_1)^{(\gamma / (\gamma - 1))}$
- $P_2 = 7 * (537.4 / 469.67)^{(1.4 / (1.4 - 1))}$
- $P_2 = 11.21 \text{ psia}$

To Find the Pressure at State 3:

Use:

- $P_3 = \text{Ratio} * P_2$
- $P_3 = 13 * 11.21$
- $P_3 = 145.73 \text{ psia}$

To Find the Temperature at State 3:

Use:

- $T_3 = T_2 * (\text{Ratio})^{(\gamma - 1 / \gamma)}$
- $T_3 = 537.4 * (13)^{(1.4 - 1 / 1.4)}$
- $T_3 = 1118.32^\circ\text{R}$

To Find the Temperature at State 5:

Use:

- $T_5 = T_4 - (T_3 - T_2)$
- $T_5 = 2400 - (1118.32 - 537.4)$
- $T_5 = 1819.08 \text{ }^\circ\text{R}$

To Find the Pressure at State 5 (Part (a)):

Use:

- $P_5 = P_4 * (T_5 / T_4)^{(y / y-1)}$
- $P_5 = 145.73 * (1819.08 / 2400)^{(1.4/1.4-1)}$
- $P_5 = 55.24 \text{ psia}$

To Find the Temperature at State 6:

Use:

- $T_6 = T_5 * (P_6 / P_5)^{(y-1 / y)}$
- $T_6 = 1819.08 * (7 / 55.24)^{(1.4-1/1.4)}$
- $T_6 = 1008.09 \text{ }^\circ\text{R}$

To Find the Velocity of Exhaust Gasses (Part (b)):

Use:

- $V_6 = \text{sqrt}(2 * C_p * (T_5 - T_6))$
- $V_6 = \text{sqrt}(2 * 0.24 * (1819.08 - 1008.09))$
- $V_6 = 3121.9 \text{ ft / s}$

To Find the Work Done by the Turbine (W_{Turbine}):

Use:

- $W_{\text{Turbine}} = (V_6 - V_1) * V_1$
- $W_{\text{Turbine}} = (3121.9 - 900) * 900$
- $W_{\text{Turbine}} = 1999710 \text{ ft}^2 / \text{s}^2 = 79.87 \text{ Btu / Lbm}$

To Find the Heat Added to the System:

Use:

- $Q_{\text{in}} = C_p * (T_4 - T_3)$
- $Q_{\text{in}} = 0.24 * (2400 - 1118.32)$
- $Q_{\text{in}} = 307.6 \text{ Btu / Lbm}$

To Find the Propulsive Efficiency (N_p) (Part (c)):

Use:

- $N_p = W_{\text{Turbine}} / Q_{\text{in}}$
- $N_p = 79.87 / 307.6$
- $N_p = 0.2597 = 25.97\%$

Problem 1-135

Consider an aircraft powered by a turbojet engine that has a pressure ratio of 9. The aircraft is stationary on the ground, held in position by its brakes. The ambient air is at 7°C and 95 kPa and enters the engine at a rate of 20 kg/s. The jet fuel has a heating value of 42,700 kJ/kg, and it is burned completely at a rate of 0.5 kg/s.

Neglecting the effect of the diffuser and disregarding the slight increase in mass at the engine exit as well as the inefficiencies of engine components, determine the force that must be applied on the brakes to hold the plane stationary.

Given:

- Ratio = 9
- $T_{\text{Ambient}} = 7^\circ\text{C} = 280.15 \text{ K}$
- $P_{\text{Ambient}} = 95 \text{ kPa}$
- $\dot{V}_{\text{Enter}} = 20 \text{ kg/s}$
- $Q_{\text{in}} = 42,700 \text{ kJ/kg}$
- $\dot{V}_{\text{Burn}} = 0.5 \text{ kg/s}$
- Use Textbook Appendix to find Air Properties at Room Temperature:
 - Specific heat ratio (γ) = 1.4
 - $C_p = 1.005 \text{ kJ / kg}\cdot\text{k}$

To Find the Pressure at the Exit:

Use:

- $P_{Exit} = P_{Ambient} * \text{Ratio}$
- $P_{Exit} = 95 * 9$
- $P_{Exit} = 855 \text{ kPa}$

To Find the Temperature at the Exit:

Use:

- $T_{Exit} = T_{Ambient} * (P_{Exit} / P_{Ambient})^{(y-1 / y)}$
- $T_{Exit} = 280.1 * (855 / 95)^{(1.4-1 / 1.4)}$
- $T_{Exit} = 524.75 \text{ K}$

To Find the Velocity of Exhaust Gasses:

Use:

- $V_{Exit} = \text{sqrt}(2 * C_p * (T_{Exit} - T_{Ambient}))$
- $V_{Exit} = \text{sqrt}(2 * 1.005 * (524.75 - 280.15))$
- $V_{Exit} = 22.17 \text{ m / s}$

To Find the Force Needed by the Brakes to Hold the Thrust:

Use:

- $F_{Brake} = (V_{Enter} + V_{Burn}) * V_{Exit}$
- $F_{Brake} = (20 + 0.5) * 22.17$
- $F_{Brake} = 454.49 \text{ N}$

Problem 9-167

Consider an ideal gas-turbine cycle with one stage of compression and two stages of expansion and regeneration. The pressure ratio across each turbine stage is the same. The high pressure turbine exhaust gas enters the regenerator and then enters the low-pressure turbine for expansion to the compressor inlet pressure. Determine the thermal efficiency of this cycle as a function of the compressor pressure ratio and the high-pressure turbine to compressor inlet temperature ratio. Compare your result with the efficiency of the standard regenerative cycle

Given:

- $T_2 = T_5 = T_7$
- $T_4 = T_6 = T_8$

To Find the Temperature at State 2:

Use:

- $T_2 = T_1 * (P_2 / P_1)^{(y-1 / y)}$

To Find the Temperature at State 4:

Use:

- $T_4 = T_3 * (P_4 / P_3)^{(y-1 / y)}$

To Find the Heat Added by the System:

Use:

- $Q_{in} = C_p * (T_3 - T_4)$

Or

- $Q_{in} = C_p * (1 - \text{Ratio}^{(y-1 / y)})$

To Find the Heat Lost by the System:

Use:

- $Q_{\text{Loss}} = C_p * (T_6 - T_1)$

Or

- $Q_{\text{Loss}} = C_p * (\text{Ratio}^{(y-1/y)} - 1)$

To Find the Thermal Efficiency:

Use:

- $\eta = Q_{\text{Loss}} / Q_{\text{in}}$

Or

- $\eta = (C_p * (1 - \text{Ratio}^{(y-1/y)})) / (C_p * (\text{Ratio}^{(y-1/y)} - 1))$

Note:

- Regenerative Cycles have a higher Thermal Efficiencies because they feature two stages compared to Standard Cycles which have one stage.
 - In other words, during the second “Regeneration” stage, hot gasses are directed from the turbine exit to preheat the incoming gasses.
 - Utilizing the remaining heat in the system to preheat the incoming gasses allows the next combustion cycle to be more efficient. Standard Cycles without regeneration do not feature the preheating stage