

Name: _____.

MET 350 Thermal Applications
Dr. Orlando Ayala
Spring 2026
Test 1

Take home – Due Sunday March 1st, 2026, before midnight.

READ FIRST

1. RELAX!!!! DO NOT OVERTHINK THE PROBLEMS!!!! There is nothing hidden. The test was designed for you to pass and get the maximum number of points, while learning at the same time. HINT: THINK BEFORE TRYING TO USE/FIND EQUATIONS (OR EVEN FIND SIMILAR PROBLEMS)
2. The total points on this test are one hundred (100). Ten (10) points are from your HW assignments, and ten (10) other points are based on the basis of technical writing. The other eighty (80) points will come from the problem solutions. For the technical writing I will follow the attached rubric.
3. There are 2 problems. Each problem will be worth (80/2) points.
4. What you turn in should be only your own work. You cannot discuss the exam with anyone, except me. Call me, skype me, text me, email me, come to my office, if you have any question.
5. I do not read minds. You should be explicit and organized in your answers. Use drawings/figures. If you make a mistake, do not erase it. Rather use that opportunity to explain why you think it is a mistake and show the way to correct the problem.
6. You have to turn in your test ON TIME and ONLY through CANVAS. You must submit only one file and it has to be a pdf file. For the ePortfolio (which is optional) you are supposed to upload this artifact to your Google drive. I will provide more instructions later.
7. Do not start at the last minute so you can handle anything that could happen. Late tests will not be accepted. Test submitted through email will not be accepted either.
8. Cheating is completely wrong. The ODU Student Honor Pledge reads: "I pledge to support the honor system of Old Dominion University. I will refrain from any form of academic dishonesty or deception, such as cheating or plagiarism." By attending Old Dominion University you have accepted the responsibility to abide by this code. This is an institutional policy approved by the Board of Visitors. It is important to remind you the following part of the Honor Code:

IX. PROHIBITED CONDUCT

A. Academic Integrity violations, including:

1. *Cheating*: Using unauthorized assistance, materials, study aids, or other information in any academic exercise (Examples of cheating include, but are not limited to, the following: using unapproved resources or assistance to complete an assignment, paper, project, quiz or exam; collaborating in violation of a faculty member's instructions; and submitting the same, or substantially the same, paper to more than one course for academic credit without first obtaining the approval of faculty).

With that said, you are NOT authorized to use any online source of any type, unless is ODU related.

1. An engineer designed a Brayton cycle with regeneration (heat exchanger) for an automobile engine. In this design, air enters the compressor at 100 kPa and 30°C. The compressor pressure ratio is 10, the maximum cycle temperature is 800°C, and the cold air stream leaving the regenerator exits 10°C below the temperature of the hot air stream entering the regenerator.

A second engineer reviewed the design and identified a major issue, concluding that the regenerator should not be used under the current operating conditions. Determine why the regenerator is ineffective in the original design and explain the reasoning behind this conclusion

The reviewing engineer stated that proper use of the regenerator would require either: reducing the compressor pressure ratio to 8.58, or increasing the maximum cycle temperature to at least 895.48 °C.

Assume: isentropic compressor and turbine processes, and constant specific heats evaluated at room temperature. Determine the following performance parameters: Net work output, Heat addition, Heat rejection, Thermal efficiency, and Regenerator (heat exchanger) effectiveness for each of the following cases:

- a. Original design (with regeneration)
- b. Original design without regeneration
- c. Original design with pressure ratio reduced to 8.58 (all other parameters unchanged)
- d. Original design with maximum cycle temperature increased to 895.48°C (all other parameters unchanged)
- e. Original design replacing the single-stage compressor with a two-stage compressor with intercooling

Discuss which option represents the best overall design. Your discussion should consider: expected manufacturing cost, system complexity, and feasibility of implementing the regenerator heat exchanger. For evaluating heat exchanger feasibility, use the provided effectiveness chart with $C_{\min}/C_{\max} = 1.00$, which applies to this problem. Note that the chart indicates a maximum physically achievable effectiveness; any required effectiveness above this limit is thermodynamically impossible.

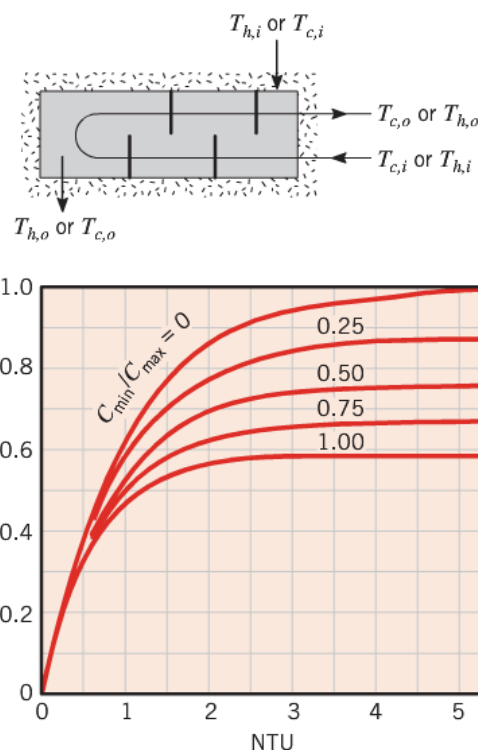


FIGURE 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc.,

2. A turbojet aircraft flies at a speed of 900 km/h at an altitude where the ambient air conditions are $T_0 = -35^\circ\text{C}$ and $p_0 = 40$ kPa. The combustion gases enter the turbine at $T_3 = 950^\circ\text{C}$. The turbine delivers 500 kJ/kg of specific work, and all of this work is used to drive the compressor (i.e., turbine work output equals compressor work input). Assume compressor isentropic efficiency: $\eta_c = 0.80$, turbine isentropic efficiency: $\eta_t = 0.90$, variable specific heats (use air/gas tables or variable- c_p relations as appropriate), and neglect kinetic energy changes across the compressor and turbine unless otherwise stated. Determine:

- (a) the pressure at the turbine exit
- (b) the nozzle exit velocity
- (c) the engine thrust, given that the diffuser inlet diameter is $D = 1.6$ m.

Problem solution rubric

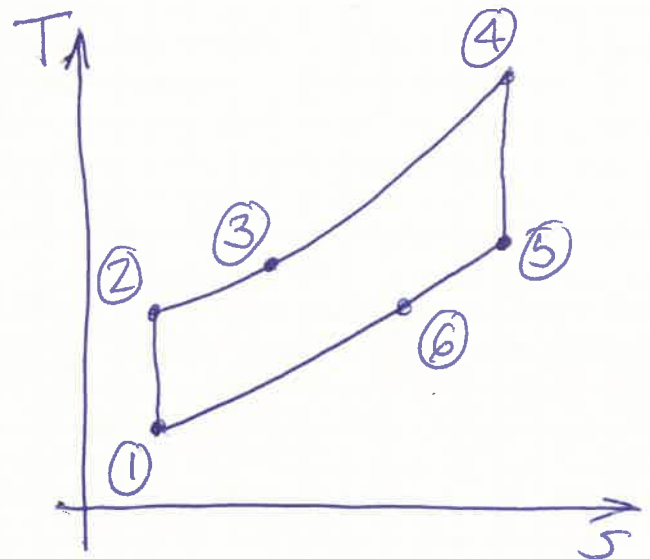
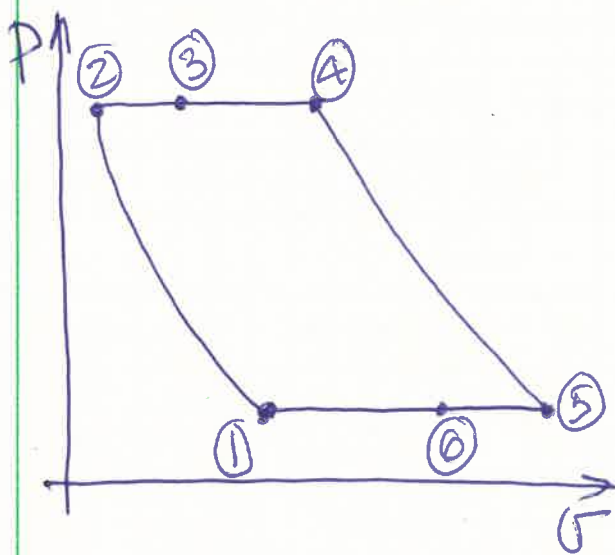
	Exceeds Standard		Meets Standard		Approaches Standard		Needs Attention	
	4		3		2		1	
	10 points		7 points		4 points		0 points	
1. Purpose 5%	The purpose of the section to be answered is clearly identified and stated.		The purpose of the section to be answered is identified, but is stated in a somewhat unclear manner.		The purpose of the section to be answered is partially identified, and is stated in a somewhat unclear manner.		The purpose of the section to be answered is erroneous or irrelevant.	
2. Drawings & Diagrams 10%	Clear and accurate diagrams are included and make the section easier to understand. Diagrams are labeled neatly and accurately.		Diagrams are included and are labeled neatly and accurately.		Diagrams are included and are labeled.		Needed diagrams are missing OR are missing important labels.	
3. Sources 5%	Several reputable background sources were used and cited correctly.		A few reputable background sources are used and cited correctly.		A few background sources are used and cited correctly, but some are not reputable sources.		Background sources are cited incorrectly.	
4. Design considerations (assumptions, safety, cost, etc) 10%	Design is carried out with applicable assumptions and full attention to safety and cost, etc.		Design is generally carried out with assumptions and attention to safety, cost, etc.		Design is carried out with some assumptions and some attention to safety, cost, etc.		Assumptions, safety and cost were ignored in the design.	
5. Data and variables 5%	All data and variables are clearly described with all relevant details.		All data and variables are clearly described with most relevant details.		Most data and variables are clearly described with most relevant details.		Data and variables are not described OR the majority lack sufficient detail.	
6. Procedure 25%	Procedure is described in clear steps. The step description is in a complete and easy to understand short paragraph.		Procedure is described in clear steps but the step description is not in a complete short paragraph.		Procedure is described in clear steps. The step description is in a complete short paragraph but it is difficult to understand.		Procedure is not described in clear steps at all.	
7. Calculations 20%	All calculations are shown and the results are correct and labeled appropriately. The units of all values are shown.		Some calculations are shown and the results are correct and labeled appropriately.		Some calculations are shown and the results labeled appropriately.		No calculations are shown OR results are inaccurate or mislabeled.	
8. Summary 5%	Summary describes the design, the relevant information and some future implications.		Summary describes the design and some relevant information.		Summary describes the design.		No summary is written.	
9. Materials 5%	All materials used in the design are clearly and accurately described.		Almost all materials used in the design are clearly and accurately described.		Most of the materials used in the design are clearly and accurately described.		Many materials are described inaccurately OR are not described at all.	
10. Analysis 10%	The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.		The design is discussed and analyzed. Argumentative predictions are made about what might happen in case of change in the operation.		The design is discussed and analyzed. No argumentative predictions are made about what might happen in case of change in the operation and how the design could be change.		The design is not discussed and analyzed.	

① PURPOSE

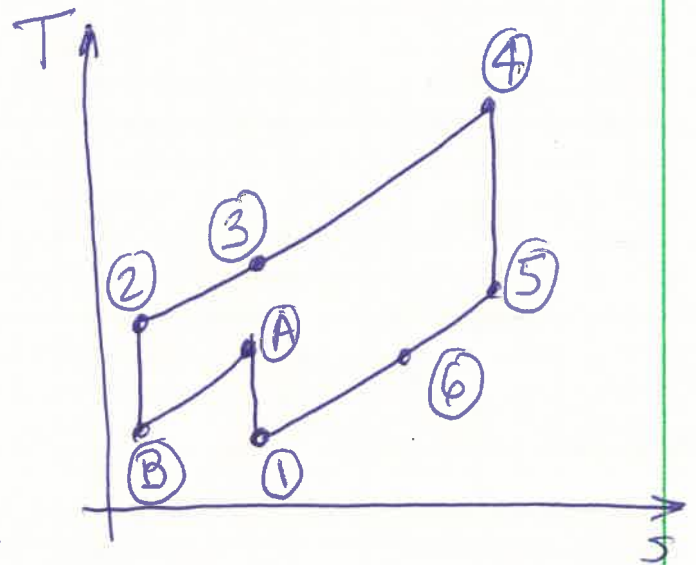
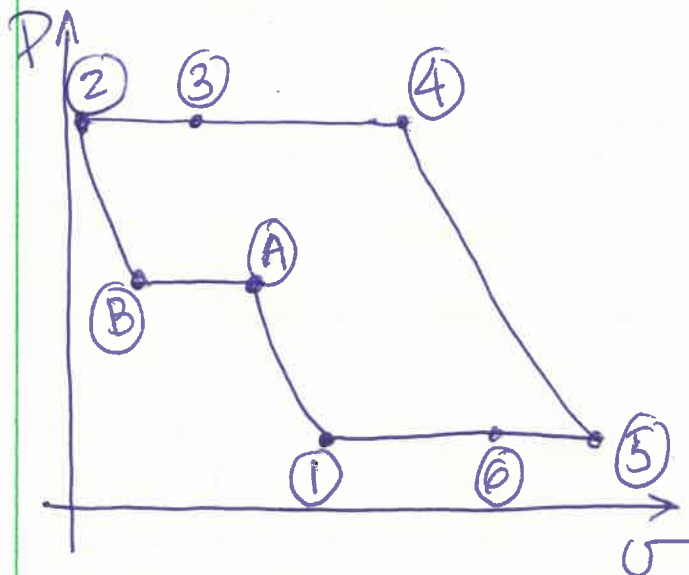
A

Determine what the best modification option is for a Brayton cycle with regeneration that is not operating correctly.

DRAWINGS & DIAGRAMS



For two stages compression



SOURCES

[B]

Cengel & Boles. "Thermodynamics - An Engineering Approach." 8th edition, McGraw Hill, 2015

DESIGN CONSIDERATIONS

I assume the following

- 1) Air behaves as ideal gas
- 2) C_p & C_v are constant
- 3) Isentropic compression & expansion
- 4) Constant pressure heat addition & rejection
- 5) $C_p = 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}}$, $k = 1.4$ (Table A-2)

DATA & VARIABLES

- * $P = 100 \text{ kPa}$, $T = 30^\circ\text{C}$ at air inlet
- * $r_p = 10$ or $r_p = 8.58$
- * $T_{\text{max}} = 800^\circ\text{C}$ or $T_{\text{max}} = 895.48^\circ\text{C}$
- * $T_5 = T_3 + 10^\circ\text{C}$

PROCEDURE & CALCULATIONS

Let us start by the case without regenerator.
The first step is to get all the states



$$P_1 = 100 \text{ kPa}$$

$$T_1 = 303.15 \text{ K}$$

$$P_2 = 10 \text{ Pa}$$

SAME AS ②

$$T_4 = 1073.15 \text{ K}$$

SAME AS ⑤

$$T_2 = T_1 r_p^{\frac{k-1}{k}}$$

$$= 303.15 \text{ K} (10)^{0.285714}$$

$$T_2 = 585.29 \text{ K}$$

$$P_2 = 1,000 \text{ kPa}$$

$$P_4 = 1,000 \text{ kPa}$$

$$P_5 = 100 \text{ kPa}$$

$$T_5 = T_4 r_p^{-\frac{k-1}{k}}$$

$$= 1,073.15 \text{ K} (10)^{-0.285714}$$

$$T_5 = 555.83 \text{ K}$$

Note that T_5 is lower than T_2 . Therefore, a regenerator cannot be used

$$\dot{W}_{\text{net}} = \dot{Q}(T_4 - T_5) - \dot{Q}(T_2 - T_1)$$

$$= \dot{Q}(T_4 - T_5 - T_2 + T_1)$$

$$= 1,005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (1,073.15 - 555.83 - 585.29 + 303.15) \text{ K}$$

$$\dot{W}_{\text{net}} = 236.36 \text{ kJ/kg}$$

$$\dot{Q}_{\text{in}} = \dot{Q}(T_4 - T_3) = 1,005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (1,073.15 - 585.29) \text{ K}$$

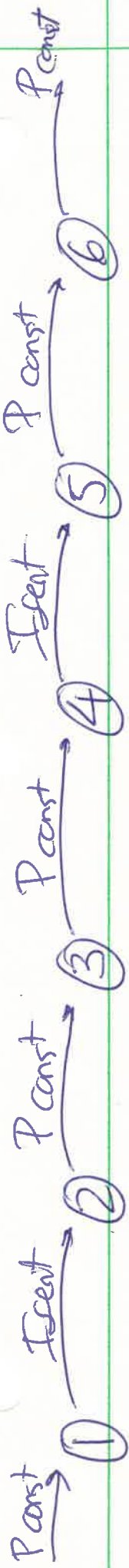
$$\dot{Q}_{\text{in}} = 490.30 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}}$$

$$\eta_{\text{th}} = 0.482$$

Now, let us study the case with regenerator

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$$P_1 = 100 \text{ kPa}$$

$$T_1 = 303.15 \text{ K}$$

$$P_2 = 100 \text{ kPa}$$

$$T_2 = 545.83 \text{ K}$$

(same as before)

$$P_3 = 100 \text{ kPa}$$

$$T_4 = 1023.15$$

$$P_5 = 100 \text{ kPa}$$

$$P_6 = 100 \text{ kPa}$$

$$T_2 = 545.83 \text{ K}$$

(same as before)

$$P_4 = 100 \text{ kPa}$$

$$T_5 = 545.83 \text{ K}$$

(same as before)

$$T_3 = T_5 - 10$$

$$T_3 = 545.83 \text{ K}$$

The air was cooled down! We want to heat it up! You should not have used regenerator

$$T_5 - T_6 = T_3 - T_2$$

$$T_6 = T_5 - (T_3 - T_2)$$

$$T_6 = 545.0 \text{ K}$$

$$W_{\text{net}} = C_p (T_4 - T_5 - T_2 + T_1)$$

$$W_{\text{net}} = 236.36 \text{ kJ/kg}$$

(same as before)

$$Q_{\text{fin}} = C_p (T_4 - T_3)$$

$$Q_{\text{fin}} = 529.96 \text{ kJ/kg}$$

more than before!

$$\eta_{\text{th}} = \frac{W_{\text{net}}}{Q_{\text{fin}}}$$

$$\eta_{\text{th}} = 0.446$$

It reduced!

$$\epsilon = \frac{T_3 - T_2}{T_5 - T_2}$$

$$\epsilon = 1.339$$

This is impossible

The reason we got such impossible value for ϵ is because the equation should be modified as the HX is not doing what it was meant to do.

Now, let us try reducing the pressure ratio:





$$P_1 = 100 \text{ kPa}$$

$$T_1 = 303.15 \text{ K}$$

$$P_2 = 8.58 \text{ Pa}$$

$$T_4 = 1073.15 \text{ K}$$

$$P_2 = 858 \text{ kPa}$$

$$P_3 = 858 \text{ kPa}$$

$$P_4 = 858 \text{ kPa}$$

$$P_5 = 100 \text{ kPa}$$

$$P_6 = 100 \text{ kPa}$$

$$T_2 = 303.15 (8.58)^{0.285714}$$

$$T_5 = 1073.15 (8.58)^{-0.285714}$$

$$T_2 = 560.23 \text{ K}$$

$$T_5 = 580.69 \text{ K}$$

$$T_3 = T_5 - 10$$

GOOD!

$$T_3 = 570.69 \text{ K}$$

$$T_6 = T_5 - (T_3 - T_2)$$

$$T_6 = 570.23 \text{ K}$$

Using same equations as before
for W_{net} , q_{in} , q_{out} , ϵ :

$$W_{net} = 236.557 \text{ kJ/kJ}$$

$$q_{in} = 504.97 \text{ kJ/kJ}$$

$$q_{th} = 0.468$$

$$\epsilon = 0.51$$

Now, the case with higher T_{max} :



$$P_1 = 100 \text{ kPa}$$

$$T_1 = 303.15 \text{ K}$$

$$P_2 = 1000 \text{ kPa}$$

$$T_2 = 585.29 \text{ K}$$

$$P_3 = 1000 \text{ kPa}$$

$$T_4 = 1168.63 \text{ K}$$

$$P_4 = 1000 \text{ kPa}$$

$$P_5 = 100 \text{ kPa}$$

$$P_6 = 100 \text{ kPa}$$

$$T_5 = 1168.63 \text{ K} \quad \text{---} 0.28514$$

$$T_5 = 605.29 \text{ K}$$

$$T_3 = T_5 - 10$$

$$T_3 = 595.29 \text{ K}$$

$$T_6 = T_5 - T_3 + T_2$$

$$T_6 = 595.29 \text{ K}$$

Using same equations as before
for $\dot{W}_{\text{net}}$, $\dot{Q}_{\text{in}}$, η_{th} , ϵ :

$$\dot{W}_{\text{net}} = 280.48 \text{ kW}$$

$$\dot{Q}_{\text{in}} = 574.08 \text{ kJ/s}$$

$$\eta_{\text{th}} = 0.498$$

$$\epsilon = 0.50$$

Now, let us try the case with
two stages compressor:



$$P_1 = 100 \text{ kPa}$$

$$T_1 = 303.15 \text{ K}$$

$$r_{p_{\text{per}} 5.23} = \sqrt{r_p}$$

$$= 3.162$$

$$T_A = 303.15 \text{ K} (3.162)^{0.285714}$$

$$T_A = 421.215 \text{ K}$$

$$P_A = 316.2 \text{ kPa} \quad P_B = 316.2 \text{ kPa}$$

$$T_B = 303.15 \text{ K}$$

(same as T_1)

$$P_2 = \sqrt{r_p} P_B$$

$$P_2 = 1,000 \text{ kPa}$$

$$T_2 = 303.15 \text{ K} (3.162)^{0.285714}$$

$$T_2 = 421.215 \text{ K}$$

$$T_4 = 1,023.15 \text{ K} \quad P = 100 \text{ kPa} \quad P_6 = 100 \text{ kPa}$$

$$P_4 = 1,000 \text{ kPa} \quad T_5 = 555.88 \text{ K}$$

(same as before)

$$T_B = T_5 - 10$$

$$T_3 = 545.88 \text{ K}$$

$$T_6 = T_5 - T_3 + T_2$$

$$T_6 = 431.215 \text{ K}$$

For this case, and using the equations we used before, we get

$$\dot{W}_{\text{net}} = 280.99 \text{ kJ/kg}$$

$$\dot{q}_{T_1} = 529.956 \text{ kJ/kg}$$

$$\eta_{\text{th}} = 0.53$$

$$e = 0.93$$

G

SUMMARY

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The results for the different cases are:

CASE	$\dot{W}_{\text{net}}$ (kJ/s)	$\dot{Q}_{\text{in}}$ (kJ/s)	η_{th}	ϵ
Original design without regeneration	236.36	490.30	0.482	—
Original design	236.36	529.96	0.446	1.339
Original design with $r_p = 8.58$	236.56	504.97	0.468	0.51
Original design with $T_{\text{max}} = 895.40^\circ\text{C}$	280.48	574.08	0.488	0.50
Original design with two stages compressor	280.99	529.96	0.530	0.93

MATERIAL: Air

ANALYSIS

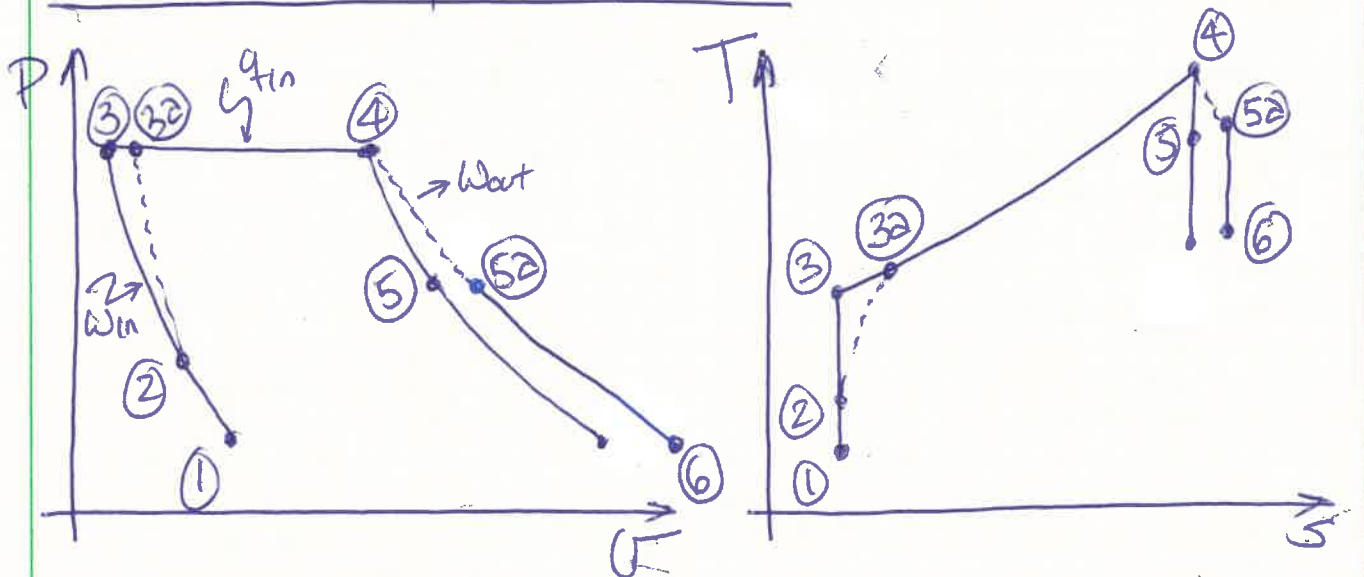
The second engineer is correct in the changes. As for the different options, the original design with regeneration is not possible because $T_5 < T_2$. The case of 2 stages compression is not possible either because you need a HX with 0.93 which is NOT possible. The max ϵ we could get is about 0.60. Thus, we are left with the reduction of r_p and increase of T_{max} options. If we find a turbine that can handle a temp of 896°C , the increase of T_{max} is the best option. It is cheaper since you do not need to buy new equipment.

② PURPOSE

I

Calculate the pressure of the combustion gases at turbine exit, the velocity of the gases at the nozzle exit, and the thrust of a turbojet in an aircraft that flies with a velocity of 900 km/h .

DRAWINGS & DIAGRAMS



SOURCES

Cengel & Boles. "Thermodynamics - An Engineering Approach." 8th edition. McGraw Hill, 2015

DESIGN CONSIDERATIONS

- 1) Air as ideal gas
- 2) $\eta_c = 0.8$, $\eta_T = 0.9$
- 3) Constant pressure heat addition
- 4) C_p & C_v variable

DATA & VARIABLES

$$V = 900 \text{ km/h} = 250 \text{ m/s}$$

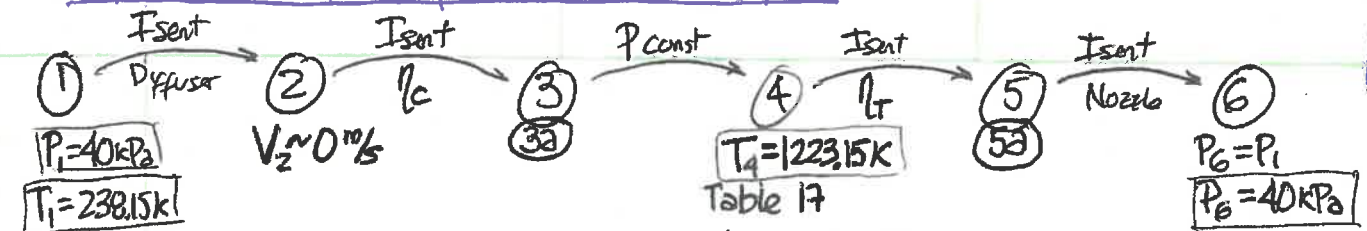
$$T_{amb} = -35^\circ\text{C}; P_{amb} = 40 \text{ kPa}$$

$$T_{fuel} = 900^\circ\text{C}$$

$$\dot{m}_f = 500 \frac{\text{kg}}{\text{s}}$$

PROCEDURE & CALCULATIONS

J



$$P_1 = 40 \text{ kPa}$$

$$T_1 = 238.15 \text{ K}$$

$$V_1 = 250 \text{ m/s}$$

Table 17

$$h_1 = 238.17 \frac{\text{kJ}}{\text{kg}}$$

$$Pr_1 = 0.619257$$

1st law

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_2 = 269.42 \frac{\text{kJ}}{\text{kg}}$$

Table 17

$$T_2 = 269.31 \text{ K}$$

$$Pr_2 = 0.95084$$

$$\frac{P_2}{P_1} = \frac{Pr_2}{Pr_1}$$

$$P_2 = P_1 \frac{Pr_2}{Pr_1} \rightarrow P_2 = 61.42 \text{ kPa}$$

Now, $W_c = 500 \frac{\text{kJ}}{\text{kg}}$

Using 1st law:

$$W_c = \frac{h_3 - h_2}{\eta_c}$$

$$h_3 = h_2 + W_c \eta_c$$

$$h_3 = 669.42 \frac{\text{kJ}}{\text{kg}}$$

Table 17

$$T_3 = 659.01 \text{ K}$$

$$Pr_3 = 23.00$$

$$\frac{P_3}{P_2} = \frac{Pr_3}{Pr_2}$$

$$P_3 = 1485.697 \text{ kPa}$$

The actual conditions at compressor and turbine exit are:

$$\eta_c = \frac{h_3 - h_2}{h_{3a} - h_2}$$

$$h_{3a} = 769.42 \frac{\text{kJ}}{\text{kg}}$$

Table 17: $T_{3a} = 751.96 \text{ K}$

Now, $W_t = 500 \frac{\text{kJ}}{\text{kg}}$

Using 1st law:

$$W_t = \eta_t (h_4 - h_5)$$

$$h_5 = h_4 - \frac{W_t}{\eta_t}$$

$$h_5 = 749.47 \frac{\text{kJ}}{\text{kg}}$$

Table 17

$$T_5 = 733.56 \text{ K}$$

$$Pr_5 = 34.35$$

$$\frac{P_5}{P_4} = \frac{Pr_5}{Pr_4}$$

$$P_5 = 198.21 \text{ kPa}$$

$$\frac{P_6}{P_5} = \frac{Pr_6}{Pr_5}$$

$$Pr_6 = 6.932$$

Table 17

$$T_6 = 473.61 \text{ K}$$

$$h_6 = 475.94 \frac{\text{kJ}}{\text{kg}}$$

Table 17: $T_{5a} = 780.0 \text{ K}$

(a) $P_{\text{ext}} = P_5 = 198.12 \text{ kPa}$

(b) $V_{\text{nozzle exit}} = ?$ 1st law in the nozzle:

or $V_6 = ?$ $h_{5a} + \frac{V_{5a}^2}{2} = h_6 + \frac{V_6^2}{2}$

$$V_6 = \sqrt{2(h_{5a} - h_6)}$$

$$V_6 = 805.10 \text{ m/s}$$

(c) $F = \dot{m}(V_{\text{exit}} - V_{\text{inlet}})$

$$\dot{m} = \rho_1 V_1 A = \frac{V_1}{U_1} \frac{\pi}{4} D^2$$

$$U_1 = \frac{RT_1}{P_1}$$

$$F = \frac{V_1 P_1}{RT_1} \frac{\pi}{4} D^2 (V_6 - V_1)$$

$$F = 163.293 \text{ kN}$$

SUMMARY

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- a) The pressure at the exit of the turbine is 198.12 kPa
- b) The velocity at the exit of the nozzle (where the propulsion occurs) is 805.10 m/s
- c) The thrust is 163.293 kN

MATERIALS

Air

ANALYSIS

If the turbine and the compressor were 100% efficient, the thrust would have been higher. Similarly, the amount of heat addition would have been less.